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DAVID KAISER: But if there are no immediately pressing questions on the assignments, then I will jump in. We can start talking some more about quantum weirdness. We can continue our discussion from yesterday, which was clearly insufficiently shocking for you. You were all very quantum complacent yesterday. Let's see if I can do a little better today.

So we'll talk about a series of developments of-- that grew out of efforts to make sense of this new set of equations, this new mathematical formalism of quantum mechanics. Some of these developments came pretty quickly in time, very soon after Schrodinger had worked out this new form of the equation, and others, as we'll see, really came together some years later.

So for today, we're going to talk about these three examples. This does not exhaust all the things one can think about under the heading of quantum weirdness, but these are some of my favorites, and I think they really show a range of the kinds of questions that this work began to open up.

So we'll start by talking about superposition and Schrodinger's cat paradox. The Schrodinger's cat is likely familiar to you, at least in outline. We'll talk a bit about what were the conceptual stakes and what was the historical moment, what was the broader context, in which that work was being developed.

Then for our second part today, we'll look at a very famous critique of quantum mechanics, now usually just referred to by the initials EPR. As we'll see, that stands for the initials of the three authors of this paper, Einstein, Podolsky, and Rosen. So we'll talk about that.

And then for the last section, we'll talk about work by the physicist John Bell, now known as Bell's inequality. And it relates very directly to something called quantum entanglement. For this last one in particular, and really for throughout today's lecture, but in particular for part three, again, I'll have to skip over some of the intermediate steps. I might show a couple of equations that might look a little difficult to parse. But there's the optional lecture notes on the Canvas site. And you can dig into that a bit more if things are not so clear. And obviously, of course, please ask questions now or email or office hours as well. So that's where we're going today. OK.

Let's quickly remind ourselves where we've gotten to so far. As we saw just in yesterday's class session, in 1926, starting in winter/spring of '26, Erwin Schrodinger developed a second, independent approach to what would be called quantum mechanics, independent from the work by Werner Heisenberg, which had been published only a few months earlier. So whereas Heisenberg had emphasized discreteness and wound up using matrices, though he didn't even know it at first, Schrodinger was developing a different approach, building much more on a continuity assumption rather than a discreteness assumption.

And in this case, Schrodinger was building very directly on work by Louis de Broglie about matter waves. Somehow, seemingly solid matter like atoms and parts of atoms have associated with them in some way that wasn't so clear some kind of inherent waviness, that that was maybe the real secret to quantum theory. So Erwin Schrodinger introduced a whole new formalism building on this continuity notion of matter waves. And he began to quantify that.

So he introduced this equation, which you see here in red. Some of you might even have that tattooed or on T-shirts. It's an excellent candidate for either of those things, the time-independent Schrodinger equation. And this governs the behavior of this new quantity that Schrodinger introduced that we now call the quantum wave function, usually given the Greek letter psi.

As we saw, solutions to this equation obeys superposition, just like most typical solutions to wave equations do. So what that means is that if you have two different solutions to the equation, we could call them psi 1 and some other solutions psi 2, then in general, it will be the case that a sum of those two solutions will also be a solution. In fact, a weighted sum. We could weight them with different coefficients in front. And that would also still be, in general, a solution.

So that's what yields all this juicy, wave-like behavior, things like interference. And so to remind you in a simple cartoon version, two waves that are each separately solutions of a particular wave equation. In areas where the crests, the peaks, nearly coincide, they'll add up to make a larger overall peak. Other places, one crest will nearly line up with a trough, and they'll almost cancel each other out. That's the kind of constructive and destructive interference, very familiar from classical waves. This was also a feature of these strange quantum wave functions that were associated with Schrodinger's equation.

That still didn't answer what psi was a wave of. It was really not so clear. Schrodinger offered several tentative suggestions or hypotheses, and even he himself showed that most of those weren't self consistent. And it was really about six months later in the summer of 1926 when Max Born interpreted Schrodinger's own work, his own equation, and said, here's what your equation perhaps might mean. We've seen this many times. The equation didn't change, but the effort to make sense of it-- that was really not so settled.

So Born suggests that the wave function is somehow related to the probability for various outcomes to be measured. So it wasn't a feature of a physical object extended in space. It wasn't a shape of some smeared-out electron through space. It was somehow, in some way it wasn't at all so clear, related to the likelihood to measure certain properties of that electron, for example. So that psi was not even the probability itself, but the probability amplitude. And the absolute square of this, in general, complex number, complex function-- the absolute square would give you the probability. And that's what leads to all these really quite delicious phenomena, like the double slit experiment that we talked a bunch about yesterday. So that's just a recap.

So pretty quickly after that, a British physicist named Paul Dirac, who was almost exactly the same age as Heisenberg and Pauli, very young, in his early to mid 20s at this point, and therefore a generation younger than Schrodinger-- Dirac learned about this work. He was very mathematically gifted and well-trained. By this point, he was at Cambridge for his PhD.

And he really began to formalize this new quantum mechanics. He was not only one of the first to demonstrate a mathematical equivalence between these very different-looking formalisms from Heisenberg and Schrodinger; he then went further, not just to show that one could build a map from Heisenberg's matrices to Schrodinger's wave equation, but in fact to formalize the entire thing in this general, abstract, geometrical way.

So it's really from people like Dirac, among the first, from whom we begin thinking about these quantum states as vectors, not in the three-dimensional space in which we move around, not in the space of x, y, and z, but in some abstract, mathematical space that we would now call a Hilbert space, so that there's a state associated with this wave function psi that would obey Schrodinger's equation.

And that's really just a vector in some abstract space, and that the vector could represent, for example, the-- it's related to the likelihood that we would measure a particular value for a property of a quantum system. You see it's already gotten very abstracted from this physical picture of real matter moving through space. It's now the behavior of these abstract pointers in an abstract vector space.

So the state vector, this vector psi, could itself be represented as some weighted sum of eigenstates. That is, states that correspond to a definite value for a specific property. So again, if this is brand new to you, please don't worry. For some of you, this might be familiar from other classes. But it's from Dirac that we start writing things like this, that there's some vector psi, some state, and that can in general be written as a sum, as a superposition, a weighted sum, of distinct basis vectors, in this case, the so-called eigenstates.

Now, the eigenstates are states that have definite values for particular property. And I'll talk about an example to make that a little more concrete in a moment. But just to pause. If this expression is unexpected or something you haven't seen before, this really is just quantifying that notion of superposition. It's just saying that if these eigenstates, these basis vectors phi, are each solutions to Schrodinger's equation, then so is their sum and so is their weighted sum. So these coefficients A, in general, could be complex numbers, but they're just a number. They're just a weight. Take five parts of phi 1 and 6 plus 7 i parts of phi 2. We're just going to add up a sum because if each of these is a solution, then is so is their weighted sum. It's just a way of formalizing superposition.

This is the kind of stuff that Dirac loved. He ate this stuff for breakfast. And he wrote this very, very influential textbook, first edition in 1930, very soon after this work was first published, trying to put this all together in this new vector space packaging.

Now, that was very abstract, what I mentioned. Let's consider a relatively simple example, again, one that might be familiar to you. Let's consider basis states or eigenstates that correspond to the property of the spin of an electron if we choose to measure that spin along some definite orientation in space. We line up our magnets along the z-direction in space. We hold up to some gradient of some magnetic field pointing along the z-direction. We could really do that. It's a real thing we could measure.

And then we can ask, if we send an electron through that region of space where that external magnetic field is, will its spin be pointing up, directly along the direction of the field, or directly opposite, antiparallel? Those are the two options. The spin could either be exactly along the specified direction of space or exactly opposite to it. There's two possible outcomes.

That means there's only two of these basis states, these two eigenstates. States with a definite property, the spin is definitely up along that direction of space, or the spin is definitely down, with definite values. And again, the actual value of that angular momentum, as usual, scaled in terms of Planck's constant.

Now, these two states are opposite to each other. They're literally orthogonal. It's spin up or spin down. And the way this gets formalized-- again, we learned this following Paul Dirac is that these vectors are orthogonal to each other. They have vanishing overlap. Again, there's a little bit of this in the associated optional lecture notes. This is just a fancy way of taking the dot product.

So we have two vectors that are perpendicular in ordinary space, say one vector pointing exactly along the x-axis, a second vector pointing directly along the y-axis. Take the dot product, and it vanishes because they're perpendicular. The same kind of properties hold among these more abstract vectors in this abstract Hilbert space. And in this case, these opposite properties-- it's either spin up or its opposite, spin down. The corresponding vectors have a vanishing overlap, or if you like, a vanishing dot product.

Nonetheless, even though the states are individually opposite to each other, we can construct a perfectly valid quantum state by using superposition. So we can build up some state psi with which to characterize this electron or a whole collection of electrons. And we could say that an individual electron would have some likelihood to be found in spin up upon performing the measurement, some perhaps different likelihood to be found in spin down. So even though the outcomes can't both be found, we'd find either one or the other, the quantum state could be this superposition, this mixture of these seemingly opposite sets of properties.

That's the kind of manipulation that people began to learn how to do once the British physicist at Cambridge, Paul Dirac, began to formalize Schrodinger's notion, building very specifically on this notion of superposition.

So again, let's stick with our little, simple example for a few more steps. If we were to perform a measurement of the spin of that particle along a particular direction in space, we'd have to orient our magnets, so we're going to choose a direction in space along which to measure spin, then we can calculate in advance the probability to get either of these answers. It will either be spin up or spin down.

And we find that the probability, according to Max Born, by asking about the absolute square of the overlap between the general quantum state and that definite outcome. And as you can see here, using some of these inner product relations, that the probability to find spin up will just be the square of this coefficient, the absolute square. The probability to find spin down is just the absolute square of the other coefficient.

So what's important to step back and recognize here before we get too lost in the mathematics is that every single time we measure spin along that same direction z, we always get a definite answer. We always find either spin up or spin down. We don't find spin at 72 degrees. We don't find some smeared-out answer or some fuzzy result. We get a definite answer.

What the equations of quantum mechanics allowed people to do starting right from the 1920s was calculate the likelihood the probability to get this definite answer versus that definite answer.

And yet what the equation seems not to allow physicists to do, and this is something that people like Max Born and Paul Dirac and Niels Bohr and Werner Heisenberg and the folks we've been talking about-- what they began to grapple with and to really reconcile themselves to, is that quantum mechanics can only be used to calculate these probabilities, that we get definite answers each time the real measurement is performed, and the equations tell us the probability to get that definite answer versus the other.

What the equations do not seem to allow the physicist to do, then or now, is get any knowledge ahead of time of what the actual value was for that property before the measurement was performed. So if we write down a valid quantum state like this, this tells us that the object has some likelihood to be measured with one property, some likelihood to be measured with some other property. But we don't know until the actual measurement is performed which property the particle really has on its own.

Bohr went even further. It's not just that we don't know. He began to suggest that maybe the particle itself simply did not have a definite value prior to its measurement. So the one statement is that we don't know for sure. And that's a statement of our ignorance, the state of our knowledge. And therefore, we average over our ignorance by calculating probabilities. Bohr and others following his line of thinking reasoned maybe the probabilities are even more radical. Maybe the world itself, so to speak, didn't know. What if the electron simply had no definite value of spin, neither spin up nor spin down, prior to its measurement?

And again, to bring that home, I like to think of human-scale examples. What would that be like? That's as if saying a person had no particular weight until she stepped on a bathroom scale. Not that she didn't know or she misremembered or it might have been one of many values. What if she literally had no weight at all, no definite weight at all, until she stepped-- until she measured it by stepping on a scale? That's a pretty radical notion, a very strong break from how we think about physics either with Newton or with Maxwell.

And not everyone liked that development. So one of the first people to really begin to raise some cautions about that approach is Albert Einstein. So he was, of course, a major contributor to some of the early work. We saw many of his very important contributions within the so-called old quantum theory epoch, even more that we didn't have a chance to talk about in this class. He was all over this early quantum theory stuff. And he was following these more recent developments very closely. And he became more and more dissatisfied, kind of uncomfortable, with this direction that people like Niels Bohr and many of the younger guard were pursuing.

So he wrote a series of really amazing letters to his very close friend Max Born, the same Born we've seen many times, who worked closely with Heisenberg who introduced this probability interpretation for Schrodinger's wavefunctions. They were very close friends. They were contemporaries, same age. They'd known each other for a long time. And when they weren't in the same city, they would write letters. And many years later, those letters were collected and published. It's a wonderful collection.

So Einstein, we know, grew frustrated with this restriction to only calculating probabilities. And he would often vent his frustration to Born and actually try to convert Born, Born himself, who had actually argued for this probabilistic interpretation.

Here's one of the most famous excerpts of quotations from the letters. There's a whole series of very juicy letters they exchange. But as you might well have heard this last part, comes from a private letter that Einstein had written to Born in December of 1926, five months after Born introduced the idea that psi relates to probabilities.

So Einstein writes that "quantum mechanics is certainly imposing." I think by that he means impressive. "But an inner voice tells me it's not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the 'old one," by which he means something like God, kind of spiritual notion. He goes on, "I, at any rate, am not convinced that He--" sorry. "I, at any rate, am convinced that He is not playing at dice."

And you've probably heard this paraphrased. Einstein claimed that God does not play dice with the universe. That's where that quotation comes from, that surely the world has definite properties, whether we happen to know them or not. That's really what he's arguing.

It turns out Einstein didn't only raise this question with Max Born. He got very-- into very animated discussions with Erwin Schrodinger. And he had many opportunities, starting very soon after Schrodinger published this pathbreaking work. Schrodinger published his work while he was still a professor in Austria, but very quickly, he was then hired to the fancy professorship in Berlin starting in 1927. In fact, he took over the professorship when Max Planck retired from it.

So now Schrodinger, starting in 1927, was the senior-- the ordinarius professor of theoretical physics in Berlin. And Einstein was in Berlin at the Prussian Academy. Now they were neighbors. They became even better friends. They would hang out all the time. We know this from their letters and from other documentation. They would enjoy these-- I love this, the [GERMAN] evenings. These are Viennese sausage parties, which we should bring back. They would also go sailing on a lake together. Einstein owned a summer home in Germany, and he loved sailing on the lake. They'd go on sailing trips together. And they would argue and talk about quantum theory on many of these visits.

And in fact, Schrodinger began to feel very much like Einstein. They became-- both became suspicious or skeptical of the direction that quantum theory was going, even though they both, Einstein and Schrodinger each, had done so much to actually build this formalism of quantum mechanics.

So one of their surviving letters-- Einstein asks Schrodinger to imagine a particular situation. He wanted to drive home why we shouldn't be content to stop only with calculating probabilities. So imagine that someone had hidden a ball, let's say a red ball, into one of two identical closed boxes. And we can't peek. We don't know where the assistant placed the ball. Prior to opening either box, we could calculate a probability that we would find the ball in either box one or box two.

So before we check, before we perform the measurement by measuring where the ball actually is, our probability would say the likelihood to find the ball in box one is 50% and the likelihood in box two is also 50%. Einstein writes to Schrodinger, "Is this a complete description? NO." All capitals. "NO. A complete statement is, the ball is or is not in the first box." There should be a value to the property of the ball's location, whether or not we know what it is. We can calculate probabilities based on our ignorance, not based on how the world is. And Einstein thought people had been too quick to conflate one with the other.

Now, these discussions, which are often very spirited and very friendly. They really came to each other very much. Those direct face-to-face interactions were pretty soon disrupted with the rise of the Nazis. And we'll talk more about the rise of Nazis again, actually, starting in the next class session. But as I'm sure you know, Hitler was elected in an open election as chancellor of Germany in January of 1933. And very quickly, the Nazi party then took over more and more elements of the German federal government and local governments.

And so by that spring, within just months of Hitler's first election, both Einstein and Schrodinger independently left Germany. Einstein-- we'll talk more about their reasons for leaving. The point is, they were now no longer in Berlin. In fact, they were no longer on the same continent. Einstein resettled to the United States. He moved to Princeton, New Jersey. Schrodinger moved at first to Oxford in England. He later settled for the wall years in Dublin, in Ireland, but he got out of Germany, out of the continent.

So now they were an ocean apart. And that was personally very disruptive. But for us historians, it was very lucky because now they could only talk to each other through letter. And most of these letters were saved and have survived. So we can see how their conversation continued, but now we have more of their conversation because we all had to be written down because they weren't sitting up all night eating their Vienna sausages or sailing on the lake.

So here's one example. Again, I write about this in one of the essays in the reader for today, one of the readings. Here's a letter that Einstein wrote to Schrodinger now about two years or so after they had both fled Germany. He says, "Imagine a charge of gunpowder, of weaponry, that's intrinsically unstable, that has even odds, even likelihood, to explode over the course of one year."

So now he's no longer talking about a kind of emotionally neutral situation, like a red ball and a blue box right. Suddenly, he's thinking about-- the world is racing toward war, at least certainly Europe is. Let's talk about worldly things like caches of weapons and what's going to happen with that stockpile over time. The nature of the discussion changes quite dramatically.

So in Einstein's letter, his private letter, he says, "In principle, the situation can easily be represented quantum mechanically." It would be a superposition of these two opposite properties, but a sum of the two, just like with any superposition. After the course of a year, this is no longer the case. This might make sense as the state of the system at the start of one year, is what Einstein is suggesting, but after a year, this can no longer be an accurate description, at least argues Einstein. Rather, the wave function describes a blend of not yet and of already exploded systems. But how could that be, right? In reality, there's just no intermediary between exploded and not exploded. Think about the nature of his examples now, even though he's still talking about statistics and probability.

Schrodinger writes back a few weeks later, and now there's a twist. This is where the so-called Schrodinger cat is first introduced, first written down. It's actually first a private letter from Schrodinger to Einstein responding to this exploding cache of gunpowder. Schrodinger sits with that. And he writes back this example, now. This is from his private letter in response. He says, "Confined in a steel chamber is a Geiger counter prepared with a tiny amount of radioactive uranium." That's over here in our cartoon, courtesy of Wikimedia. "So that so small, such a small amount of radioactivity, that in the next hour it is just as probable as not to expect one atomic decay." So it has a 50/50 chance to decay in one hour. Its half life, let's say, is one hour.

There's an amplified relay system, so if any radioactive decay is detected here, it will trigger this thing to drop a hammer to shatter a bottle. And the bottle contains what they called prussic acid, which is a certain kind of cyanide poison. And cruelly, a cat is also trapped in the steel chamber. So it is a windowless box. You put a cat inside, which is not very friendly to the cat, radioactive source with a sensitive detector with a particular half life. It's only 50/50 odds to detect even a single decay after one hour. If even one decay happens, it will trigger the thing and break the vial and kill the cat. If no decays happen to happen in that hour, the vial will not break-- be broken, and the cat will live.

After one hour, the living and dead cat are smeared out in equal measure according to quantum mechanics. Again, we can write this Schrodinger-like superposition of totally opposite properties. And yet quantum mechanics, they both argue, has no way to tell the difference or to tell us what the real state of the world is, only the likelihood.

Einstein writes back. He can say, "Your cat shows that we are in complete agreement." Now they're talking about life and death with poison and gunpowder, not about balls and urns. A wave function "that contains the living as well as the dead cat just cannot be taken as a description of the real state of affairs."

And by the way, this is the version, almost word for word, that then Schrodinger publishes in this very famous article later that autumn. It comes out in November of 1935. And that's the first published version that comes to be known as Schrodinger's cat. It's almost exactly word for word what Schrodinger had first written in his private letter to Einstein.

And there's an irony in all this, a kind of historical irony, that by the mid-1930s, by 1935, Schrodinger had become quite skeptical, actually quite critical, of some of his own contributions to quantum mechanics. It was Schrodinger who wrote down the Schrodinger wave equation. Schrodinger said, isn't this great? We know how waves behave mathematically. They obey things like superposition. And yet 10 years later, between 1926 and 1935, on the order of 10 years, Schrodinger, like Einstein, had become quite uncomfortable with the conceptual direction with which people were trying to make sense of his own equation. So he became actually an outspoken critic.

And so the irony, at least, I think, for us today, is that he invented this now totally famous cat paradox. There are internet memes up and down, not to help teach people about his equations, but actually as a critique. He invented the paradox to say there's something rotten and wrong with quantum mechanics, not as a fun way to teach the theory.

So I'll pause there. We can have a little discussion, some questions. So Gary asks, is it possible that even after measuring a definitive measure, like the weight on the scale, the electron or the person does not have at all definite values, but its elemental state probabilistic-- I'm not-- Gary, this is a little garbled. Let's see.

I think the idea is that it would be one thing to say that the answer's-- the definite answer we'll get anytime we perform a measurement is an answer drawn from some distribution with some weighted probabilities. But it's a different thing to say there was actually no value of that property whatsoever. If we had never performed a measurement, the person would never have had any weight, not that we wouldn't know which weight it had from within, say, a Gaussian distribution or a bimodal.

It's that-- the more radical interpretation that at least people like Niels Bohr began to articulate was that there was no definite value at all for that property. And that could be a property as basic as is the cat alive or dead. That's what Schrodinger was thinking about. Not just what's the value of its portfolio if the stock market closed in New York but is still open in Tokyo. There could be reasons why certain values might be subject to uncertainty.

But this was like, let's take one value. Every time we perform a measurement, we get a definite answer. It's not that the question is poorly posed. It's that there was no answer before we asked the question. I think that's what got people like Einstein and Schrodinger even more riled up.

[LAUGHS]

Alex suggests that Schrodinger better hope the cat is dead, or else the cat's going to come for him. It's true. The prussic acid is a pretty cruel-- that's not so nice.

And so coming back to Gary's point, that the idea was that what if nature simply doesn't have definite properties until we ask questions of it? And that might be perfectly comfortable for us today. It's not very comfortable for me. But that's the sharpened question that led people like Einstein and Schrodinger to begin to really question even their own work or the direction in which their work was being taken.

And again, it's for properties that otherwise could be trivially measured. So it's not like it's a hard measurement to perform. Oh, I don't know what the answer is. There's a lot of statistical uncertainty. It might have been this or that. It's like, nope, the answer was this versus that. I can tell them clearly apart. And yet until I actually do the measurement, I have no definite way of knowing which it will be. And maybe neither does the electron. That was the weirder part, that last step. Maybe it's not about our lack of knowledge about this system. What if the world itself simply had no particular value for objects in the world?

Stephen, thank you. That's a fantastic question, and a really hard one. So it's unfair to ask me. It's a good question. It's a great question. That's still a subject of research. That's still something actively worked on by experts in the field. So Bohr himself had an answer for that. It's not an answer that everyone accepts anymore. Bohr's answer was that there should be some irreversibly amplified detection event. So maybe-- and that's a mouthful. Everything Bohr said was a mouthful. He spoke very verbose. It wasn't clear in Danish. It wasn't clear in German. It wasn't clear in English. That's [INAUDIBLE].

But his idea was that maybe the electron-- some little electron interacts with some little equally microscopic thing. How would we know? To Bohr, the measurement was completed when some irreversible signal of that event, of that interaction, was amplified to macroscopic scales, irreversible being the key point. So that could be, like, there was-- the photographic film was developed and there was really a spot here, not a spot there. There was something that we can't-- that can't be washed away.

Now, let me be clear, that's not at all the taken for granted answer today, and a real question remains. The so-called measurement problem has its own name, which is, what counts as a measurement? And if the whole world is ultimately made of atoms and atoms obey quantum theory, then aren't the measuring apparatuses also subject to all these same quantum effects?

So the question has to do with, as you say, what's the scale at which the quantum mechanical effects are no longer dominant? And that's, I think, what Bohr was gesturing toward in this notion of an irreversibly amplified classical signal. And as you can see from the rate at which I'm waving my hands, clearly I'm hand waving, because it is genuinely still actively debated, even among the experts today.

That's exactly right. And it does go right back to what we called what we spoke very briefly about yesterday, the so-called quantum to classical transition. We can get buckyballs to do two double-sided interference. We can't get you or me to do that. So it's very directly related to that. So one line-- let me just go off hook a little bit, jump ahead a few decades. One line of reasoning, and Amanda hints at this already in the chat as well.

One line of reasoning is that this does apply even to you and me into macroscopic things, to you and me on bicycles, to viruses. You might know where I'm going, and that both answers are real. They're just real in causally disjointed regions of the universe. This is what's now often called the many worlds interpretation, very eye raising-- eyebrow raising title. The original title is the relative state formulation.

What if it really is quantum mechanics all the way down, and we just have consistent-- self-consistent branches? So the cat is both alive and dead, but any observer who sees it as alive, that version of that observer is now following one track in a kind of forked path coming, to a fork in the road, and that anything else that observer's path will witness will be consistent with having found the cat either alive or dead. I forgot which I said.

So one example is this is all quantum mechanics, period. And just deal with it. And that is very much like what Amanda suggested to Gary. A different, or a distinct approach, sometimes they get combined. A distinct approach is to say there's something that happens from the so-called environment, from having lots and lots of quantum mechanical stuff together and interacting together, that the essential quantumness can be smeared out. It can decohere, is the technical term, that the effect of the environment is in some sense to reduce the range of quantum possibilities to one definite outcome because there are additional physical interactions that we haven't taken into account. And that's more like how much stuff do you need before quantum behaves like classical. That's like a big box is different than a very, very tiny electron.

So it's more like, do you have enough stuff, enough environment, that it's going to somehow smear out the range of possibilities and leave only one left? And sometimes people combine the two. Maybe it's actually the decoherence plus the many worlds is the answer.

So that's-- when we go to the cutting edge, the cognoscenti today, we'll worry about the measurement problem. These are the kinds of questions they ask. They're only able to ask them because of the conceptual work being done back in 1935. The inverse is not true, right? Schrodinger and Einstein didn't have these options to think about at the time. So these are all coming downstream from worrying about exactly this question about-- the electron could have been either way. Didn't have an answer. The cat doesn't make sense, so that it could've have been either. Maybe it could, right? So that's why that's still going on today.

So let's see. Aiden asks, is it theoretically possible to accurately predict if the uranium will decay or not in short scenario, but we as humans with the technology available just don't know enough? Very good. So Aiden, that's the kind of thing-- we'll actually come to that very soon in the next part. That's the kind of thing that Schrodinger and Einstein said should be possible. They didn't have the final answer in hand. But that's what they wondered-- that's what they thought any reliable theory of physics should enable one to do.

So they never said quantum mechanics is wrong in the sense that it violates measurements. It yields the correct probabilities for definite experimental outcomes. It's accurate as far as it goes. But they said it doesn't go far enough. So some ultimate physical theory, they argued, should enable you to say in advance the uranium nucleus really will or will not decay, not just it has a likelihood. It has a definite state in advance. So the electron spin really will spin up, even though we didn't know it in advance.

And so that's more what Einstein and Schrodinger said should be possible. Let's keep aiming for that. Let's not stop short. They didn't have a worked out model that would allow them to do that. That was their aspiration.

It's a great point, Yangwei. It's tricky because on the one hand, Schrodinger and Einstein both were being kind of wildly creative and original. I mean, Schrodinger gave us this wave function we didn't have-- we, collectively, didn't have before. And yet you're quite right that his approach was, in some sense to retain as much as possible of that look and feel.

So it's not like it's just a boring repetition of what had come before. It's not-- there's real original work being done. But the character of it, some other kind of way of describing it-- it does feel different. And I think it felt different at the time to practitioners than the kind of frankly brash, young, very-- what can I say? Kind of alpha male aggressive work, at least in portrayal, by people like Heisenberg and Pauli.

Pauli loved telling everyone else they were wrong. In fact, his favorite expression was not-- he would say, that's not right. That's not even wrong. It's not even-- that's so stupid, it's not even a candidate for being right or wrong. That's not a nice thing to say to people, right? Pauli was pretty-- he had an acid tongue, even as a 20-year-old, let alone as a 50-year-old. He was just mean, aggressive, mean, "I'm smarter than all of you." There was that kind of thing going on, to be sure.

And so Schrodinger was-- and Einstein both were doing creative, original, new stuff well into their 40s and 50s. It gives me hope, frankly. But it did have a different kind of style, and in some sense, a different character than some of these folks in their mid-20s.

Let me press on before we all debate the Hugh Everett many worlds interpretation because then we'll never leave that branch of the multiverse. But let me go on now to talk about the second big critique of quantum theory by Einstein and his younger colleagues, the so-called EPR thought experiment.

So this was published in the very same year as Schrodinger's cat. It actually came out a bit earlier. And this is one of the things that Einstein and Schrodinger were writing about across the ocean. So soon after Einstein got settled at the Institute for Advanced Study in Princeton-- it's in the same town as Princeton University, though it is formally a separate institution. He began working with these two younger colleagues, Boris Podolsky and Nathan Rosen. And they developed an even more elaborate and really very compelling, very, very tricky critique, of quantum theory, as what we now call the EPR, for their initials.

And this was in the reader. So again, it's very short. It's deceptively short. It's only four pages, but it's pretty tricky so I'm going to take a little while to walk through the argument, which might not have been so easy upon a first reading. And that's OK. It's a very tricky paper.

So they're imagining a thought experiment like this, in a kind of cartoon form. There's some source, that's the capital S here, that will shoot out pairs of particles in opposite directions. We can call them particles A and B. And each particle will be measured some distance apart. There's some detector here that will perform some measurement on particle A, some separate detector some distance away that will eventually measure some property of particle B. And the physicist here can choose to perform one of two types of measurements. In the original EPR paper, the authors considered either measuring the position along the x-axis. They can form a measurement of x. Or they could choose to perform a measurement of the momentum in that direction, p sub x.

Now, according to quantum mechanics, prior to either measurement, the system, the system particles A and B, would be in some superposition, not just a single cat that's either alive or dead, but basically, it's like two cats, either of which could be alive or dead. But here's the twist. The properties we will eventually measure of one are tangled up or entangled with the properties we will eventually measure of particle two. So each particle is in some as-yet indefinite state, and yet their two states are somehow connected to each other.

And that's written quantitatively as this special kind of superposition. There's two different two-particle states that are then put in superposition. Either particle A will have one property and particle B will have some other, or particle A will have that second property and particle B will have the first. That's two-- that's a superposition of two particle states. So it's even more elaborate than a single cat.

And what got them so exercised, what got them so upset about this, Einstein, Podolsky, and Rosen, is that this quantum state, this version of psi, does not factorize. We can't write it, at least according to quantum mechanics, as some mathematical description of the behavior of particle A over here and then separately in some factorizable way, some way of describing the behavior of particle B, that as soon as we say anything about the likely behavior of particle A, in the same breath, we have to say something about the behavior of particle B and vice versa. OK.

Why was that so concerning for them? Well, let's suppose that physicists one over here at this detector happens to select a position measurement for particle A. And then from the conservation of momentum, she could immediately infer the position of particle B once she completes her measurement.

On the other hand-- sorry, again, on the other hand, she could have chosen to perform the other kind of measurement. She could have chosen just as well to have measured the momentum along the x-direction and then immediately inferred the momentum some other property of particle B. What if she waited until the very last possible moment to decide which measurement to perform?

So now there's no time for even a light signal traveling at the very fast speed of light to get from here and catch up with the particle B in time to give it a kind of status update. Oh, when you get measured, make sure your properties agree with what are consistent with what was just measured of particle A over here. So if even a single light beam couldn't have updated particle B in time, then particle B must have had all its own relevant properties on its own prior, to its measurement. It wouldn't have time to wait for an update.

So the only thing that made sense to Einstein, Podolsky, and Rosen was that particle B already had its answers at the ready. It already had definite values for those properties. It was not, like Bohr would have insisted, a person who has not yet stepped on a bathroom scale. Particle B would have had definite values, both for position and for momentum. And we didn't know what they were yet, but the particle had them.

The problem with that, according to EPR, is that quantum mechanics has no way to describe quantitatively particle B's properties on its own, separate from what's going to happen to particle A. So they conclude in this famous paper that quantum mechanics must be incomplete.

And so this assumption, the supposed incompleteness of quantum theory, that-- sorry, that conclusion rests on two assumptions. The first of them, they tell us right on the opening page. It became known as the reality criterion. This is on the first page of their paper. And quoting their own text, "If, without in any way disturbing a system, we can predict with certainty, that is, with probability equal to unity, the value of a physical quantity, then there exists an element of reality, something about the world, not just about our knowledge of the world, corresponding to that physical quantity."

So that's an assumption, that quantum objects possess their own properties on their own. They carry them around as they move through space, independent of and prior to our efforts to measure them. The person has a value of her weight, and it can be revealed upon a measurement. The measurement doesn't create her value of the weight. That's what they're responding to Niels Bohr.

A little later in the paper, they make explicit their second, equally critical assumption, that would later be called locality. It's now on page 779 of their short article. Here's their wording for that. "Since at the time of measurement of the two systems, particles A and B, they no longer interact, no real change can take place in the second system, particle B, in consequence of anything that might have been done to the first system."

That's what we think as of localism, that local causes yield only local effects, or if we think about Einstein's work on relativity, that no force or information or influence of any kind can travel across space arbitrarily quickly. Things could travel maybe as fast as light, but not infinitely fast. And that's the updating paradox that they had for particle B.

So even a dozen years later, this was still upsetting Einstein, that quantum mechanics seemed to fail this test. And he wrote, again, in a very famous letter again to Max Born in the late 1940s, "I cannot seriously believe in quantum mechanics because a theory cannot be reconciled with the idea that physics should represent a reality in time and space free from spooky actions at a distance," a phrase that has followed this field since 1947, or I guess, since the letters were published in the early '70s, that he thought anything-- any physical theory, including quantum mechanics, that says that these two particles could remain coordinated across arbitrary distance-- that's a ghost story. That's spooky action at a distance. That's not a proper physical theory in which local causes yield local effects.

So let me pause there. Again, it's a complicated article, the EPR paper. It's a classic, but it's very dense. So what I hope we would get out of it upon reading it was at least that-- those important highlights, that they draw a conclusion about the nature of explanation in quantum theory and that their conclusion rests squarely on those two separate, but equally important, assumptions. Objects have their own properties, like it or not. The cat's just dead. Get over it, right? Even though we didn't look yet, that kind of thing. That's the elements of reality. And stuff can't travel arbitrarily fast.

And Tiffany says-- she's quite right. Tiffany, thank you so much for sharing that. There's a really lovely, lovely book. It's actually a kind of graphic novel I think you'd enjoy, very creative, by a pair of authors, Jeffrey and Tanya Bub. Those are father and daughter. It's a lovely pairing. Jeffrey Bub is a very accomplished physicist and philosopher of science, now very senior, and his daughter Tanya is a really terrific and very accomplished graphic artist. And they teamed up to write this very fun and quirky and creative graphic novel all about this notion of Einstein's critique and more recent thoughts. It's called *Totally Random* is what it's called, *Totally Random.* It's a great, great book.

And what would Einstein say about current work on entanglement? I don't know, Gary. I can make predictions for what he'd say. I don't know what he'd say. But I think about that a lot, for reasons we'll talk about in the last part of today's class. It's a fair question. But there's a lot that's been done since his time with which we've been able to sharpen these questions. And so the questions that he set in motion have really energized generations of work ever since. That's pretty amazing.

Isn't it more accurate to say spooky at a distance because no physical action happens? That's right. So Alex, what Einstein was worried about was it sounded like telepathy. It sounded like some occult connection, where somehow these two things at arbitrary distance-- it might not just be between Princeton, New Jersey and Dublin, where his friend Schrodinger was. It could be between Princeton, New Jersey and the Andromeda galaxy. And somehow instantaneously, at least according to quantum mechanics, instantaneously, these objects will behave in a rigidly coordinated way, even though there was no time to let one know what the other was going to do. So it really is this-- no physical mechanism. Nothing could have traveled through the intervening space to connect them.

And that's what he thought was spooky. And spooky was not meant to be like, oh, cool, it's spooky. I'm intrigued. Like, that's garbage, right? Think back to what we saw in the previous class, Schrodinger and Heisenberg calling each other's work disgusting and repulsive and trash. It's in that spirit that Einstein is writing to his very dear friend Max Born, still trying to convince him in 1947, still trying to convince him in 1955, right up to the time that he died. He just couldn't let this go.

Any other questions on the EPR thought experiment or the role of those two assumptions and all that?

Yeah. Thank you, Stephen, so much.

AUDIENCE: [INAUDIBLE]

DAVID KAISER: So on the one hand, that's a great preview for the last part of today's lecture. So on the one hand, hang on. There's also a lot of optional stuff. I know it's, like, midterms. It's a busy, busy time of the term. You're all working on paper, too. Hint, hint. So you're busy. But there's optional stuff on the Canvas site for today's class session as well just for whenever you have time and curiosity. And there's plenty more. I'd be delighted to talk more about this individually as well.

But one of the optional things for-- on today's group of readings, totally optional, is a chapter I wrote over the summer, still a draft, on the current status of experimental tests of quantum entanglement and ways to try to really check, have we closed down every possible objection, even far-flung but logically consistent objection, that someone like Einstein, or let's say, an Einsteinian, might raise to the existing battery of tests, and what would it take to close down every possible alternative explanation?

And so there's a lot of work that's going on around the world to this day, some of it with my own group and, of course, many groups around the world. So that's on there. Not that I'm asking you to read another 40 pages during a busy time, but there's some resources. And I'd be glad to chat with Stephen, and really, frankly, anyone. I love this stuff. About these bigger questions.

So let me give you the bumper sticker version. I think every-- well, I don't know if I'd say every. The vast majority of working physicists today agree that quantum entanglement is a fact of the world that particles seem to betray this very strong correlation across arbitrary distance. And I'll talk more about some experiments lately that have shown that more clearly. Is there a physical mechanism for that? Maybe there's some more exotic structure of space time. Maybe there's a wormhole that would give an Einstein description for a local physics that just was traveling a more complicated path through space and time than our naive picture. I don't think that's the best answer, but that's a legitimate answer. Maybe it's some other thing.

So people are still trying to get a physical mechanism that might fill in the dots. But there seems to be little room for doubt anymore that the phenomenon of entanglement is robust and can be reproduced at will. The idea that maybe-- a separate possibility is the one that I think is not the leading-- not the best way to go, would be to say that everything comes down to initial conditions, that the Big Bang, everything was preordained.

It's not just that everything's determined, that A leads to B, but everything was so-called super determined, that even my choice to teach this lecture-- I didn't have that much choice, but you know, my choice of what to do on my slides, the choice of the National Science Foundation to give my group our grant, the choice-- the fact that all these cascading, seemingly random decisions since the Big Bang itself, have all been preordained.

That's a-- I think a not very compelling and yet logically self-consistent explanation for all these phenomena. But what's amazing to me in the light of experiments like my own, but many others, is that's what's left. You get to say, it's quantum mechanics. Like, just, it's quantum mechanics, right? Sorry, Einstein. Or it's something that sounds, frankly, even more bizarre and potentially not even really subject to experimental tests, right?

So what we've been trying to do in the years since Einstein, Podolsky, and Rosen, and really since John Bell, whom I'll talk about in a moment, is to clear out all-- as many intervening, logical possibilities that could have been left. Oh, it could have been this, or this, or this, or this, or this. OK, well, let's design updated experiments to clear out as many of those other possibilities as we can and see what's left.

And Stephen, the short answer today is what's left, is quantum mechanics. There's some nonmechanical connection that leads to correlations that are measured reliably all around the world all the time. That's not because it's like two tin cans connected by a piece of twine. It's not a mechanical connection. We've ruled out all those kinds of models, or many, many of them. It's either that, or some, really, more even strange-sounding possibilities. Strangeness is, of course, a surjective judgment.

But not-- they're no more appealing to what Einstein would have been arguing for, either. The Einstein-like options we've really been able to pretty well put in a box and subject them to test. That's really what I'll talk about in this next part of the class. But it's a big, juicy question. I'd be delighted to talk more about it, now or whenever any of you has a chance to catch your breath, maybe after midterms and all that.

Let me press on, then, to the last part. Because this now comes to some of the more recent developments in exactly this kind of discussion. So almost exactly 30 years after that EPR paper, late in 1964, the Irish physicist John Bell went back to that question. He really scrutinized the EPR paper. You can see he even titled his own paper "On the Einstein-Podolsky-Rosen Paradox."

And he went back to those two critical assumptions that the authors had made. And just to remind you, the first one is what they had called the reality criterion, these elements of reality, the assumption is that each particle has definite properties on its own, prior to an independent measurement.

And to my mind, that's like saying that each particle carries with it a kind of logbook or an instruction manual that anywhere it goes through space, it has an answer to the question that might be asked of it. It has a definite value for its weight. It has a definite value for its spin along the z-direction. We might not have measured it yet, but it had a real value. That was their assumption one. And assumption two, remember, is this locality, that no force or information or influence can travel across space arbitrarily quickly.

What Bell argued is that these are actually assumptions. Any conclusion is only as good as the assumptions that went into its derivation. And this pair of assumptions became called local realism. The local part is this notion consistent with relativity. There's a speed limit before A could affect B. And the realism part refers to the object having real or definite properties on its own, independent of us and measuring.

What Bell suggested in this paper late in 1964 is that these are assumptions. Maybe we should try to test them instead of just conclude that quantum mechanics is or is not correct. So we introduced what we now call Bell tests. And you see these are very closely modeled on that original EPR framework. So we have some source of particles that are prepared in a special way, some source that shoots out pairs of particles A and B, two identical measuring devices at-- apart from each other in space.

And these have-- these boxes are now kind of generalized. There's some choice of what measurement to perform we call that the detector setting. I'll use lowercase letters, little a and b. That's like saying we could choose to measure the spin along the x-direction. We set the dial. That's how we orient our magnets. We could choose to measure the spin along the y-direction. We change the dial, rotate the magnets, choose to measure some intervening angle. So we choose the question to ask. We choose the measurement, the specific measurement, to be performed. And that's encoded in these detector settings, little a and b.

We set the question. The particle encounters the device. The device gives us an answer. That's the measurement outcome. We'll call that capital A and B.

Now, to be a little more quantitative, one of the things that Bell did was to consider certain kinds of properties, certain questions we could ask, for which the answer is always of the form either plus 1 or minus 1. If we measure position, the answer could be 1.07 meter, 1.0732 meters. It could be anywhere along a continuum. Likewise for momentum.

So Bell clarified, let's stick with measurements for which the answer is discrete and one of two possible discrete outcomes, like spin or like polarization of a photon of light. It's either polarized exactly parallel to the polarizing filter or perpendicular, not in between. So the answers we can always be put in the form of a kind of yes/no question, spin up, spin down, heads or tails, that kind of thing.

And then we're going to introduce these correlation functions. This is a fancy way of just saying what's the product of those two outcomes. No matter what question we ask, no matter how we set the detector setting, little a and little b, the outcome, capital A, was either a plus 1 or a minus 1, no matter what question we asked. Spin along this direction, spin along this direction, spin along this direction.

The answer for any of those questions will either be spin up, plus 1, or spin down, minus 1. So we can compare the answers, even as we ask different questions. And we'll average that, the brackets here, and we will take an average over many, many trials in which we ask a certain pair of questions, spin along this direction here, spin along some other direction here, and see how often the answers were exactly the same or opposite.

OK. And then Bell derived this really innocuous-looking expression. This is a comparison of the correlations. So how do those outcomes of measurements behave as we vary the pairs of questions being asked? That's all this means. And again, I'm going to go a little quickly here. There's the optional lecture notes that focus just on Bell's work in particular on the course on the Canvas side.

So this quantity S is really just saying, as we do each of these comparisons with question a and b, question a prime and b, question a with b prime-- you get the idea. How do these answers line up? Are they giving the same answer, plus 1 with plus 1, minus 1 with minus 1, or are they giving opposite answers?

Now, Bell goes on-- and again, a deceptively simple or a short paper. It's only six journal pages, very brief. And yet again, it's very dense. He goes on to show that any theory that would obey this local realism of the Einstein-Podolsky-Rosen framework, no matter what the specific model is, it should be put-- one should be able to put it in this general form. Bell argues there must be some properties of those particles that come from how they're prepared. So the lambda is some of shared properties because of some way of preparing particles A and B together.

There's some likelihood to get this set of properties versus another, so we're to average over all those. But then there's some probability to get a definite answer here when we subject particle A to a measurement. We choose to perform a particular measurement, and the outcome we get should depend on the question we asked and the shared properties of the particles. That's what this is encoding very abstractly. Likewise, we should be able to predict what answer we'll get at the other device. The answer capital B should depend on the question we ask there, our detector setting, the orientation of our magnets, say, and of these shared properties lambda.

And then Bell shows that any framework like this, any model that is consistent with local realism, will have an upper limit. There'll be a limit to how correlated these measurements can be. So they can line up a bunch of the time. They can't line up arbitrarily often. This becomes known as Bell's inequality. This combination of comparisons, combination of correlation functions, should be bounded for any theory, he argues, any possible theory, that is consistent with the local realism assumptions of EPR.

And let me just dig in here a little bit. And again, there's some more in the lecture notes. Why does it say that? Look at the form that he's chosen to write. He's encoded this locality, that the measurement outcome here, whether we get spin up or spin down, depends on the question we ask locally and on the shared properties that the particle could have carried with it. It cannot depend in this framework on either the question that's asked very far away or on the answer that's found very far away.

That's this local realism. The particle must have had its own properties to answer, so to speak, its own question. And that's like saying the answer we get here can depend on local things local to the box, things that-- properties that particle brought with it, the particular question we chose to ask. The answer here shouldn't depend on something that could be done arbitrarily late in the process arbitrarily far away, OK?

And yet, again, the calculation is pretty short. I go through it with all the intervening steps just to make it very explicit in those lecture notes. Quantum mechanics predicts really unambiguously that for certain kinds of systems, certain pairs of particles we might prepare in certain ways-- these measurements should be more strongly correlated than Bell's limit would allow, that the answers at these distant devices should line up even more often than any local realist theory could accommodate or could account for.

And not by a little bit, by as much as 40%, that when you do this comparison of the correlations, this single number S could be as large as 2 times the square root of 2. That's like roughly 2.8, 2.81, 2.82, something like that, which is considerably larger than this upper limit from Bell's inequality of 2.

So that still sounds pretty abstract. And Stephen, this might be the talk you were referring to. You might have heard this last spring. But for others who might not have heard it. I like to think of this entanglement, the conceptual stakes, in terms of the following parable. Again, I find it helpful to bring this up to human scale.

So I imagine a pair of twins, Alice and Bob, who work very hard. They're both very bright and diligent and hard working. And when it comes time to go to college, Alice, who is frankly, just more diligent and more smart, she gets an MIT, and so she comes to Cambridge, Massachusetts. And her very affable but somewhat duller brother Bob goes to Cambridge, England.

Alice-- this is the part that's now hard for us to imagine. Alice physically comes to campus. That's hard for some of us to imagine. She then goes to restaurants. What is this crazy story? So imagine prepandemic, Alice comes to MIT. She goes to her favorite local restaurant near campus.

And after dinner, the waiter asks, what would you like for dessert? So he's going to pose a question to her, and she'll give an answer. He's posing a question to her about baked goods. He says, tonight, your choice is you can have either a brownie or a cookie. She has one of two possible choices. That's like the spin either being spin up or spin down. The waiter has specified the question. That's the little a. I'll measure spin along this direction. I'll ask about baked goods preferences. He's setting the question, and Alice can give one of two possible outcomes, brownie or cookie, plus 1 or minus 1.

Some other nights-- she likes this place a lot. She goes back there very often for dinner. On other nights, the waiter asks her a different question. He now asks a different kind of question by rotating that dial. And he measures in some different measurement basis, like he's rotating the magnets for spin. This new question is actually about frozen dessert. She can either have on that night an ice cream sundae or frozen yogurt. Again, he poses the question. She has one of two possible answers that she can give, a plus 1 or a minus 1.

The waiter selects which question to ask by some local process. Maybe he flips a coin back in the kitchen. Alice has no clear preference. She likes all these options. And we know that because if you look at the tally of all her dessert orders after she keeps going for more and more fatty desserts. I don't recommend this experiment too often.

But she winds up choosing each option on average equally often. So when she's offered question one about baked goods, on average, half the time, she orders a brownie. Half the time, she orders the cookie, in an order that looks totally random. It's not clear which night she'll order a cookie or brownie, but averaged over the whole set, she has equal numbers of those choices, and likewise for the frozen desserts.

Unbeknownst to Alice, an ocean apart, her brother Bob is now going to a pub in Cambridge and being posed the same questions at the same time. The question's drawn from the same pair. So in some visits, Bob's waiter will ask him about baked goods, and he has one of two possible choices, either the brownie or the cookie. On other evenings, the waiter asks Bob about frozen goods. He can choose one of two options. Again, the waiter chooses by some random process. Bob has no clear preference. He, again, is choosing each option in equal frequency, but in what looks like a random order.

They come back for Thanksgiving, and the first thing they want to do after being away for the first semester of college is compare their 10,000 dessert orders. That's what I would talk about with my sibling. So they realize there are some pretty surprising correlations that get found when they compare these sets of answers.

When they both happen to have been asked the first question about baked goods, they both gave the same answer. They both gave either the brownie answer or the cookie answer at the same time, even though they might have ordered the opposite choice the next time. So when they were both asked the same question, their answers lined up, even though they seem to be answering them independently.

When they were asked different questions, there, again, was a correlation, that when Alice was asked about baked goods, she took the brownie at the time when Bob took the frozen yogurt, but not the ice cream when he was asked about frozen goods, and vice versa. There was a correlation in the cross question answers as well. And then when they're both asked question B about frozen goods, their answers once again align. When they both happen to be asked about frozen goods, they both ordered either the ice cream sundae or the fro-yo at the same time.

Why should this be surprising? If we just look at how Bob is answering all alone in this British pub, his answer to the same question about frozen goods seems to change and seems to in some sense depend, or at least be correlated, with something that's happening an ocean away for which he's not supposed to have any information. That's what Einstein said shouldn't happen, that the world shouldn't have that kind of telepathy, that kind of spooky connection. And in particular, if Bob is answering his questions out of some kind of local logbook or instruction manual, how-- he doesn't have enough information at his location, at his British pub, to know how to get his answers to line up with those that Alice will give at the same moment thousands of miles away.

So how could we subject that kind of human parable to experimental test? And there's many, many ways to do it. I'm going to talk about an effort that my own group did just about two years ago with this absolutely gorgeous mountaintop observatory in the Canary Islands just off the coast of Morocco at the Roque de Los Muchachos Observatory. We were using some of the largest optical telescopes on the planet. It was fantastic.

OK. So here's-- we had three main parts to our experiment. I'll go through very quickly. We're almost at time. I'd be glad to chat more about it. We set up a temporary laboratory here that was actually a shipping crate that my colleagues from Vienna shipped over. And so that was our transmitter station. That's our source of the entangled particles. That's where particles A and B are created and then emitted. And then quite a distance away, about half a kilometer in each direction, we had our receiving stations, our detectors.

So let's talk about what's going on in this transmitter station. What we're doing there is we have a pump laser, which is just awesome. It's a laser that's actually tuned to emit light in the visible range. It actually looks very much like this color of purple. You could see it with your own eyes. It's in the visible portion. It's in the purple region of the visible spectrum.

We shine that laser light into a very special crystal. It's only about a square inch long. It's not very big. It's what's called a nonlinear crystal. It has the fantastic property of absorbing light of a very specific frequency that comes in. And when that particular frequency combs in, it absorbs that one photon of light and emits pairs of particles, pairs of photons. So they conserve momentum. They conserve energy. Each of these particles has less energy than the incoming particle. They conserve all the things they should. But basically, one comes in. Two come out. And that's a property the crystal. Not any old time you shine any light on it, light in; only light of a very specific frequency. So we tuned our laser very carefully in conjunction with our crystal.

So that's like creating our twin. That's how we create our twins, Alice and Bob, and then we beam them half a mile in each direction across the island. You can see now the guide lasers here to line up our optics across the island.

So once the particles travel at the speed of light, half a kilometer in opposite directions, then we subject them to measurements. And that's like these dessert-- these waiters about to take their dessert orders. We're going to measure properties of those particles after they've been created together they have some common history and common properties but then shoot them off and ask random questions of them and see how often the answers line up.

So one thing we able to do is close down that locality explanation. As we'll see, the answers line up a lot, just like quantum mechanics says they do. And we wanted to be sure it couldn't be because they were somehow sharing information en route. And we ensured that by the space time arrangement of the experiment. The time it took to create the particles, beam them across the island, and complete a measurement, not just have it arrive, but actually complete a measurement in this case of its polarization, took just over 2 microseconds, 2 millionths of a second, for-- in each direction. And yet the light travel time between the detectors was nearly 3 and 1/2 microseconds.

So there was no time to get an update from, say, Alice's side to Bob's, either to have said my waiter just asked me about baked goods or to say I just ordered a brownie, right? Both the questions that were asked at that particular moment and the actual measurement outcomes, the answers given, were space-like separated from each other. There was no way that information traveling even as fast as light could have updated one side about what's happening on the other side.

What else is happening at these receiver stations? Here's where these amazing instruments become so important. I can't believe they let us play with these. I didn't get to touch them. They knew better than me. These are each 4-meter telescopes, some of the largest anywhere, the Galileo National Telescope on one side and the William Herschel Telescope on the other.

So these telescopes were so big, 13-foot polished mirrors, polished really to perfection, that they could gather light in a fraction of a second, less than a microsecond, even from very, very dim very distant objects, like very distant extragalactic objects like quasars, some of the most distant galaxies in the universe, distant from us.

So what we were doing is performing real-time measurements of the color of the light from those quasars, one telescope pointing in one direction of sky, the other pointing in the opposite direction of sky, and taking new measurements of that light every-- roughly every microsecond, every fraction of a second, while the entangled particles were in flight.

So you emit the particles first. They don't know what questions to be asked of them. They don't know, in a sense, what properties are about to be measured of them. And while they're traveling, we perform a real-time measurement of some of the oldest light in the universe. Sometimes in that very narrow window, the light will be more blue than average, sometimes more red than average.

And so on the times when it's more blue, that activates local electronics to perform measurement of the earthbound entangled particle in one basis. We measure the polarization in one orientation in space. That's like the waiter choosing to ask about baked goods. On those microsecond windows when the light happened-- the astronomical light happened to be more red than average, we perform the other measure. Let's say we ask the other question.

We did this now across the island. We did this with 20,000 pairs of particles. And we found that the measurements line up exactly as quantum mechanics says they would. We get almost exactly the maximum value of the correlation that quantum mechanics predicts. We violate Bell's inequality by more than nine standard deviations. The statistical likelihood that was due to a fluke is 1 part in 10 to the 20. Take that, Gary Gensler. I'd like to see you guys do that with your finance stuff. Anyway, huge statistical significance, even though the decision of what question to be asked came from events on opposite sides of the universe 8 and 12 billion years ago.

So on the basis of experiments like this one and many others that I described in that optional reading, many, many kinds of experiments now, the world really does seem to be as spooky as Einstein feared or suspected, even when we close down the kinds of Einstein-like local realist explanations that otherwise might have been able to account for those correlations.

So let me very, very rapidly sum up. I know we're basically at time, just to summarize today's material. Schrodinger's equation brings up this property of superposition. It is a wave equation, and therefore, solutions obey the wavelike property of superposition that the sum of any two solutions is itself a solution.

But that-- unlike with water waves, that led to some pretty jarring or unexpected questions when we apply that to the outcomes of events. If the wave functions can obey superposition and the wave functions are about likelihoods of events to happen, then what does that do to the nature of explanation? And then finally, what happens if multiple things are in superposition in this entangled way? So the outcome over here is not only a heads or tails that we don't know yet, but is somehow bound up with something that might happen arbitrarily far apart.

So let me stop sharing. My apologies for running a little bit long today. But we do have time for a few more questions now, or, of course, if you have questions, please feel free to email, come to office hours. There's lots more stuff I'm glad to send your way if you're curious.

Gary is an identical twin, he tells us, and so even though he answers questions at a distance, you thought you had some free will. Exactly. So with the twins, it's not just that you would share a DNA and a similar upbringing. It really is like, would you answer randomly posed questions when you don't know what question you'll be asked in advance? And even when you're asked different questions, have your answers line up? That's what-- people often say, oh, they're twins, they share so much. Of course they give the same answers.

And that's what these Bell tests really force us to say. It's not just that they have similar proclivities, but really that answering questions they couldn't know in, advance even different questions, in ways that their answers still line up, even if one is in Cambridge, Massachusetts and the other is in Cambridge, England, or on Alpha Centauri.

If that doesn't raise the hair on the top of one's head, for those of us who-- for those of you who have hair on your heads. That discounts both Gary and me. If that doesn't make you really stay up late at night, then I don't know. Then I'm done. I got nothing else, right? You guys weren't upset about the double slit. I did my best. Entanglement should just-- kind of ups the ante a little bit.

Any questions on that? OK. I've stunned you into silence. It's also a busy time of the term, so don't worry about that. We'll meet again at our regular time on Monday. In the meantime, I encourage you, please, please do start thinking about paper 2. It's a chunkier assignment. You don't want to leave it to the end. And of course, as always, please don't hesitate to contact me or the teaching assistants for any questions in the meantime.