[SQUEAKING] [RUSTLING] [CLICKING]

DAVID KAISER: So today, we are talking about the reception of Einstein's work from 1905 on the electrodynamics of moving bodies. So we'll talk about the reception of what we have come to call the Special Theory of Relativity.

So as a quick reminder, we're going to-- sorry, excuse me. The class today has three main parts, like they usually do. We'll talk about some of the early reactions to Einstein himself and to this body of work. And then we'll look at two specific examples of those reactions in a bit more detail.

Get more of a sense of what ways were people could have engaging with Einstein's paper from 1905, what did they think it was really about, what did they get excited about, what did they ignore that otherwise seem to have been really important to Einstein himself.

So we're going to talk about not just did people read the paper and make use of it, but in what specific ways? How were they creatively making this work relevant from where they sat intellectually and institutionally?

And we'll see that sometimes that that diverged-- sometimes in dramatic ways-- from what Einstein himself thought was most relevant or most interesting or most important about his own work. And that's a theme again we've seen already a few times so far in this class. Different people engaging with Maxwell's equations in different ways. It's a theme that we'll see really throughout the entire semester.

And here's a class I like-- this is pretty fun material because I think we get to really sit with some examples in some detail and really watch people wrestle with what was a fairly new material, Einstein's paper from 1905.

So just a reminder. What did we cover in our previous discussion just on Monday? Einstein had many different threads that he brought together in this paper from 1905 entitled, "On the Electrodynamics of Moving Bodies." He was a recent college graduate. He was working off-hours on his PhD thesis. He had a full-time day job working at the patent office in Bern, Switzerland at the lowest level, entry-level job of patent officer third class.

And in the evenings, he would hang out with his friends like Maurice Sullivan and Conrad Habicht and another friend, Michelangelo Besso, in a small circle. And they would read physics and philosophy together, and they called themselves the Olympia Academy even though it was really just three recent graduates drinking beer in the pub talking about things like the work of Ernst Mach. It's not a bad way to spend your time after college.

So one of the people they were really interested in was Ernst Mach, and again, we have this great reading that was on the Canvas site I think designed for this for Monday's-- this past Monday's class by our own Amanda Gefter, one of our TAs, who helps us really even more understand what was Mach all about, what did Einstein take from Mach's work? And so I encourage you to go back and reread Amanda's piece as well.

So Ernst Mach is this Viennese polymath who introduced this philosophy of science in the later years of the 19th century, really during Einstein's childhood, that came to be called positivism. And as far as Ernst Mach was concerned, the best way to make progress scientifically was to focus on objects of positive experience-- that's where the positivism comes from. Things we could really concretely observe or measure. And not focus on things that could never, even in principle, be subject to these kind of empirical inputs.

So we had this extensive, often very biting, very significant critique of Newtonian physics, which Mach was-- as far as Mach was concerned, was riddled with these things that could never ever become objects of positive experience. And that really was very, very inspiring for young Einstein and his circle of friends.

So one of the lessons that Einstein took from that, among several, in these years leading up to around 1905 was that it would make sense to focus on kinematics, the motion of objects through space and time, things we could, at least in principle, really measured and observed, rather than starting our inquiry with dynamics, the study of forces, that was really turning on its head, what had become quite a standard way to approach topics like electrodynamics.

We saw when Hendrik Lorentz considered electrodynamics. He would start with the forces exerted by the ether on objects and so on. Einstein said, let's start with kinematics first.

It was that kind of thinking that led him to postulate these opening postulates in his 1905 paper, which hopefully is now quite familiar to you in these opening few pages. This is the subject for your first paper assignment. And to paraphrase these two postulates that Einstein elevates in those opening paragraphs, the first is a extension or generalization of an idea that was actually centuries old by this point. He was really building on Galileo's work.

And Einstein says, not only should the laws of mechanics be valid for any observer who is moving at a constant speed, as long as they're not speeding up or slowing down-- they're not accelerating, then all the laws of mechanics of ballistics of a ball you might toss and play catch, all those laws are valid whether you're on a boat moving at a constant speed down the river or standing still on the shore.

Einstein says, that's true-- that should be true for all of physics. For electricity, for magnetism, for optics, for thermodynamics. So part 1 is elevating or generalizing what was already a fairly familiar notion to many, many physicists. And part two, at least at first, he says, it looks apparently irreconcilable with the first.

The second postulate, according to Einstein, is that the speed of light, c, is a constant independent of the motion of the source. And we now know-- he came to that in his own private thinking, or in talking with a small circle of friends-- by a series of these thought experiments like, what would it be like if you could catch up to a light wave like this surfer riding at the same speed as the ocean wave? You would see this frozen field configuration which, Einstein was convinced, could never really happen.

So how do he make sure no one gets caught in that absurdity, makes sure no one could ever catch up to a light wave? Well, if light's always going to travel at a fixed speed no matter how fast you're chasing after it.

If I'm on a supersonic jet plane, I'm still going to see the light wave travel past me at this fixed speed of light, then no matter what fancy engines I sup up on my means of transportation, I will never, ever be like this surfer, I will never catch up to and move at the same speed as that light wave.

And that was, Einstein considered, really central as a postulate. He doesn't prove it, he doesn't even really motivate it in this paper very strongly. He elevates it to an assumption then sees what will follow.

And then it's from those starting postulates where, again, just in the opening few pages of this article, where he comes to what he calls the relativity of simultaneity.

That events that we might consider to have happened at the same time, to have been simultaneous, will not appear to have been simultaneous to an observer who is otherwise fully legitimate, meaning all the laws of physics should hold perfectly valid for her, she just happens to be moving at some constant speed with respect to us.

We will disagree on what counts as simultaneous. And from that, Einstein was convinced, would lead these other phenomena like length contraction and time dilation, not because of forces exerted by an ether, but because we disagree on when to implement our measurements, because we disagree on what counts as simultaneous events.

And that comes for him-- from starting with kinematics, not dynamics. And again, that's the form of reasoning that Einstein puts into those opening pages of his paper from 1905. OK.

So what were the early reactions to that body of work? Remember, it was published in, really, the leading, or certainly one of the leading journals in Western Europe for professional physicists. It was a perfect venue to try to get some attention. And yet, we now know historically the first reaction was no reaction at all. The baseline summary for today's class is no one paid attention or very few people paid attention.

We now know Einstein's name. We might own Einstein's swag-- T-shirts and coffee mugs and bumper stickers, but that simply was not the case in 1905 or 1906 or 1907. It really was quite a delay.

So at the time Einstein, as we saw, was this really little-known patent clerk. He was not employed as a research scientist. He was not teaching in a university. That had been his ambition. He just was not able to get that kind of job at first.

He was publishing, before then, perfectly adequate, not very special articles on other topics in the main journal. He was building up a kind of portfolio of perfectly competent, not very earth-shattering work. So no one was hanging on, waiting at their mailbox to get the latest issue of the journal to see what Einstein had done lately. Very few people noticed anything at all.

So this next point is one of my favorites. Not only did Einstein still not get the kind of job he had most wanted, an academic position, he didn't even get a raise or promotion at his current job. He remained a patent clerk third class even after publishing all of these articles even in the space of just one year within the leading physics journal. Not just the one we've looked at so far, even others we'll look at soon. He didn't even become patent clerk second class. This was really, really a complete shattering silence in response to his work.

So one of his close friends, an exact contemporary of his, Max von Laue, whom he happened to meet, who was following a more traditional physics research and academic career-- von Laue was the same age as Einstein, but was making advances of the sort Einstein had hoped to make but was not yet making. Von Laue did his PhD quickly, he got a very prestigious postdoctoral position, he was soon teaching at a central research institute.

Von Laue had befriended Einstein, they would talk, and because of his direct connection, Max von Laue and a very small number of others began trying to get a little more attention to Einstein's work on what we would now call special relativity.

So for example, from his better-established academic position, von Laue wrote a review article that people did pay attention to. He was seen as a rising person to follow. He wrote an article fully four years later in 1909 which helped to bring Einstein's work to others' attention, Einstein's work and that work of others as well.

So a few years later, some more people begin to pay attention not because they were following Einstein, per se, but a little bit of a kind of percolation starts. So reaction one is really no reaction at all. The second reaction didn't make Einstein much happier.

So when people did notice the work, the minority, the exceptions to that rule who paid any attention at all, they tended to read Einstein's paper, the one that for all immersed in now, as a clever rederivation of what was, by then old news. They saw this as a really kind of smart way to get back to the results that leading physicists, like Hendrik Lorentz and Henri Poincare in Paris and others, had been finding for a decade since the 1890s.

Remember, we even-- we saw this in our own class discussions a few sessions ago. Hendrik Lorentz had already published on length contraction. He was directly responding to the Michelson-Morley experiment in Lorentz's case. He hadn't really argued about there should be a contraction.

He had derived quantitatively an expression involving this factor gamma that we've already seen many times, 1 over the square root of 1 minus v over c squared. That was the exact same quantitative result to which Einstein had arrived by very different forms of reasoning.

Lorenz was a very senior, very established, very productive mathematical physicist. People really did want to see what was Lorenz's latest work. They really would hang on his latest articles.

So the small number, the exceptions who noticed Einstein's work in 1905 said, oh, I've seen that before. It's a clever way of getting back to what was by then 10-year-old results. And so if anyone gave Einstein credit in the early years, it was as like second fiddle. They would refer sometimes to Lorenz's theory, Lorenz's electrodynamics, or a few of them would refer to the Lorenz-Einstein theory. No one referred to the Einstein-Lorenz theory.

So the second reaction didn't make Einstein much happier. Either people ignored it or they somehow, as far as he was concerned, misunderstood this as a trivial reworking of more established results.

So even when people gave Einstein's work any attention within this rubric of the framework that Lorenz had been establishing throughout the 1890s, few people seem to notice that the conceptual underpinning was really quite distinct. Some of the equations were identical, the same factor of gamma, the same coordinate transformations and so on. And yet, they seem to be based, as we would now see with hindsight, on very, very different starting assumptions.

Lorenz's work was all about the ether. The ether wasn't only central, it was exerting forces, these dynamics according to Lorenz's view. It was literally squeezing the atoms and molecules in the arm of Michelson's device. It was all about the elasticity and the forces exerted by a physical ether.

And as we've seen in Einstein's case, in the opening few paragraphs, he's dismissed the ether as superfluous. His work is not about the ether. He says the ether was a century-long distraction, which is a very different starting point even though they both arrive at some very similar equations. So even when people gave it attention, they misread it or they read it in ways different from how Einstein probably had hoped.

And again, this lasts not just until 1906 or '07. Even after 1909, review articles by people like Max von Laue. Here's a great example that I take from Andy Warwick's work. We had some more of Andy's work to read for today, and I'll talk about that later in today's class.

But he has this great example in his article. That even as late as 1913-- now we're talking nearly a decade later. So fully eight years later. A leading British physicist had written about what he called "The abstruse conceptions" of Einstein's 1905 paper, which were, quote, "most foreign to our habits of thought," and "as yet scarcely anyone in this country," by which he means Britain, "professed to understand or at least to appreciate."

So even the people who knew there was this person named Einstein who worked on the electrodynamics of moving bodies, this was not kind of commanding attention in the way that we might expect because we know how the story ends. We know what Einstein would go on to be recognized for. It was a pretty slow transformation.

And we see this reflected in Einstein's own career as well. So as I mentioned, one of my favorite factoids for the entire class today, Einstein didn't-- not only didn't leave the patent office, he didn't even get promoted within the patent office in the light of this quite extraordinary work, his first break, the first opportunity for the kind of career we know that he really wanted-- we know from his personal letters that have survived and are available, he had been hoping for an academic position from his early, early days.

His first big break along that new career path comes actually in 1909, around the time that von Laue was writing that review article and others, where he was offered an assistant professor position in Zurich, which was actually very exciting. That's where he had done his own school-- his own university training.

Then two years later, he's given a promotion, an offer to move now to Prague. He spends about one year there, then he's hired back at a different institution, back in Zurich. And then finally, in the spring of 1914, in April or May, he's invited to join the very, very prestigious Prussian Academy of Sciences in Berlin.

So roughly a decade later, he's promoted to the kind of position he'd been really angling for all along, and it comes not one year, not two years, not three years later, but nearly a decade later. And after a lot more work has come out, not just on the basis of this 1905 work.

So has a slowly accelerating academic career starting from this very unusual starting point. So I'll pause there. Any questions on that early discussion of the reception of Einstein's work? This is a good moment to remind you, try not to annoy all your professors in college. You'll get better letters of recommendation that will help you on the job market.

That was one of many things Einstein had done so poorly because he was kind of a dummy. Einstein, as I like to say, was no Einstein when it came to his own schooling. So that's life lesson number one for 8225. Write that down. Any other questions about Einstein's-- the early reception of either his own career or this body of work? If not, I'll gladly press on.

OK. If questions come up, put them in the chat, we'll have more time to talk about them. But I'll go back to the slides. We'll talk about now this next part. We can dig in a bit more to-- an example of a bit more significant engagement, a stronger reaction and response to Einstein's work.

And that came actually somewhat surprisingly from one of Einstein's former mathematics teachers, Herman Minkowski. So Minkowski was a very elite established professional mathematician. He was a professor at the ETH-- that's the university that Einstein had worked so hard to get into, the Eidgenossische Technische Hochschule, which I never tire of trying to pronounce, we can just call it ETH. That was the Swiss Federal Polyatomic Institute in Zurich, kind of like MIT.

Minkowski was a mathematics professor, not a physicist. And through a kind of roundabout way, Einstein's work was brought back to Minkowski's attention a few years later post-publication.

So it turns out, Minkowski was certainly not looking for Einstein's work, and we have, again, some wonderful, juicy, colorful letters that survive of Minkowski basically talking behind Einstein's back and vice versa. So we know that when Einstein was a university student, he'd worked so hard to get into ETH. What did he do? He began cutting classes, especially classes of faculty for whom he had little respect, and that happens to include Herman Minkowski.

So even though he was registered and took classes for credit from Minkowski in the Math Department, he rarely showed up, and Minkowski noticed that. So Einstein used to borrow notes from several friends, from his, at the time, girlfriend, who became his first wife, Mileva Maric.

She was also a physics student at the ETH and doing actually quite well in her mathematics classes. She would very dutifully attend, take very careful notes, share them with her good-for-nothing boyfriend Albert Einstein. He would cram before the exam and kind of do OK.

Another one of Einstein's close buddies from undergraduate days, Marcel Grossmann, did the same thing. Grossmann went on to a career as a mathematician himself. So Einstein would basically cut classes from people like Minkowski and then catch up and just skate through. He would just like barely pass by cramming the night before. Don't do that on paper 1, please.

So a friend of Einstein's encouraged the professor, Minkowski, to read this 1905 paper a few years after it'd come out. And Minkowski wrote back, I really wouldn't have thought Einstein capable of that. He wrote-- Minkowski wrote to another colleague, Einstein's paper came as a tremendous surprise, because in his student days, Einstein had been a lazy dog. He never bothered about mathematics at all. So much for Minkowski having a high opinion of young Albert Einstein.

So once Minkowski was convinced to even look at this again a few years post-publication, he quickly became convinced that Einstein was still a lazy dog or at least was still making things unnecessarily complicated. So Minkowski thought Einstein had a few interesting ideas in this paper from 1905, but had really messed things up as far as Minkowski was concerned. That he had really missed the main point of his own work.

Minkowski was convinced that Einstein's understanding of Einstein's work was not the best understanding at all. So in fact, Minkowski set about reformulating this work in his own way in around 1907, it was actually published posthumously in 1908. Minkowski died rather young, so he gave this famous lecture and it was published soon after he passed away.

Who was this person, Herman Minkowski? He wasn't only a mathematician at the ETH in Zurich, he was, in particular, a geometer. His specialty within mathematics was pure geometry. In fact, he was like an evangelist for geometry. He had written a book, first published in 1896-- the cover here is from a later edition-- called the *Geometry of Numbers.* He actually wanted to remake abstract fields of pure mathematics like number theory, make those part of geometry as well.

So he wasn't only an expert in geometry, he thought geometry would unlock the secrets for all of math and that's all he cared about. That was the key to everything. So he really was a geometer all the way through.

So when he then later turned to Einstein's work about two years after it had been published, he did so as a geometer. Not as a physicist, not as a philosopher interested in math or positivism. To Minkowski, Einstein's work held lessons that were best understood through using the tools of geometry.

So one of the first things Minkowski did is something we all take for granted today. Many of you have probably seen this before. We now will call these things space-time diagrams, or often we'll call them Minkowski diagrams. These were one of the things that Herman Minkowski introduced once he grudgingly began to pay attention to this paper of Albert Einstein's.

So what are we going to do with these space-time diagrams? To make things simple, we'll consider motion in only one direction of space, let's say the x-axis. So quite typically, the convention very quickly became, we'll measure locations in space along the horizontal axis or the x-axis. And we'll measure changes in time along the vertical axis. So now we have a two-dimensional plot to measure-- with which to make sense of motion in space and time. These are space-time diagrams.

What are the next things that became very, very common very quickly with these kinds of graphs was to use very convenient coordinates, coordinates in which light waves would travel one unit of space, one tick along the x-axis, for every one tick, for every one unit of time.

So for example, if we're going to measure time in seconds, a very common unit with which to measure time, then we better not measure spatial distances in either meters or feet or kilometers or even parsecs. We're going to measure them in light-seconds, which is the distance light will travel in one second.

So we're going to measure the distances in units such that light will travel to one of those spatial units in one unit of time. For example, seconds and light-seconds. So what we're really doing, in effect, is scaling the speed of light to be 1.

And therefore, when we go to plot the motion of light rays or light waves on these space-time diagrams, they'll follow these very simple 45-degree diagonals. Their slope is fixed to be always inclined at 45 degrees because we've chosen our coordinates such that they will always traverse one unit of space in one unit of time.

Now why will they always travel at that slope? Because if we take Einstein's second postulate seriously, light will always be traveling at that single universal speed, the speed of light. Therefore, they'll have a single fixed slope. They could travel off to the left, and therefore, then we'd have a worldline like this inclined at 45 degrees pointing off to the left. They can travel to the right, but they can't change the slope. That's the impact of Einstein's second postulate.

OK. So far, so good. We can do things like we can plot more than just the path of light. Here's this-- this is real data. This is how I spend my time these days in quarantine. I don't go anywhere.

So my position, the plot of my motion through space and time is really easy now. I just sit in one location in space, like this chair, and I don't move. So all that happens is time ticks inexorably by, I just get older and older and older, but I'm not moving around, or barely.

So we can plot what's called my worldline. That's the vocabulary that gets used now with these space-time diagrams, dating really back to the era of Herman Minkowski. So for any object, we can chart its motion through space and time in one of these plots. If it's not changing its location, but only moving through time, it will have a straight line heading straight up the page. It will move-- if it's light, it's going to move at this fixed inclination or slope.

And of course, if my family forces me to get off the chair and go out for a jog, then I'll have a more complicated worldline. I'll be moving, I'll be changing my location over time, not nearly as fast as light, not nearly as fast as shown here. This is like supersonic jetpack.

But I will sometimes be moving to the right at some speed. I might change directions later and move to the left. But whatever I do, I'll be moving at a speed slower than light, that's for sure. And therefore, the slope of my worldline is limited to never be as sharp as inclined as a 45-degree line.

So I might have a nontrivial worldline, but it's a way, nonetheless, to chart my motion through space and time. Many of you have probably seen that before. This comes from the geometer Herman Minkowski trying to make sense of Einstein's work. OK.

Now let's go back to one of the scenarios that Einstein introduced in those opening pages, the part that you have for your assignment of his 1905 paper. Let's imagine that I'm standing still-- so I'm back to my very trivial straight-up-the page worldline, I'm back in my chair. And I'm standing exactly equal distance from two colleagues who are at locations A and B.

So I'm an equal distance along the x-axis from each of my colleagues. I've asked them to please shine their lanterns at me at the same time. If they've really done that, then I'll receive the light waves from each of them at the same time. The light wave traveling from B toward me and the light wave traveling from A toward me will arrive at my location at the same time-- sorry.

And if so, then I know that A and B emitted their light waves simultaneously because the speed of light is fixed. So the light wave from, say, the lantern held by person B couldn't have either sped up or slowed down and had no choice but to travel along this worldline at exactly 45 degrees. Likewise for the light wave from my friend at position A.

So if I know I'm equal distances along the x-axis from A and B And I receive both light waves at the same time here, then they must have been emitted simultaneously there. So now I can establish lines of simultaneity. Every point along this x-axis would have the same value of time in my coordinate system.

The value of t on my t axis is the same. So the way I've set things up here, event A corresponds to the time t equals 0, event B corresponds to time equals 0 at time equals 0 sitting here on my chair. Likewise, every other moment line of simultaneity is parallel to the x-axis, they're all parallel to the line t equals 0.

So I can imagine every event in my set of coordinates that had the assignment t equals 1 instead of t equals 0. All of these were simultaneous with each other, not simultaneous with events A and B, or every point along the line t equals 2 and so on. This is, I'm sure, very, very familiar.

This is how we use graphs all the time. Minkowski's trying to just give us a more straightforward geometrical interpretation for what Einstein belabors in those opening pages about what do we mean by simultaneous events in our frame of reference. These are events that occur along a line of simultaneity.

Well, now we can use our set of coordinates, our x and t, and map the motion of some other object that's moving with respect to us. For example, Einstein's favorite object, a train. So now we can chart the worldlines of the back of the train, the middle of the train, and the front of the train as the entire assembly moves past us while we stand still on the train platform.

So this is our mapping of the moving object's motion from within our set of coordinates with respect-- the coordinates mark our frame of rest, and we're going to watch this moving train move through our coordinates.

Now remember, we have a partner who rides in the middle of the train, again, very much like what Einstein was describing. So we have a colleague who's sitting in the middle of the train. She knows-- she's exactly equal distance from the front to the back of the train. She's asked friends on the train to shine their lanterns toward her and she'll be able to determine whether or not her colleagues A and B shine their lanterns toward her simultaneously.

Well, light can only travel along 45 degrees on these diagrams for me or for her. Even on the moving train, the light wave that's emitted from point B doesn't speed up, it can only but go along a 45-degree angle. That's the real force of Einstein's second postulate.

Likewise, the light wave that's emitted from point B can only travel at the speed of light. So in my coordinates, as much as in my colleagues coordinates on the train, those light waves must travel at 45-degree lines, at the constant speed of light. So our colleague who's moving on this train sitting in the middle, she receives the light beams from point A and point B at the same time. So now she knows those emission events must have been simultaneous.

How could they not have been? She's an equal distance, the light waves couldn't have either sped up or slowed down. So therefore, all the points along this new line, we can call the x prime axis, all those events occur along a line of simultaneity for her.

In the moving train, she knows exactly how to establish which events are simultaneous. She can take advantage of light waves, they travel at a universal speed. It just turns out that what events are simultaneous for her no longer match the set of events that are simultaneous for us.

She has a different set of lines of simultaneity. And Minkowski was saying, in effect, you idiot, Einstein, it's just a coordinate rotation. You're just establishing a new set of coordinates like any student of geometry should be able to do. He said it only slightly more nicely in his published article.

Meanwhile, the t prime axis is nothing other than the worldline of the zero point of space for the moving observer. That is to say, it's the worldline of the back of the train. The origin point of her coordinates is, say, the back of the train, x prime equals 0. She can measure any distance she wants with reference to the back of the train. She's moving with the train. She can always lay out meter sticks and measure how far the front of the train is from the back.

So the origin, the spatial origin of her coordinates is the back of the train, how does that move through space and time? That just becomes her t prime axis. Much as our location of the point x equals 0 as it just sits still and moves through time, that maps out for us the t axis.

The next thing Minkowski showed very easily for him as a geometer is that the angle between the x and the x prime axis, some angle theta, is exactly the same angle as that between the t and the t prime axis. In fact, that angle is directly related to this ratio v over c.

So the faster the relative motion between us and the train, the more steeply these lines are inclined, the larger the angle. As the relative speed v rises with respect to that constant universal speed, the speed of light, this angle becomes larger, the x prime axis becomes tilted even more far away from x, the t prime axis gets tilted even more inward away from t.

But all we're doing is establishing a new set of axes with respect to which we can mark the motion of objects through space and time according to the geometer Herman Minkowski.

So now we come to some of these strange-sounding ideas that Einstein, again, had belabored as far as Minkowski was concerned. Einstein belabored with all this lengthy discussion of Ernst Mach-inspired measurement procedures. Minkowski says, it's just more geometry. Let's take the example of length contraction.

We all agree, the procedure for measuring lengths is to measure the location of the front of the object and the back of the object at the same time and then take the difference. OK. So we measure the length of the train to be this length I've called L prime.

We have our friend at the back of the train at position A, we have our friend at the front of the train at position B prime. We know those are simultaneous because they lie along a line of simultaneity for us, or a little bit more concretely, both the event A and the event B prime share the coordinate t equals 0. They have the same time coordinate in our coordinate system, in our reference frame.

They lie along a line of simultaneity. So we've therefore marked the position along the x-axis of the front of the train and the back of the train at the same time in our reference frame. Now we can just take the difference, subtract the location of this point B prime along the x-axis, subtract it from the origin. OK. That's our length of the train. Front and back, same time, length L prime.

Meanwhile, for our friend who's riding along the train, she knows exactly which events are simultaneous because she can have her friends test this with the exchange of light signals. And she knows very, very clearly that events A and B are simultaneous. For her reference frame, A and B lie along a shared line of simultaneity, just not our line of simultaneity.

So her friend at the front of the train can mark that location, her friend at the back of the train can mark that location. We know they're doing it at the same time in the moving reference frame. Take the difference, it's the difference L, which, as you can see now, just trivially is longer than L prime.

So what Einstein called-- and for that matter, Lorentz called length contraction, our measurement of the moving object is short compared to the measurement conducted by someone moving with that object. L prime is demonstrably shorter than L. And Minkowski says, in effect, of course it is.

We're just projecting our measurements onto different sets of axes. We are geometers, we reckon objects with certain sets of coordinates, and we just happen to have different coordinates with which we're making sense of the world. He doesn't just derive the qualitative phenomenon of length contraction, he gets exactly the answer that both Einstein and Lorentz had gotten, even though they had very different forms of argument.

He even gets the exact form of this gamma, because as you may remember, the angle here between x and x prime is directly related to v over c, as is the factor gamma, a few lines of algebra, not very hard, to see the relationship between this line segment and that line segment.

What's important, again, to underscore is not the algebra-- I'm confident we can all do that. Is to say that for Minkowski, this is not about the ether, it's not about Machian observations that are subject to positive experience, this is just geometry as we transform between different coordinate systems. And you can almost hear him saying, "No duh!" to Einstein who cut all his classes.

Minkowski goes on in this article, first worked out in 1907. Using these space-time diagrams, he then shows that the full Lorentz transformation was nothing but geometry. Remember, to Minkowski, everything was geometry. This was just a rotation in space-time.

Now to warm up for that, again, we'll do something probably a little bit familiar to start. We'll talk about just an ordinary x-y plane. Two directions of space that are perpendicular to each other, the Cartesian plane, or, if you like the Euclidean plane. We can lay out our coordinates.

x will be our horizontal axis, y is at right angles heading up the page. These are two directions of space. We can identify some points in the plane. We'll call that point P. We can label the location of that point in our coordinate system. We'll project what that means. Really, is we're projecting the location of point P onto our perpendicular axes.

So we label the x-coordinate of point P as its projection along the P-- excuse me, along the x-axis, it has some value x1. We identify the location of point P, the projection along the y-axis, that's its y-coordinate y1. Fine.

Now we all know-- even Einstein knew-- we're allowed to rotate our coordinate systems in the x-y plane. What if a friend of ours had used an inclined set of coordinates, x prime and y prime, that are rotated by some fixed angle away from our original coordinates?

The point P hasn't moved. All we've done is change the labels we assigned to the coordinates. In our new coordinate system, in the rotated coordinates, the same point P, which itself hasn't moved, is assigned a different value of its x prime coordinate projecting now to the new axis and a different value of its y prime coordinate projecting to the new y-axis.

These are related, quite straightforwardly, by a rotation matrix, by-- if we want to be a little fancy pants, we could say it's a one-parameter group of the rotation in two dimensions. But it's just a set of sines and cosines to take into account how the x and x prime axes are related to each other by this angle theta and the y and y prime axes.

So it's actually very straightforward to relate the coordinate labels in one set of coordinates to those in the other. Nothing magic has happened. We can see that what I had called x prime was related to x in this very straightforward fashion, taking into account the different coordinate systems and their angle between them, and likewise, y prime in terms of x and y.

So Minkowski says, that's all that's happening with space-time diagrams as well. Now let's imagine some event in space-time. So now we're going back to these Minkowski diagrams, the spatial direction runs along the horizontal axis, the time direction runs vertically, we have some event that happens in space and time.

That is to say, we assign to it a time that happened at 12:00 noon at my house, a spatial location, some point. Now the observer on the train is going to label that exact same event, but give it different coordinate values because her coordinate system is different. The lines of simultaneity for her frame are different, the worldline of x equals 0 for her is different.

So she has the x prime, t prime coordinate. She's still perfectly capable of labeling the event P and assigning to it values in her coordinates x1 prime and t1 prime. And again, remember, the angle of inclination here is directly related to the relative speed between these two sets of coordinates, these two reference frames.

Moreover, what Lorenz had derived because of the-- what he assumed to be the elastic forces of an ether acting on the molecules, what Einstein had derived quite differently by thinking about the relativity of simultaneity, Minkowski says is just the set of rules for making rotations in this only slightly more complicated geometrical space. It's still basically like making rotations in the x-y plane.

We have, again, to be fancy pants, a one-parameter family of rotations. The one parameter that changes now is the relative speed v, or if you like, the ratio of v over c, but of course, c is a universal constant. The only thing that really varies here is the speed little v.

We have some other set of rules. These take the place of our sines and cosines of the relative angle. So once again, we can now trivially, geometrically relate the labels that the moving-- the person on the moving train had assigned to the same event P, relate those to the labels that we assigned standing still on the station platform.

He gets precisely the Lorentz transformation, not because of the ether, not because of Einstein's arguments about how we perform measurements, but because he's just a geometer making rotations in a particular kind of two-dimensional space.

So to Minkowski, the Lorentz transformation itself was nothing but geometric rotations in space-time. Again, so not surprising since we know Minkowski is a geometer above all. OK.

Now this gives us something that's actually new. He's no longer just rederiving stuff in a new way. Now he gets stuff that neither Lorentz nor Einstein had actually recognized or realized. This next part comes really from Minkowski taking geometry very seriously.

As he knew as a geometer, even in ordinary x-y plane-type geometry, when we can perform rotations, the coordinate labels of a given point will change, but some quantities remain unchanged even under rotations. For example, the distance. So what's the distance between the origin of my coordinates and the point P? We'll call that distance d. That doesn't change even if I change the labeling of my coordinates.

So when I rotate by some angle theta, I change the labeling the description of x and y versus x prime and y prime. I didn't pick up the point P to move it further from the origin. The distance between, say, the origin and point P is invariant under those rotations.

And again, it'll take you about 28 milliseconds to figure that out for the geometrical case just using sine squared theta plus cosine squared theta equals 1. Use the usual trigonometric identity between these trigonometric functions, and then it's very straightforward to see that the distance is invariant even though the coordinate labels have changed under rotation.

That's what now becomes really new that Minkowski pushes forward as the first real concrete advantage as far as he's concerned of this geometrical approach. There's a generalization of distance that Minkowski introduces. It comes be called the space-time interval, often abbreviated by the letter S. It's like a version of distance for space-time instead of only a spatial problem, only space-space.

So what is this space-time interval that remains unchanged even though the coordinate labels might be switched under these coordinate Lorentz transformations? So again, we come back to this way to relate our x and t coordinates to our x prime and t prime, that's what Minkowski is rederived from his rotation.

And now he shows, using the definition of gamma, that just for the exact same reason that the Euclidean distance, little d, in the previous example remains unchanged, just because of the definitions of sine and cosine of a given angle, given the definition of gamma, that way of relating the kind of rotation, the degree to which x prime is inclined with respect to x, we get an unavoidable outcome.

That whether we had chosen to label the event P in our x and t coordinates or x prime and t prime coordinates, the space-time interval, little s, has remained unchanged. Something remains invariant even though our different descriptions of spatial distances and temporal durations have changed.

So to Minkowski, this shows that really, geometry-- in particular, the geometry of space-time, that, to Minkowski, becomes the only thing that matters. Not the ether, not this Mach-like show me how I can taste it or measure it or make it subject of positive experience, it's this geometrical feature of space-time. And in particular, not space and time separately. It's actually Minkowski's work, not Einstein's at first, that really introduces this notion of a union of space and time that very quickly becomes called by a single name, space-time.

So that's what Minkowski is convinced is the only really important insight or outcome of Einstein's work, it is geometrical. So as he famously concludes in this article-- it was published soon after he passed away, laying out all this work on space-time diagrams, the invariant space-time interval and so on.

He says, "Henceforth space by itself and time by itself are doomed to fade away into mere shadows." These are really just projections on a kind of idiosyncratic choice of coordinate axes. These are mere shadows. "And only a union of the two--" a single union of space-time-- "will preserve independence."

So let me pause there again and ask for any questions. Any questions on the Minkowski stuff and on where he was coming from? So Silu asks, what were Einstein's reactions to Minkowski? Good, very good. So the first reaction of Einstein to Minkowski was also thoroughly predictable. He hated it. He never liked Minkowski as a person, he cut his classes. He was like, oh look, this guy still understands nothing important. The feeling was totally mutual.

So Minkowski thought Einstein had made a mess of things and only grudgingly paid attention. Einstein read this paper and said, oh, this guy understands nothing, he's not reading Mach, he missed the real conceptual innovation. He's just playing with axes.

Now the difference was, Minkowski died and Einstein didn't in 1908. So Einstein later, over-- and in fact, we'll look at this in our upcoming class session. Over the next several years-- again, about four years after Minkowski's work was published, did Einstein slowly, grudgingly, with other kind of inputs and nudges, begin to appreciate the geometrical view that Minkowski was putting forward.

Einstein's original reaction was, this guy still doesn't get it. I would cut his classes again if I were still there. I mean, really, it was just this mutual, like, blah. And it's really about four years later, as we'll talk a bit about in the next class, that Einstein grudgingly comes along to say, geometry is really cool, space-time as a unified notion actually seems important.

And then he says, oh, I wish I had cited all that math after all, and he goes back to one of his friends, Marcel Grossmann, and says, oh, not only can I borrow your notes, can you teach me all the stuff I should have learned when I was in college? Lesson number 3 is don't act like Einstein.

And Gary asks, if Minkowski had lived, who would have been deemed the greater genius? Oh, that's an interesting question. I don't know. That's a time-varying question. In 1909, still Minkowski. In 1915, I'm not sure. In 1945, I don't know. It's interesting to see. And I imagine the mathematicians then, as now, would herald Minkowski's work for a lot longer. We'd see, I'm sure, some different opinions continue to vary.

Jesus writes, it's scary to realize that nobody cared too much about these groundbreaking results for so long. Makes me wonder what kind of groundbreaking work may have actually fallen through the cracks and never gotten a full exploration at all. Yes, that's a fair point.

That's something we can keep in mind throughout this whole semester. Not only-- there's all kinds of gatekeeping about who determines at what time what work is worth paying attention to. And institutions play a role, individual rivalries play a role. Soon, we'll come to examples where worldly events play a role, well beyond the control of any given individual or institution. Sexism plays a role, racism plays a role.

I mean, there's all kinds of things that factor into who determines this is of value and worth paying attention to at any given time. And that's a general historical lesson we grapple with all the time, not just in the history of science, but much more generally. And we'll have a lot of examples of that. Really, actually, more examples even pretty soon in our own class, alone examples we might think of from other instances.

Here's an example where, to his credit, Einstein later, grudgingly, came around to say, oh, that stuff is important, Minkowski does deserve credit. I now see it was really valuable. But it took Einstein himself quite a few years to get there. Doesn't always happen, or there can be a much longer delay. Any other questions on Minkowski's reaction to or remaking of Einstein's work?

So Minkowski actually thought this was really a big deal. Minkowski thought his own work was a big deal, which is not shocking. It's also not unique in the history of science. And in particular, he thought it wasn't actually only about coordinate transformations and rotations of axes. He thought that was the most obvious thing to do, and anyone who doesn't do that is a dummy. He had very clear ideas about the proper tools of research.

But even in that first article that gets published soon after he passed away, he develops in the later sections a larger worldview. Literally, a view of the world that's based on this union space-time notion. It gets very wrapped up with certain trends in idealist German philosophy. He's very inspired by the work of Immanuel Kant, for example. Einstein was, too, but in different ways.

And so as far as Minkowski is concerned, he thought he'd found a new deep level of reality. The reality wasn't the ether to Minkowski. The reality was there was this single thing, space-time, and there are certain things on which all people will agree, these invariances, even as we disagree separately on measurements of length or measurements of time.

And so that, for him, reintroduced a kind of absolute, an absolutism. Not absolute rest like in Newton, not absolute space, because space and time, he says, are now just mere shadows. But there is an absolute past and an absolute future. There's an absolute away, the things that we would now call, if you've had some more coursework in this area, you'd recognize this things like spacelike-separated and so on.

He starts mapping sets of events that could not possibly have exchanged even a single light signal yet. Those are absolutely separated from each other. And he gives these very absolutist language to them. So he's actually mapping what he considers like the ultimate absolute structure of space-time. And as far as he's concerned, that's the real lesson that the geometry leads us to.

So he actually thinks it's not a theory of relativity at all. Relativity is like an accident and boring and trivial and the stupid thing that no one should pay attention to. Relativity-- oh, we differ, the person on the train says this wasn't the same. He says, that's just accidents. Who cares? What's actually important are these underlying invariances or absolutes. We all agree on certain combinations of things, and that's the real lesson.

So for him, this becomes a re-injection of the absolute. Not because for all sitting in a physical ether. He couldn't care less, he's a geometer. Because there is a unique structure to space-time that the tools of geometry are uniquely well-adapted to help us understand. So does that makes sense? Does that address your question? OK, cool. Thank you.

So let's see. Fisher asks the question in the chat. We now think of space-time as something is deformable. Hey, that's cheating. That's-- we're going to get there next week. You don't know that yet. It's 1905 or 1908. But certainly, we'll talk about quite a lot starting in this coming-- the next class session where Einstein himself gets to is to begin thinking about space-time as one thing that also can respond, to be deformable.

Originally introduced as a mathematical artifact, was a point where we moved from thinking about it as a mathematical concept when general relavity was published? Yes, very good. OK, so Fisher, that's a great question. Thank you for that. And we're going to come-- we'll get to sit with that actually quite directly on our next class, so I'm going to I'm going to punt on that for now.

The short answer is it's not-- it doesn't come only when Einstein publishes general relativity. That comes very late, 1915. But it's an evolution in Einstein's own thinking with some colleagues, like Marcel Grossman and a small circle, over the intervening decade.

So starting around 1911, '12, '13, he has a lot of mistakes, a lot of blind alleys. He starts wondering about that more in the terms as you describe it. Starting well downstream both from his own work in 1905, downstream by four years even from Minkowski's work. It's a real intellectual journey. So that's a preview, we'll get there next time. Great.

Gary asks, how did folks react to Minkowski's work? Again, I think maybe-- sorry, maybe I mentioned that or you asked earlier, who was the-- who made the greater contribution? Yeah, Minkowski was on people's radar screen. He was a big fancy senior mathematician, very widely published, very influential at a central-- center of learning in Western Europe.

So his work was paid attention to mostly by other mathematicians and by some mathematical physicists like Hendrik Lorentz. Not every single physicist, and certainly not every experimental physicist, but it was not buried and forgotten for years and years. It was appreciated in certain specialist communities right away, and then as we'll talk about later, even other people like Einstein belatedly come to admire it as well.

So it becomes very, very well-known, partly, I think, because Einstein goes back to it. And Einstein himself becomes, then-- eventually someone that everyone pays attention to, many people. So it has a bumpy few years. But it was not launched into obscurity because Minkowski himself was actually already quite well-established and well-known.

Abdul Aziz asks, what was Lorenz's stance with regard to this? Good. Did he jump on the space-time wagon? Great. So the short answer is-- we'll see an example of the kind of reaction actually the next part of today's class. Not Lorentz's, per se, but an example.

Lorentz, really, for the rest of his career was still pretty convinced that the ether exists. And he had a great appreciation for some of these very clever mathematical techniques. He certainly liked it that Einstein's, and now Minkowski's work, led back to equations that he himself had derived from his own set of arguments.

But he really was convinced that there's something physical about this elastic medium in which for all immersed for a long, long time. So he thought these were more cool tools, more mathematical tools with which to keep asking questions he already considered important. Those weren't, as we've seen, the same set of questions that Einstein thought were important or that Minkowski thought were important. And in that sense, it's a lot like the example we'll look at now from Cambridge.

Yeah, very good. So again, we'll look at this much more directly in the upcoming class session next week. The short answer is, it wasn't really Minkowski, per se, because he just had no respect for Minkowski, he kept cutting his classes.

It was actually an independent series of thought experiments and conceptual puzzles and calculations that Einstein got immersed in, and he kept getting stuck, and that led him back to realizing geometry might be really important, by which point, Minkowski had died, he went to his former classmate, his buddy, Marcel Grossman, and had to do a crash course on the stuff that he kind of should have learned when he was an undergraduate.

So it was-- he came to an appreciation of the geometrical approach, of Minkowski's work in particular thinking about a single object called space-time. He gets there. But not because he reads Minkowski's paper and says, oh, my great professor, I'm totally convinced. He's like, that's dumb, it's wrong, it's boring, he's still irrelevant, I would cut his classes again. That's the reaction for like four years, three or four years.

And it's a separate set of lines of thought that-- and interactions with other people, including friends who were better-versed in mathematics, that bring him his own thinking back-- or bring his thinking towards a more thoroughly geometric approach. And then he never leaves. Then he becomes a card-carrying geometer himself once he gets a crash course from real geometers and gets a newfound appreciation because of his own questions have led him there.

So he comes to greatly admire Minkowski's work, and indeed, to try to build upon it, as we'll see next time, but not because he thought, oh, this is a cool work by a smart person. It was, again, a kind of roundabout thing. And so-- and we'll look at that in a bit more detail in the coming class. Any other question on this stuff? If not, I'll go to this last part of the lecture on some of the pretty wacky work that the Cambridge gang starts doing with this as well.

Yeah, thank you, Fisher. So as far as Minkowski was concerned, Einstein's paper, especially the first few pages, which you are all dutifully reviewing and maybe even rejecting, or not, or asking at least for revisions, the first few pages, at least, are really, to someone like Minkowski, a jumble word salad.

This is like, of course I know how to calculate distance traveled if the person's traveling at constant speed. That's like middle school. This is not like, why are you belaboring this paragraph after paragraph? Why are you using this Mach-inspired language without even citing Ernst Mach about how objects should be subject to empirical measurement?

Why are you telling me how to have my friend shine a lantern in my face? It's like, get to the point. Show me equations, show me what you're doing, show me the measurements you performed in your laboratory, show me the missing factor of 2 pi in Maxwell's equation. Do something. Calculate something, you lazy dog!

I think what Minkowski was reacting to was this kind of-- not just a kind of everything's in a specific coordinate basis, it's not very unified, we would learn fancy techniques later. It's not just that he doesn't-- that Einstein doesn't scale out the speed of light to make things more dimensionless. These are all things that we would admire or come to take-- to consider a natural move to make because-- on the benefit of work by people like Minkowski.

I think to Minkowski, this was just a lot of philosophical blah, blah, blah that wasn't advancing any new mathematics. It wasn't analyzing concrete experiments in any detail. It wasn't telling us anything new about things that other researchers had done. It was like, hey, I read some stuff and I think it's confusing, why don't people talk about it? I think that was the kind of response from many readers, which is why many people ignored it.

Those who bothered reading past page 5 said, oh, I've seen that equation before, that's just Lorenz's stuff. Oh, it's the Lorentz-Einstein theory. I think it's that kind of response. And for Minkowski, I think he would have seen an insufficient precision and clarity. And now that's a value judgment, and that depends on our own personality, on our own training, our own toolkit.

So for Minkowski, if you're not talking about what happens when you perform rotations, there should be something invariant. Like that should be less than 0 in geometry. The distance between my house and the town square hasn't changed even if I rotate the street map. Like, "Who doesn't think of that?" I think would have been Minkowski's response.

Once you think in a geometrical way, according to Minkowski, there should be unavoidable follow-up questions, and this lazy dog is too busy talking about, he doesn't like the way textbooks describe Maxwell, like I got no time for this. I think that would have been a pretty reasonable approximation to his first reaction.

And then he's like, oh, actually, OK, the speed of light is constant and I can perform these very clever sets of rotations, and of course, therefore, there's an invariance, and of course, there's an absolute away. So he has his own-- he's on his own train. He's on a different reference frame, so to speak. And certain questions are the obvious next ones to ask because he has a certain set of tools and starting point.

And that's true of-- I think of all of us, and we'll see a contrasting example in this next part of today's class, we'll see that over and over again throughout the term. Not only about relativity, but quantum theory, about cosmology, about particle physics.

And here's a kind of early example where we can sink our teeth into it and say, these people weren't just misreading it, they were doing really cool productive stuff, some of which we still take advantage of today, and yet, what they thought they were doing is neither what we think we do, nor what the original authors thought they were doing.

So we have this plasticity of meaning and interpretation even when our equations agree perfectly, let alone when we come up with different equations. I find that wonderful. I love that. And in fact, they're all writing down the same factor gamma, that's fantastic, because then we can really map what is the same and what's not the same conceptually, let alone in the exchange of light signals.

So keep that kind of question in mind. We'll come back to that theme, really, over and over this semester, it's a great question. Let me go on to the last part and look at our favorite Cambridge Wranglers, or at least my favorite Cambridge Wranglers.

So, Minkowski wasn't the only person who slowly, sometimes kind of grudgingly, began to read Einstein's work on the electrodynamics of moving bodies. Another group that really did dig in in pretty substantive ways was a subset of these Cambridge Wranglers, the folks that now we've talked about a few times at Cambridge University.

For them, much like for Minkowski, there were elements of real value in Einstein's work, they just weren't the elements that Minkowski considered valuable or that Einstein considered valuable. So let's take a little look at the kinds of things that they did with Einstein's paper.

As a warm-up to make sense of this-- and by the way, for this part, of course, I'm drawing very heavily on the article in the reading for today from Andy Warwick. And I'll talk through some of the specifics-- some of the details of that article might be pretty confusing, so we'll talk through what were they doing at least a little bit more.

The historical point I want to clarify is really just like Minkowski in the sense that very smart professional researchers were making sense of Einstein's work differently than Einstein did, and that didn't make them wrong, it made them-- it indicates that they were finding elements of value and of interest in Einstein's work different from Einstein's own and sometimes different from our own.

That's the historical lesson. And if you get a little bit lost with image charges and conformal transformations, that's OK. In this class especially, that's OK. But I think it's still fun to see what were they really doing, so that's what we're going to talk about a little bit here.

So many of you, again, might have seen, even maybe in high school classes on electricity and magnetism, there are all these really clever tricks that we still take advantage of to try to make pretty complicated problems simpler and yet can still get exact solutions.

One of those that we learned pretty early on is called the method of image charges in electrostatics. So let's say some meanie, one of my mean old colleagues in the Department of Physics, assigned to you the problem to solve for the state of the electric field between a stationary point charge with a positive charge, some charge plus q, and some infinite in extent, some infinitely long, perfectly flat conducting plate that grounded.

That's a pain in the neck to find the exact value of the electric field. It's a vector quantity to worry about how it's changing through space. So one thing that you may remember from studying Maxwell's equations is that for this kind of scenario, the field lines of the electric field all must intersect the plate at right angles.

That's to remain consistent with the fact that this is a grounded conducting plate. If they didn't all intersect at right angles, you would actually build up a voltage difference, it would not satisfy the boundary conditions of the problem. It's a pain in the neck. Meanies.

So there's a pretty clever trick. We can solve a much, much, much simpler mathematical setup to get the exact solution for the original problem. Forget about the plate. Throw the plate away for a few steps. And instead, insert a second point charge of equal but opposite sign. Equal-equal magnitude, but opposite sign.

So now in this new scenario, we pretend there's nothing physical here, but we have an equal distance away from that where the plate had been in the original problem. We put down an imaginary image charge that has a negative charge, but at the same magnitude as the original positive charge.

Now we can easily solve for-- much more easily solve for the value of the electrostatic potential. It's just two bodies. It just goes like a 1 over r between them. We can take the gradient, see how that changes through space. We can do everything we would ask to do to solve for the exact behavior of the electric field by replacing one problem with a simpler but equivalent one.

That's the method of image charges. You can learn about that on Wikipedia or on any textbook. It's a standard technique that goes back really centuries. It's pretty powerful.

Well, that's baby stuff. We're talking about Cambridge Wranglers here, or for that matter, MIT students. We don't have to do one single point charge behind what would have been an infinite unbending conducting plate. That is boring. Let's do some crazy stuff. And that's what the Wranglers got very good at doing with their personal tutors for the Tripos exam.

So a very similar related technique became known as the method of inversion. And that's, again, really just mapping a difficult problem into a simpler one. And again, this illustration is taken from Andy Warwick's article, which then is elaborated upon in this book of his that I really can't recommend strongly enough. I love this book, *Masters of Theory.* But the article itself really gets this part across.

Let's first do a little geometry, a little mapping exercise, which is what the Wranglers would have learned to do very early with their coaches. Take any single point A. Here's the point a for our case. Any point that happens to be inside a circle of radius k. So the point A is contained within that circle.

We can then do a unique mapping to an inverse point, kind of like that-- where we place our image charge from the past example. There is a one-to-one map for a point corresponding uniquely to that point A that's now outside the circle.

And so the way we determine where that point will be is by requiring the distance from the center of the circle to our original point, that distance OA, times the distance between the center of the circle and the new point, the inverse point. The product of those distances must equal the square of this radius. That's enough to do this mapping. It's actually called a conformal mapping.

Now what if there were some point that point A is actually moving along this curve, this orange curve between points P and Q? We can imagine, there is a whole set of points, each of which lies entirely within this original circle. For each of these points along the orange curve between p and q, we can do the same trick. We can map them to each of their corresponding inverse points.

For every single point along this orange curve, we require this relationship to hold. And we map them to an inverse arc, a collection of inverse points that are entirely outside the circle. So if we imagine swooping from point p to q within that circle, the mapped motion, the uniquely corresponding motion would swoop from point p prime to q prime.

Now this transformation is called a conformal transformation. We use this all the time-- I use these all the time, even to this day in my own physics research. They're really, really helpful in topics like general relativity and cosmology. The Wranglers are doing this for electrostatics to get ready for their exam.

So this transformation preserves angles, the angles between any of these arcs, any of these line segments, this angle marked w here. Those angles are preserved, the lengths are not. You can see, the line segment pq is clearly a shorter overall length-- the distance between points p and q is shorter than the distance between the corresponding inverse points p prime and q prime.

So very generally, conformal transformations preserve angles, but not lengths. If that sounds confusing to you, you're in good company. The point is, that's the kind of stuff these Wranglers would have learned with their coaches pretty early. There are properties of these geometrical transformations, the upshot of which is to make complicated problems easier.

So now let's get to an actual Tripos-type problem. Now let's say, because you're at Cambridge in 1880 or 1890, they don't ask you to solve for the field between a static positive charge and an infinite conducting plate. Come on. We are Wranglers. Instead, we want to solve for the electric field lines, or let's say the equipotential surfaces where the electrostatic potential is equal-- so equipotential surface.

Due to a single charge at location O that's near a grounded conducting sphere centered at B. So here is the actual physical parts of our problem. Some charge here sitting at rest, electrostatics, some electric charge at point O that's near some spherical grounded conducting sphere-- there's a harder version than our infinite plane.

Well, let's use this method of inversion to treat what turns out to be a much simpler problem. So we have to map the simpler version first. Instead of considering either point O or this conducting sphere B, we introduce the equivalent of a kind of image charge.

We consider a charged conducting sphere centered at point A. We put in this-- like our image charge-- our new geometrical simplified system some positive charge and a sphere at A. Then we draw the field lines and the equipotential surfaces for that. Now that's very easy. It's just a sphere. They're just going to be concentric circles around it for the equipotential surfaces. The field is just radial. It goes back to Faraday's lines of force. It couldn't be easier for these Wranglers.

Now we use that method of inversion to map it back to the original problem. We're going to close this imaginary object A in a sphere of radius K. And they will map every point along this orange curve to the corresponding image points outside of our circle. And now we've mapped the equipotential surfaces for the original problem based on this much simplified problem.

If that's confusing to you, first of all, good. You've led a healthy life. This is not on the exam. This is just meant to be an example to make sure that the kind of reasoning that these Wranglers would have been immersed in of the sort we've seen a few times. This is like standard Tripos stuff.

And remember, unlike us here at MIT, the undergraduates at Cambridge in the 1880s would have done all this on a timed exam that determined their graduation rank and would be published in the national newspaper, so we won't to ask that of you.

So now let's come back to what Andy is writing about in this article. This method of inversion and these geometrical techniques, like conformal transformations, that was like daily stuff for these Cambridge Wranglers by 1900. These techniques apply to electrostatics.

Imagine, you have a fixed charge sitting still at point O, a fixed grounded sphere, nothing changes over time. And that is to say a little more quantitatively, you're always solving for situations in which the electric potential is not varying. It's a problem in electrostatics not varying over time even though it varies over space.

So what these young recent Wranglers wanted to do, Ebenezer Cunningham and Harry Bateman, was to actually use Einstein's work to tackle a problem they cared about, which is to generalize these inversion problems and conformal transformations to situations where objects might be moving around to electrodynamics of moving bodies, not to electrostatics. You can see why Einstein-- the title of Einstein's paper would have caught their attention.

They wanted to understand how to generalize these Wrangler-like techniques to the question of the electrodynamics of moving bodies. And they thought Einstein's paper had some pretty clever tools in there to do that.

So they could identify, using Einstein's work, all the transformations, which we would now call the Lorentz transformations, lambda-- Einstein had separately rederived those in a later part of the paper that I didn't assign for our class, but it's in Einstein's paper, too. These are the transformations that would leave the original wave equation invariant.

So then they could use these inversion, these mapping techniques, the characteristic Wrangler techniques to map into new dynamical solutions that could vary in time and not just space, keeping in mind that now the time coordinate, as well as the spatial coordinates, might need to be shifted thanks to this Lorentz transformation.

So they wanted to use Einstein's paper to find the most general class of transformations lambda that would leave this harder problem invariant. To do for electrodynamics what they had already learned with their coaches to do for electrostatics. That's what relativity was all about for these folks. They were reading Einstein's work very carefully. They were doing extremely productive, hard, original work. They just were doing things that Einstein didn't notice or care or think was important or that Minkowski did.

So they weren't trying to understand the ether, per se. Again, they weren't trying to understand Machian positivism. They weren't even really that concerned about Minkowski space-time. For them, it's about wrangling most of the manipulations of mapping complicated problems into simpler ones, and Einstein's paper then gave them additional tools with which to do their problem.

So they weren't ignoring Einstein's work, but they also weren't card-carrying Einsteineans. So that's what I wanted to go through just to make sure that the real historical point of Andy's paper would be clear. It gets pretty complicated, the details of these inversions and all that. I'd be glad to chat more about that. But the main point-- the main lesson for us is really that they're also doing real work with Einstein's paper, just different work than Einstein. So let me wrap up for today.

Researchers didn't just read Einstein's paper and become convinced, they didn't become like converted Einsteineans for a long, long time. Most people ignored the paper altogether. Those few who paid attention often thought it was just a minor elaboration of previous work.

The few who really did pay attention to it much more squarely did so from within their own context. We can even jokingly say, within their own frames of reference, much like Einstein's favorite example of observers on the platform or on the train. They do different stuff with it. Different parts of the paper are relevant to them and of value.

They're not misunderstanding Einstein's work, they're like utterly understanding it. They're doing real stuff with it, just not what Einstein had first intended.

So we saw Minkowski reinterpreted it in terms of a certain kind of geometrical vision. The Cambridge gang, Cunningham Bateman and some of their immediate circles, they do other kinds of geometrical things with it because they have other concerns more related to the properties and-- transformation properties of differential equations.

The point is, none of these readers seem to care much about what was most important to Einstein. As far as Einstein was concerned, none of them were getting the point of his article even though these were among the people who paid any attention at all. Einstein had argued in those opening paragraphs that the ether was merely superfluous. Who cared? They didn't-- that wasn't the part that landed with them.

Moreover, Einstein had insisted that we start with kinematics instead of dynamics. Forget-- that didn't wash, that didn't register. Instead, as we've seen before, and we'll see again, even the exact same equations could inspire quite different interpretations or different meanings.

So I'll stop there. Any other questions on that last part about the Cambridge gang? And again, if the particulars of conformal transformations or inversion went by too quickly or Andy's paper was confusing, please don't worry, I'd be glad to chat more if it's of interest for you, but the historical point is just to say they were doing real work.

It's still in our textbooks. It was valuable work. It was just different work than Einstein's or Minkowski's or Lorentz's and so on. That's really the point I think we can hold on to in this class.

Any other questions on that? Anyone want to volunteer to do more inversion problems for conducting spheres and-- I hate that stuff. That stuff, yeah. Julia says no. I'm with Julia on this. That stuff makes me-- that really makes my head hurt.

And you want to do that for five hours before lunch like a good Wrangler? That's another question for you. Your math homework before lunchtime. OK. If there's no more questions for now, then I'll pause there. We'll pick it up early next week.

And we'll-- and we'll get into many of the questions that were raised even for today. What is Einstein himself do with all this work in his march toward what we would eventually call the General Theory of Relativity? So we'll come to that. That will occupy us, then, for our next class.