[SQUEAKING, RUSTLING, CLICKING]

DAVID KAISER: If there are no immediate questions, that's good. And if so, then I'll go ahead and share screen. And we'll launch into our continuing adventure with quantum theory, which is, frankly, one of my favorite adventures ever, speaking just for myself.

OK, so you will hopefully remember on our previous class session, we looked at Werner Heisenberg's introduction of what came to be called matrix mechanics in spring-summer 1925, and what brought him to it, what was he reacting against, and a little taste of what people thought it might all add up to. And today, we're going to look actually at a kind of complementary set of developments that were unfolding very close in time, very rapid periods of period of development. But now looking at what became known as wave mechanics and some of its features that people began to really puzzle about.

So for today, we'll look at some of Erwin Schrodinger's most famous work. Again, some of this might already be familiar to some of you, but probably not to all. And I think it's fun to just go back through and zoom out a bit and say, what did Schrodinger think he was doing and why? Let alone, what's the form of the equation that he put together?

So that's the part one. Then, we're going to look in parts two, and very briefly, a shorter part three, at some of the implications that physicists began already to grapple with implications of Schrodinger's wave mechanics really pretty quickly. And for both part two and part three, as often happens for class session, I'll try to cover some of the main highlights, some of the most-- what I think, most interesting or salient points. And then, some of the details, some of the extra derivations, and so on are in those optional lecture notes.

Strictly optional. They are available on Canvas. But if something goes by too quickly or I just had to skim over some of the intervening steps as sometimes happens, then hopefully those lecture notes will help fill that in for those of you who want to dig in a bit more. So that's our plan for today.

So just to remind ourselves again-- looking back to the previous class session-- starting in the spring of 1925, after that bout of severe hay fever that chased Werner Heisenberg temporarily out of Copenhagen and to this really quite lovely-looking island of Heligoland in the North Sea during that span of spring and summer '25 Heisenberg introduced a whole new way of trying to think about quantum theory.

This was now something like 25 years into various physicists grappling with something having to do with quantum theory, going back to Max Planck's work that was first published in 1900.

We saw through our several moments of sitting with what was called "the old quantum theory" that it wasn't quite so obvious that that collection of ideas was adding up to any one new thing or that they'd even fit together. And that was a concern that people like Heisenberg and some of his young colleagues really sought to tackle.

They wanted to have a first-principles approach to quantum theory-- a quantum mechanics-- rather than what looked already to them, as well as to many of us today, this grab bag which had characterized the so-called "old quantum theory" where people would begin with quite traditional expressions for the energy of this or momentum of that, take either Newtonian or Maxwellian expressions and then just kind of staple on at the end some new, usually not very well explained, quantum conditions and kind of ad hoc new rule.

And Heisenberg and his very good friend from grad school days, Wolfgang Pauli, and a lot of these younger folks were-- by the mid-1920s-- were looking to do something different. They wanted to have a first principles theory all its own that might account for phenomena at the atomic scale or smaller.

And we saw last time Heisenberg, in particular, was guided by a kind of approach that sounded very much like what Albert Einstein had announced back in 1905, in thinking about the electrodynamics of moving bodies. Heisenberg starts sounding a lot like a acolyte of the physicist and philosopher, Ernst Mach.

So we saw in the opening pages of Heisenberg's article-- though we had it in the English translation in our reader-- but even in the opening paragraph he says, we're going to keep fooling ourselves or chasing our own tails, basically, if we keep trying to talk about things that are not even, in principle, observable. Examples of which he mentioned, would be things like the orbit of an electron and an atom.

We don't watch the electrons zooming around like a planet in the solar system. Let's stop trying to calculate those features and focus instead on things that really are objects of positive experience like the emission lines, these spectral lines that come out when certain gases of atoms are excited.

So Heisenberg began looking at things like the observable features of these spectral lines. OK, so he reasoned then, we saw in particular that since the frequencies of these emission lines, especially for very simple spectra-like for a hydrogen atom, the frequencies obey this law of addition, that one line in the spectrum could be written the actual numerical frequency, the number of cycles per second, obey these kinds of relationships.

Where there are two other distinct emission lines, the sum of whose frequencies would add up to this third one and it could be it could be extended beyond just pairs, 3's, 4's, 5's. That there were these addition relationships between the frequencies of light that really genuinely was emitted by these excited hydrogen atoms.

And then Heisenberg reasoned that when we characterize lights, Maxwellian or otherwise, the frequency appears in the exponent, which suggested to Heisenberg that the amplitude should multiply. If the frequencies are adding, according to this law of addition, and the frequencies are one way of characterizing the light that comes out, then shouldn't the amplitudes of those associated light waves multiply?

And that's what he found, to his great surprise, alone on the island of Heligoland, that these amplitudes didn't behave the way he expected them to. In fact, the outcome of multiplying two of those things together depended on the order in which they were multiplied.

And so he returned, or started very soon afterwards, a position in Gottingen. And his senior advisor there, Max Born, clarified these are matrices. The fact that the outcome of the multiplication depends on the order is a generic feature of matrices, whereas mathematicians would say, matrices do not commute. And so soon after that, his work became known as matrix mechanics.

And then Heisenberg continued working on that, sharpening the formalism, working directly with Max Born and some other colleagues. But also trying to figure out conceptually, what might it mean if the world is characterized, the atomic world is characterized, by these strange arrays of numbers whose mathematics was, at least to him, rather unexpected and foreign? And where it seemed to be like the outcome would depend on the order in which you did stuff.

Could it be that our operations on the quantum realm could have an impact on what we could ever see or measure? And in thinking along those lines, about what would it take to perform certain kinds of measurements of things like an electron scattering high-energy light off of it and so on, that Heisenberg began to articulate his uncertainty principle that there seemed to be a consequence of the fact that these matrices don't commune with each other.

There was an irreducible uncertainty in the sharpness, the accuracy, or precision with which we could ever specify at the same moment the position of, say, an electron and its simultaneous momentum along the x direction, that there was a trade off whose scale, like so many of these things, was set by Planck's constant.

So that was what we mostly looked at last time. And as I gave a kind of foreshadowing for today, right around that same time, just a remarkably close span of weeks and months, there was a really quite different approach to some of those very same questions being worked out by Werner Heisenberg.

And we had an excerpt from this really quite wonderful biography by Walter Moore. A part of today's readings was a little section of Moore's book, talking through how did Schrodinger think about what became known as wave mechanics? I want to talk a little bit about that.

So this was happening not in the spring summer of '25 but in winter/spring '26, but really just a few months later. Aaron Schrodinger began introducing a different first-principles approach to quantum theory, much like Heisenberg and Pauli and these other kind of young researchers.

Schrodinger also thought there should be a kind of way of building a new theory from the ground up to treat the behavior of atoms and parts of atoms. But he followed a different conceptual path. He was working quite independently of Heisenberg in these early steps. So Heisenberg had built upon these ideas about discreteness. He was influenced by Bohr's work and ultimately tried to extract away from Bohr's work.

But Schrodinger actually went a different way. He was most directly inspired, his work, not by the kind of Bohr atom or this notion of very rigid, discrete quantum jumps but rather by Louis de Broglie's very suggestive idea from 1924 about matter waves.

And here's this, again, kind of cartoon version of the bottom here, trying to show de Broglie's suggestion for why these very discrete orbits that Niels Bohr had identified of, say, the behavior of an electron in a hydrogen atom. Why might those special orbits be picked out as stable? Maybe there was an inherent waviness of the electron and you had to have a constructive interference for a stable orbit. Anything else would suffer destructive interference.

Those are the kinds of things that really caught Schrodinger's imagination very soon after de Broglie himself had introduced them. So Schrodinger was coming from a bit of a different community. He was originally from Vienna. He was, at the time, teaching in Zurich. He wasn't from Copenhagen. He wasn't working in Gottingen. It was a little bit kind of different network.

Perhaps more important, he was also a generation older than Heisenberg and Pauli. He was basically very nearly the same age as Albert Einstein. They became very close friends, closer in age to Niels Bohr. He'd been working as a professional theoretical physicist for decades by the time this stuff came up, unlike Heisenberg and Pauli, who were working very soon after their own PhDs.

Schrodinger was a established researcher. And his approach to this question, I think, shows that. We could characterize his approach to the new quantum mechanics, the new quantum realm, as an effort to retain as much of the kind of look and feel of the toolkit of time-tested physics as he could.

And I'm just going to pause briefly and mention-- some of you might know-- so for the tool kits, Heisenberg and Schrodinger look very distinct with Schrodinger seeming to be a little bit more, almost, conservative, in the sense of let's retain as much as possible. And Heisenberg saying, it's a new world. We need a conceptual revolution. We'll use new techniques.

And yet socially and personally, they were almost inverted. So Schrodinger, even in his day, was known as a remarkably Bohemian, leading a Bohemian lifestyle, not the kind of traditional family values. He had open relationships with multiple women at a time. He raised children that he had with one person, he raised them with his wife.

And it was a really unusual arrangement that, actually, his own contemporaries would talk about. It stood out. And Heisenberg, as we know, was pursuing, actually, a very fairly traditional, stereotypical personal family life. So who's conservative and who's radical actually, different about what kinds of aspects of their life and work were trying to characterize.

And, again, Moore's biography, I think, is really quite astonishing for this rounded view of Schrodinger, not only how he got to a few equations. So I recommend that-- more of the book you might enjoy beyond just the excerpt that I gave. OK, what did Schrodinger do?

So we know from his own private notebooks that have survived, this work was happening in December 1925, January, 1926, really just roughly half a year after Schrodinger's first publications. Schrodinger began with the usual expression for energy, just the Newtonian expression for the kinetic energy and the potential energy. But then he immediately, immediately tried to pick up on what de Broglie had suggested about these matter waves.

And so Schrodinger, who was again, very senior, very seasoned mathematical physicist by this point, reasoned that if something about matter waves is going to be so important moving forward, let's use what we know about how to quantitatively characterize the behavior of waves. So he did. He went in a kind of wranglerish way and said, well, let's consider a wave equation. How would we characterize, for example, the behavior of a standing wave?

And we would write down this very simple-looking expression simple certainly to someone like Schrodinger, very familiar. And as many of you probably know-- and we saw briefly in some earlier lectures-- this parameter K here is called the wave number. It's inversely proportional to the wavelength.

So we can characterize the behavior of any kind of wave in terms of a wavelength and ask, how is the amplitude of that wave changing across space? How is the wave behaving as we move from one position to the other? And that's what this expression is quantifying.

But now de Broglie had said that there was something very specific about a wavelength for matter, the de Broglie wavelength. It should be proportional, de Broglie suggested, to Planck's constant and inversely proportional to the momentum of that particle, of that quantum object.

So now if the wave number is related to a wavelength and de Broglie's wavelength is related to a momentum, it was actually not a very difficult series of steps for Schrodinger to say, maybe this k, this wave number that appears in any old wave equation for classical waves of a guitar or ocean waves in the water, maybe that k is actually deeply related to the momentum in a way that actually is new, that comes straight from de Broglie's suggestion.

And if so, if we can relate K to the momentum as usual, scaled by Planck's constant here, then we go back to this quite standard equation for a standing wave of any kind. And now we want to characterize something specifically quantum theoretic then we can substitute the k for the momentum with keeping track of our h bar.

And now it looks like we actually have a whole new way of thinking about the momentum of any quantum object. Maybe the momentum is just a measure of the kind of spatial gradients of some of associated matter wave.

So maybe there's a whole new operator. Maybe there's a new object that tells us that relates the momentum of some object to the gradient of its associated matter wave with, as usual, the scaling, the kind of sense of scale or a sense of proportion provided by Planck's constant.

So what Schrodinger was doing here was trying to build up an equation where Planck's constant was kind of built in from the start, not appended at the end. Not, let's solve for some solutions and then smack on some new constraint but trying to build a first principles equation that would describe how matter behaves where Planck's constant is immediately setting the scale from the start.

And so, with that series of reasonings, Schrodinger, arrived what we would now call the time-independent Schrodinger equation. And it really was. You can see where it comes from. It's starting from the typical relation between kinetic and potential energy but trying to plug-in these insights and quantify these insights from de Broglie right from the start.

OK, so many of you have probably seen this before, that we still call it the Schrodinger wave equation. And let's just talk for a few minutes about the nature of that expression. First of all, it's all continuum. This wave function PSI seems to behave just like any other wave quantity. At least there's no reason to think it wouldn't from this expression. It should be varying continuously across space, gently having a slightly different value here than there.

There's nothing of a kind of inherent radical discreteness that faces us when we look at Schrodinger's equation. It looks like, other than the presence of Planck's constant, the form of the equation the mathematical form, looks quite familiar. It's a differential equation for how some quantity is going to change over space.

And he later, as many of you know, he introduced a time-varying version where the same quantity PSI could be continuously smoothly varying over time, as well as across space. So it's, again, retaining the look and feel of continuum-based differential equations. It doesn't have these ad hoc discreteness features that seem to be put in by hand by someone like Bohr in the Bohr model.

So one of his first tasks-- and, again, we know this from his own private notebook-- he was having a ski skiing vacation with, not his wife, but with two very much younger twin women. And in the midst of this he would take these breaks and try to calculate. It's quite astonishing.

And so one of the first tests he knew he had to apply this formula to was the now time-honored hydrogen atoms. So he said, what if the potential energy is not any old form but is the specific form he'd need to characterize the mutual attraction between an electron and a proton in a hydrogen atom put in the Coulomb potential?

And then he found, again, just on his own in this kind of ski chalet right around the holiday-- right around New Year's-- that solutions to this equation will correspond to very specific values for the energy. You notice here, the first in this whole series of articles that he wrote were called "Quantization as an Eigenvalue Problem." And, again, that's a hint.

He wants to work with familiar differential equations. What he's done here is he has some differential operators, some combination of terms that quantify the rates of change of some continuous property. That's this side. And there should be some eigenvalues of that differential operator, some allowable numerical kind of coefficients, that will characterize the changing across space of that quantity.

And so he finds just by putting in the coulomb potential for the hydrogen atom that this very kind of generic looking or familiar kind of format of a differential equation will yield exactly the same energy spectrum, exactly the same allowable values for the eigenvalues of the energy, the same coefficients here that will satisfy the equation, exactly the same series as what Bohr had found from very different starting points.

So therefore Schrodinger's equation, as well, would reproduce the successes like the Balmer spectrum. It should predict precisely the same frequencies of those emission lines from excited hydrogen atoms. So Schrodinger, again, had a sense that he was maybe on to something important here.

How does that work? And, again, many of you have probably done this calculation in problem sets or seen it done in lecture class. For others of you, it's an exciting thing to look forward to. So I'm not going to go through the whole calculation. But I just want to make sure we understand the kinds of steps that Schrodinger was following.

How does this continuum-based equation, this differential equation for some smoothly varying quantity PSI, how would that ever give us a discrete energy spectrum? What would this integer n that Bohr had argued came from some fundamental discreteness in the behavior of electrons, how does that come from Schrodinger's equation?

So let's step back and think about a simpler case involving standing waves. Let's just consider motion in one direction, so the x-axis. And let's imagine we have a guitar string or a violin string where the endpoints are anchored.

So whatever the legitimate solutions are for this wave equation, PSI being sort of the amplitude of the wave in this case, the wave has to vanish at the origin and at the length L, where the other end of the rope or the string is anchored. So how do we combine that with the general solutions for this differential equation? Again, here's our differential operator, so to speak.

Well, if we ignore the boundary conditions, if we ignore the fact that this string is anchored at both ends, then we can very quickly write down the most general series of solutions of this wave equation. It will oscillate across space like sines and cosines, with some characteristic wave number and different coefficients.

But now, if we apply the boundary conditions, then not all of these solutions are viable anymore. Some of them no longer solve the relevant equation. And in fact, if we apply these particular boundary conditions to this very general equation, differential equation, then the allowable solutions have a discrete set of eigenvalues.

The wave number k, in this case, becomes not any old continuous value but, in fact, kind of snaps into place with only integer allowed, a series of allowable wave numbers. So the wave number could be 1 times some basic reference wave number, or 2 times that, but not 1.8 times that. We get a quantum discrete spectrum by solving a continuum-based differential equation and applying the appropriate boundary conditions.

And so for the hydrogen atom, Schrodinger reasons similarly. It wasn't that the electron was somehow glued in space and could only be here versus there, like Bohr had suggested in this kind of ad hoc way. But, rather, Schrodinger said, well, we're going to solve this differential equation for some general smoothly varying quantity PSI. And let's apply some reasonable boundary conditions that the amplitude should vanish at the origin. The electron should not be found in the nucleus.

So the wave function somehow describing the electrons' motion will vanish at the origin. And the electron shouldn't be found infinitely far away from its host nucleus. So the amplitude for whatever's waving, whatever kind of de Broglie-like matter wave is associated with the electron, that should have vanishing amplitude when you're arbitrarily far away from the rest of the atom.

If you impose those boundary conditions, a little more complicated. But, conceptually, it's like this kind of standing wave on the string of fixed endpoints. That's what forces a discrete spectrum for the allowable energies, not restricting the kind of motion of the electron per se.

So Schrodinger now has retained the look and feel of manipulating differential equations familiar for a long time by that point. He has a way of reproducing this kind of inherent discreteness for things like the emission of these spectral lines but by building up this kind of waviness of nature from the start, rather than imposing a kind of ad hoc quantum condition. That's what Schrodinger was working toward in that winter of '25-'26.

He also then quickly realizes that solutions to his equation are wave functions. They should obey something called superposition, which we've seen briefly in previous class sessions, even this term. If there was one solution to that equation and a separate solution, then the sum of them will also be a solution, for that matter, the difference.

That you can take legitimate solutions to the wave equation and add them and the resulting sum will also be a solution. And that means that these wave functions could do all the things that waves do. They could undergo interference, for example.

And, again, here's just a quick example. If you have two waves of the same amplitude but different wavelength, then in places where the amplitudes happen to line up, where crest is near crest and you add those waves together, we'll get constructive interference. The resulting wave will have a height nearly twice as tall as either wave alone.

On the other hand, in other locations where a crest is nearly lined up with a trough, they'll just about cancel each other out. And the resulting amplitude will be much smaller. You can have destructive interference. All those things that are familiar from water waves, from sound waves, from light waves in a Maxwellian context, all those things seem to carry over to solutions of Schrodinger's wave equation as well, even though it's seemingly describing something like an electron.

So that really forced the question, if this wave function PSI has definite wave-like properties, what was it? What was it a wave of or in? It starts to sound like the questions about light that we began this class with.

So what is doing the waving if PSI behaves like a wave? And then it became even more bizarre. Here's when it began to break from just the familiar physics of classical waves. And, again, Schrodinger-- to his credit-- was very quick to notice this himself and really point it out and begin to try to puzzle it through.

If you consider a system with more than one electron, let's say as simple as just two electrons, not just one, then the equation suddenly depends on many, many coordinates. The interaction potential, that key portion within the Schrodinger equation, will now, in general, depend on the location of both electron one and electron two. They're each existing in three dimensional space, x, y, and z. They could be at different locations.

So the potential energy of that two-body system will now, in general, depend on six spatial coordinates, not just three. So now solutions PSI, solutions to Schrodinger's wave equation, will themselves depend on six dimensions, six coordinates of space, not just three. That's not like a water wave, or a sound wave, or a light wave.

So even though the mathematics was, in many ways, very familiar and Schrodinger used that to great effect to find solutions, to figure out why there was a seemingly quantized or discrete spectrum of eigenvalues and all that, it wasn't actually just classical waves.

And there were even, mathematically, some unexpected features that Schrodinger himself began to elucidate, that PSI, whatever it was, was not just like a water wave. In fact, it seemed to live in some higher dimensional abstract mathematical space that would quickly become known as configuration space rather than the space in which we seem to move around every day.

So that was what Schrodinger began publishing as early as January 1926. He wrote a series of papers very, very quickly, one after the other in a four-part series throughout that winter and spring of 1926. And so before long, both Schrodinger and Heisenberg themselves, and soon many members of this kind of tight-knit community, were beginning to do a kind of compare and contrast exercise.

There were now two quite different looking first approaches to a new quantum theory on offer. Heisenberg's matrix mechanics, Schrodinger's wave mechanics, they looked nothing alike. They seemed to make very different starting assumptions. And they had different kinds of strange features. They were strange in different ways.

So Schrodinger eventually caught up with some of Heisenberg's papers. And he added a footnote into one of Schrodinger's own follow-up articles in that first series. I think this was not in his very first article but some time later that spring in a footnote. He writes, as you see here, my theory was inspired by Louis de Broglie and by short, but incomplete, remarks by Albert Einstein.

No genetic relation whatever with Heisenberg is known to me. I knew of his theory, of course, but felt discouraged, not to say repelled by the methods of transcendental algebra, what we now call matrices, which appeared very difficult to me, and by the lack of visualizability. So that's about as rude as you can get in a footnote in a physics journal, at least at that time. I was discouraged and repelled by my colleagues' work.

Heisenberg was a little less restrained. He wrote just in a private letter to his friend, Wolfgang Pauli, around that same time, just a little correspondence exchange. Heisenberg said, the more I reflect on the physical portion of Schrodinger's theory, the more disgusting I find it. Where Schrodinger writes on the visual ability of his theory, I consider it trash. After all, these waves live in six or more dimensional space. That seems no more visualizable than these abstract matrices.

So this is really a remarkable clash between two people working on very similar topics at the same time and finding each other's work clearly lacking, not just, say, repulsive, disgusting.

And so it was all the more surprising and really unexpected when several other physicists, actually Schrodinger himself and then two others, Pascual Jordan, who was another young physicist about the same age as Heisenberg and Pauli. Jordan was also a young physicist in Gottingen. So he began working directly with Heisenberg to elaborate matrix mechanics.

And then separately still, Paul Dirac, a young British physicist who at this point was on a fellowship in Cambridge, England. All independently within a very short span of time, all three of those physicists demonstrated by the summer of 1926 that these two quite different-looking approaches, Heisenberg's matrix mechanics, Schrodinger's wave mechanics, were actually mathematically equivalent.

You could actually build a mathematical kind of one-to-one mapping to any expression you might try to formulate in matrix mechanics and put that in wave mechanics or vice versa. So it looks disgusting, and repulsive, and un visualizable, one to the other, between Schrodinger and Heisenberg. It looked like these were really just remarkably different ways of trying to carve up the world.

And yet, again, within just a matter of a few months, people had found at least a mathematical bridge between them.

And it was that last work that really made a kind of a map between matrix and wave mechanics that really convinced people already by 1926 to begin referring to a new single thing, a new thing called quantum mechanics, not just matrix mechanics, not just wave mechanics, but now one thing called quantum mechanics and to then start using the phrase, "old quantum theory" to refer to that kind of grab bag of techniques that we looked at that had come before 1925-1926.

So the new quantum mechanics was heralded, really, by that name almost in real time, just remarkable kind of convergence by summer and fall of 1926. So I'm going to pause there. I see a couple of questions come up in the chat. Time for some other discussion. Any questions on that stuff?

So Alex asks, is this is Heligoland the same island the British tried to blow up after World War II? I have no idea. Well, that's news to me. It might be. Wikipedia at least could inform us. I don't know.

Did configuration space ever become generalized to Hilbert space, Fisher asks. Yes, I'm not sure I'd say generalized to separately, and also actually pretty soon afterwards, by people like Paul Dirac began trying to formulate this new quantum mechanics in a more mathematically first principles way, thinking about vectors and vector spaces and so on.

And that comes not only from Dirac but people like Dirac and John von Neumann, among others. So they weren't replacing configuration space per se. But they were certainly trying to think more carefully about the kinds of spaces, the kinds of mathematical spaces in which something like a quantum state might reside.

So if you're thinking about a wave function, that's in configuration space. If you're thinking about a quantum state as a kind of eigenvector in some abstract Hilbert space, then that's the direction that Dirac especially starts to formalize. He writes an extraordinarily influential textbook. In fact, I showed its cover on the previous slide. First edition 1930, that's pretty quick to have a textbook on stuff that was really just a few years old.

And then with the Hilbert spaces, people began to realize these could be of almost arbitrary dimension. So Hilbert spaces could be two dimensions. They could be 10,000 dimensions. They could be infinite dimensional. And so, in that sense, they're similar to configuration space in not being limited to only x, y, and z. But they're not exactly the same kind of thing. It's really people like Paul Dirac, who started pushing very heavily in that direction.

Silu asked, did their opinions on each other's work ever change? That's a good question. I think they stopped calling each other's work disgusting in print. So that's a plus.

And I think Schrodinger himself-- since he was one of the people who did find this mathematical bridge, a kind of equivalence between the two approaches-- Schrodinger, I think, after that began to realize there are kind of conveniences of either approach. So it's not one versus the other, since they could be mathematically mapped.

But as each of you have probably learned just from your problem sets, for certain kinds of questions we might choose to ask, for certain kinds of calculations, one starting approach, one coordinate system, one way of characterizing the balance of forces and all that, that's often more convenient than others, even if, in principle, we could have mapped it into a different format.

That's how people begin to talk more and more about the compare-and-contrast exercise between wave mechanics and matrix mechanics. If they are fundamentally mappable, then use whichever makes the most sense for a given problem, as opposed to say, choose one versus the other.

So Lucas helpfully tells us that Heligoland was the site of one of the largest non-nuclear explosions ever. I had no idea. My goodness, thank you. I didn't know that. Thank you both to Alex and to Lucas. I've never been to Heligoland, obviously hard to travel now, to put it mildly. That would be an awesome field trip that we-- we should at least take a virtual field trip. We can look into that.

Good, so my understanding is Heligoland, it has been contested territory. Again, Wikipedia can tell us who. It's a territory of what nation has been an unstable question, Germany, Denmark, probably others. It seems to sit in the North Sea pretty close to the coasts of both Germany and Denmark. So I knew it's been contested.

And I thought it was also a lovely kind of vacation spot for much of the 19th century. I didn't realize it was subject to such a bombardment. So that's a preview.

As you may have guessed, we're going to be coming to the Second World War in this class pretty soon, actually. So we'll be getting to similar kinds of themes, actually, within about two classes. So keep that theme in mind, I guess. Any other questions about Schrodinger, the wave equation, superposition, discrete eigenvalue spectra? They're fun. If not, then I think I'll press on.

We can look at the next part of class. Of course, obviously please keep the questions coming, if more questions come up. So let's look now at the second part for today.

And again here, I just remind you there is the optional lecture notes that go into this a little bit more explicitly, as well. So let's talk about something called the double-slit experiment which, again, might be something that at least some of you have heard about before. Some of you might have even been able to do a version in junior lab, for that matter. I'm not sure.

This is just a remarkably fruitful experiment. It's the pedagogical gift that keeps on giving. It's not just me who thinks so. Look at this remarkable series of people who have come back time and time again to the double slit.

So Heisenberg lectured on it as early as 1929, perhaps earlier. But that's the first case we know about. Schrodinger was lecturing on this as early as 1936 and maybe earlier. But we have summer school lecture notes from as early as 1936, where Schrodinger featured it, as well.

Niels Bohr featured this very famously in his very well known discussion of his long debate with Albert Einstein over quantum theory. That was another one of the readings for today, was an excerpt, a kind of shortened version of Bohr's very famous characterization of his long debate with Einstein.

Coming closer to our own time in the 1960s, Richard Feynman declared that the double slit has in it the very heart of quantum mechanics. He said this is basically all you need to need to know, practically, about quantum theory.

And even really much more recently, readers of*Physics World* magazine, which as you may know, is the kind of physics today of the UK physics society, they voted this double-slit experiment the single most beautiful experiment in the history of all of physics-- not the history of 20th century physics-- since the Earth cooled. And humans have done experiments to learn about nature.

Readers of *Physics World* magazine concluded the single most beautiful effort of any kind ever was a double-slit experiment. So let's spend at least a few minutes giving proper, well-deserved attention to a double slit.

So here's my very bad cartoon version of the double slit. I'm going to really follow what Richard Feynman's approach in what became in the canonical way of introducing the double slit. I've really lifted much of this from Feynman's very famous version of this in the Feynman lectures from the early '60s. He would have had better pictures. But the ideas are pretty similar.

So let's imagine, first, the behavior of classical particles. These could be baseballs, or basketballs, or bullets fired from a gun, or any kind of projectile. So let's imagine we're firing bullets one at a time toward a bulletproof wall. So this wall here, this thick wall, is meant to be bulletproof, sorry.

And yet there are two narrow slits in the wall. We'll just label those slits, this space here is slit A, this slit, B. The slits are a distance D apart. And then we have some backstop here where the bullets will lodge.

And then, when it's safe to, and the gun's put away, we can go out and count up the number of bullets that landed here versus here versus here. So we're to ultimately try to add up. We're going to make a kind of histogram and plot the number of bullets per location as we march up in this direction of the backstop.

So we want to then figure out, in a sense, the probability distribution where our bullet's most likely to wind up on this backdrop when, either, one slot is open, the other slot is open, or both are open. That's our exercise here with classical objects like baseballs, or in this case bullets fired from a gun.

So let's first imagine that slit A is open but slit B is closed with some firm bullet-proof kind of shielding. So then our number of bullets will look like this. It's been smoothed out. Let's say if we shot 10,000 bullets, then it would actually be a pretty smooth curve. It would be basically a Gaussian. It would look like a kind of bell-shaped curve centered around the open slit.

Most bullets would be found directly behind the open slit. But there'd be some modest scatter, some width to the distribution, so to speak.

Likewise, if we then sealed up slit A with some bullet-proof shielding, and opened only slit B, and performed and shot another 10,000 bullets at it one at a time, and then did the same counting exercise, we could make a separate histogram. It looks remarkably similar, not surprising. It's a symmetrical problem.

So if only slit B is open, then the vast majority of bullets are found directly behind that open slit. And there'll be some kind of bell-shaped curve, basically kind of Gaussian distribution. On either side there'll be some modest scatter.

And then, if we then open both slits, fire another 10,000 bullets at the bulletproof wall with both slits A and B open, do the same exercise and make our histogram, the resulting distribution is just the sum of the previous two. So now there are a fair number of bullets that wind up in the middle because some are coming from the tail of this distribution. Some are coming from the tail of this distribution.

And so some of them wind up in the middle. It's the sum of those two. But, nonetheless, the distribution is clearly still peaked in the positions directly behind the two open slits. So we have the resulting probability distribution. The histogram, when we add up when both slits are open, really is just the sum of those two separate ones.

And so for classical particles, these probability distributions are of independent events. And therefore the probabilities in this case simply add. . We can get the probability distribution for the combined series, slits A and B open, simply by adding two independent possibilities.

Each bullet either went through slit A, in which case it followed this distribution. Or each bullet, or an individual bullet, went through slit B. Add up those two options. Here's our resulting distribution, not too surprising.

Now let's try a second version with waves. And this could be macroscopic waves, for example, ocean waves. In my little write up, I talk about doing this near the coast of Australia with waves approaching the Great Barrier Reef. But it could be even in an artificial wave pool with some barrier.

So now we have some reef or some structure that can block waves that are coming from further away. They're approaching. Here's the shore. And we have some barrier that will block waves from passing through here. But the barrier has these two narrow slits which, again, we can label A and B. And they're a distance D apart. So now what we want to do is measure the intensity of the waves that lap up on shore. Clearly some water gets through slits A and B.

So what's the pattern of waves that we find on the shoreline? And how does the intensity of that wave vary as we march along in location? And, again, we'll project the positions of slits A and B on the shore and ask about the varying intensity of the resulting wave as a function of distance or location.

So once again, let's start with only one of those slits open. So we're going to close slit B with some kind of dam. We're going to block one of those slits so water can only get through that one narrow slit bay. And therefore we'll have a clearly peaked distribution.

The wave that results, that we find on shore, is virtually zero, far away from that location of slit A, the projection onto the shore of the open slit. It's very clearly peaked, right directly behind the open slit. In fact, we can characterize the intensity of the wave as the square of some amplitude. It's a wave. It has some amplitude and some phase.

And the intensity goes clearly as the square of this height, of this amplitude. And it basically is very clearly dramatically peaked behind the open slit. Now we place the dam in front of slit A, open slit B, same kind of story.

But as many of you probably know, once we open both slits A and B, we get a totally different story. Now we get one of these characteristic interference patterns on the shore, not merely the sum of these two distributions together. And if we do a little more work, we can realize we can still characterize the resulting intensity by summing something together, not by summing the two intensities, actually by summing the two amplitudes first and then squaring that.

And that's what allows for these interference patterns, much like we saw with the standing waves earlier. In regions where two crests happen to align, we'll get a sharp constructive interference, a heightened peak, in regions where there might have been one large peak. But a very small trough on the other side will get destructive interference.

So in fact, when we add the amplitudes and then square, the resulting intensity looks nothing like the sum of the two intensities because that would overlook, that would discount all these ways that the waves can actually interfere with each other. And one indication of that is this characteristic feature of the interference pattern.

The point on the shore with the largest intensity is a point between either two slits, whereas when either slit is open alone, that's a point of actually virtually vanishing intensity. So the greatest intensity is at a spot where neither scenario on its own had much power at all.

And so, again, this is a characteristic of classical wave behavior, that the intensity goes as the square of the sum, not the sum of the squares. And since the amplitudes can be both positive and negative, they can even be complex numbers. They can be all kinds of situations in which the sum in which the square of the sum is quite different than the sum of the squares and, in particular, has this characteristic wave-like interference pattern. Again, probably pretty familiar.

So now what happens when we now move to the quantum realm? Now let's imagine some cathode ray like JJ Thomson had, or of a sort that was by this point quite familiar, even by the 1920s. Things like what Germer and Davisson used in 1927 to think about de Broglie waves. This was already literally a feasible experiment as early as the mid 1920s. And people tried to do it.

So let's imagine shooting out single electrons one at a time from some source, some cathode ray, let's say, some electron gun. We're going to send one electron at a time. We're not going to have a continuous spray of them. We'll send one, shut down the device. Let it cool. Go out for a coffee. Come back an hour later, send out one more electron. We can really take our time.

So at any given moment, there's only one electron in play in the device. We're going to aim those one-at-a time electrons at a device. This is, by the way, taken-- this illustration is taken from Bohr's essay about his debates with Einstein. You hopefully recognize it from your reader.

So we're going to send these electrons one at a time toward a device that has two slits. The electrons will be blocked if they hit anywhere here. But they can get through either this slit or that slit. And we have the opportunity to open or close each slit independently. And then behind the wall with two slits, we have an array of very closely-spaced detectors, electron detectors.

And so we can measure with very good resolution where electrons are detected as a function of position along this kind of backstop, as array of detectors. We also can arrange things So that the distance between the two slits, the distance little D, is 10,000 times larger than the characteristic waviness scale, the de Broglie wavelength of those electrons.

So the electron, if it's approaching, if it's anywhere near this slit, it should be 10,000 electron distances away from that slit and vice versa. It should have a localized behavior as it encounters the slits. Moreover, each of the detectors is going to detect the location of electron with a very, very small region of space.

So we're not going to expect to find a splayed out wave of electrons. Each one is detected as a point-like discrete particulate detection because we've these very high resolution detectors in the back.

So we're going to release one electron at a time. Wait one hour between and do that 10,000 times. Which, as I like to say, is why we actually have graduate students. Who's going to have patience for that, with many thanks to our teaching systems. So this is going to take months. But we care about the nature of the universe. And so we're going to just go plow through and do this.

Now we're going to do the same kind of accounting exercise as we've done with both the classical particles and with the classical waves. We're going to make a kind of histogram of the number of electrons that are found in each of these bins, each of these detector bins, as a function of location.

So once again, we'll start by keeping one slit closed so the electrons can only pass through the other slit. So if only slit A is open, we get this very strongly peaked distribution directly behind the open slit, just like with the bullets. If slit A is closed but only slit B is open, again, a very strongly peaked distribution.

And so it looks a lot like the classical particles. In fact, if you've taken junior lab, you probably would learn to say those little wiggles are clearly instrumental error, that statistical insignificance. They never rise above about 1% of the central peak.

It really looks like electrons are behaving like bullets. They're each emitted like a tiny little discrete particle. They're encountering one slit or the other because we've arranged the distance to be so extreme. They're all being detected as one little blip on that detector screen. They look like little baseballs or bullets.

And yet, when we then look at the pattern when both slits are open but only one electron went through at a time, we get right back to this very familiar wave pattern. How this can happen has kept people up at night for, now, very nearly 100 years. And I hope we're not too blase about it even today.

It looks like particles when only one slit's open. It looks like waves when both slits are open. And yet we've been super careful to only send one electron through at a time. Unlike a water wave is clearly an extended continuous object, clearly the wave can interfere with itself. It is literally an extended object which has many moving parts.

These electrons are tiny little pellets. We emit them as little discrete bundles. We only have one in play at a time. It's not like two electrons could have influenced each other by repelling each other and made some kind of characteristic pattern because they were interacting with each other. We sent only one through at a time and waited a whole hour in between.

We've done that 10,000 individual times. And yet the pattern that builds up, one individual point-like detection at a time, is this undeniable, very familiar wave pattern where, much as we would expect now from the ocean waves, the resulting probability distribution goes like the square of the sum and definitely not like the sum of the squares.

It is not the sum of this distribution plus that one. It is absolutely emphatically not like 10,000 baseballs thrown at a wall with two holes, or bullets shot at some barrier. It looks just like the water wave, even though this built up over time from 10,000 independent events.

Now, if I were to unmute you all, I should hear you screaming with either joy and ecstasy or real deep despair. Either way this should not be an emotionally neutral statement.

So what comes from all this is Max Born, the Gottingen theorist, a contemporary of Einstein and Schrodinger, Heisenberg's mentor who taught mentor that these were matrices, Max Born then suggests from exactly these kinds of reasoning as early as the summer of 1926 that what is this wave PSI, the thing that is a solution to Schrodinger's wave equation?

It seems to be not a physical bit of the electron. It's not some electron density that's spread out in space. It seems to be related to the likely behavior of the associated particle. It's not a physical bit of the particle. It's a description of the way that particle is likely to behave.

In particular, it's what comes to be called a probability amplitude, not the probability, not the peak that has this value on our histogram but actually the object whose absolute square gives rise to this thick black-line pattern.

So the probability, Born says-- we now call this Born's interpretation-- the probability to find an electron here versus here versus here goes like the absolute square of this wave-like quantity, Schrodinger's wave function. That's what gets worked out within six months of Schrodinger's first publication on this by thinking about what is actually observable in experiments involving these wave-like features.

So, again, that was a thought experiment in 1926. By 1920-- literally by 1927, there were efforts to try to do this experimentally. Nowadays we have very fancy equipment with high-precision sources and detectors. So here's a series of images courtesy from my colleagues, Robert Austin and Lyman Page at Princeton, this is not something we can do as an in-class demo or as an undergraduate laboratory.

So this is real data. This isn't simulated. These are snapshots of the kind of fluorescent screen of an actual double-slit experiment done at different moments in time as the pattern builds up. So individual photons were fired out of a screen with two slits, literally one at a time. And now we can do single photon experiments, which is quite extraordinary. And then you can detect where each individual photon was detected.

So these are little pinpricks of light where an individual photon interacted with the fluorescent photo detector. So you have a mostly blackened screen because we've only sent a couple of photons through, one at a time so far, very localized individual detections here and here, but not here.

That's after the first 1/30 of a second that's shooting rapidly one photon at a time and only one at a time. After a whole second has gone by, you have accumulating pattern. Now you have several hundred photons individually detected.

Wait 100 seconds and you see very clearly this wave-like interference pattern, exactly the pattern described by that Born interpretation even though this is built from thousands of otherwise independents located in time and space detections of individual quanta.

So we shoot out individual quanta, particulate-like. We detect individual quanta, particulate-like. And they can't interfere with each other because we make sure only one's sort of in the device at a time. And yet, over time, they somehow build up this very characteristic wave-like interference pattern. That is what's called the double-slit experiment. So let pause there.

How do we go from one electron every hour to 100 every second? Oh, good. So, Alex, good. So the data I just showed you is actually with photons, the Princeton group. One can do it with electrons, as well. They were using, let's say, a photon source that emits single photons rapidly. So they really were shooting out only one photon at a time. But they can produce many photons per second.

So the timing when we're dealing with that particular source of individual quanta, just what's called the duty cycle is shorter. We also have very rapid electronics now, unlike 1927. So the electronics can actually resolve individual detections in a tiny fraction of a second.

So, conceptually, it's as if we had gone back to my cartoon, turned on our machine, shot out one particle, turn it off, went away. It's just now we can do that with the kind of high-duty cycle rapid electronics. But, conceptually, it to my mind is quite similar.

Vittorio asks, as you do the experiment with progressively heavier particles-- oh, good question-- is there a certain point where the resulting pattern begins to look like what you'd expect from two bullets? Oh, thank you, Vittorio. What a wonderful question. That is a question that is still occupying, literally, many of my friends, let along colleagues around the world. The short answer is yes. The longer answer is why and how.

And Jade's quite right. Jade puts in the chat, this experiment has been done, a comparable experiment, with objects as large as buckyballs, C60, molecules of 60 carbon atoms in one bound state, not just individual electrons.

So an object as large as a buckyball which is, I think, not a billion, but several tens of millions of times larger than an individual electron, we still find this characteristic quantum interference very clearly. Again, it's well within statistical significance. And, in fact, that's an experiment that one of my colleagues, Anton Zeilinger, and his group had done quite some time ago.

So the question is, where does that stop? I had a very beloved physics professor of my own, when I was an undergraduate. He was very senior. He's actually just retired. He was still teaching just for fun. And he used to joke that when he was in graduate school, he and his buddies would wonder, could they get themselves to interfere if they rode a bicycle through two standing posts sufficiently quickly, something when they don't interfere with themselves.

An individual human on a bicycle doesn't undergo, at least not in any obvious way, this kind of interference. It doesn't seem to happen with things we can shrink down and look at microscopically at the scale of, say, amoebas, which people have tried. But it does happen with enormous molecules, enormous on the size of parts of atoms. So where is that line? And what causes the transition in behavior?

If we're all made up of atoms and electrons, and electrons and atoms behave in this fundamentally quantum mechanical way, why don't we? Why didn't my professor on his bicycle get diffracted when he biked through between two standing posts? So that's what's often now called the quantum to classical transition.

If you have nothing better to do with yourself for the next 17 years, I suggest you start googling that and read every paper on it. It's enormously complicated and still very much on the forefront of the research frontier for many people. And we just genuinely, we collectively, the community genuinely doesn't know.

We're getting better at characterizing where the break is, where the line is, but not why, and not what would cause the behavior to transition from one very specific kind of behavior to the other.

So other colleagues are trying to do this with living objects. So it turns out viruses, some kinds of viruses, might be just small enough for this to work and yet much, much bigger than buckyballs. So people are trying to get living things, seemingly living things like viruses, to quantum interfere. That's not been done yet. But that's one thing people are trying.

So this question, how big could it get and still show this inherent quantumness? That is still very much a live issue. And as I said, there's two facets to it. What's the scale? But even more subtly, what's the reason why that stops working as we continue considering larger objects? So thank you for that question. Great question.

Any other questions about the double slit, about this, what should have been the scream. I mean, all of you should have just been unmuted and just howling at the moon. At least that's how I feel. And I still think about this. Maybe that's my problem. I just find that just astonishing.

It's not just a thought experiment. We can do that now with individual quanta with super careful control on our source, on our detectors, have really extraordinarily high statistical significance on all the measurements, and fire what looked, for, all intents and purposes, to be like little tiny baseballs. And somehow they know. Somehow their behavior is guided in a way that baseballs and bullets just aren't, or basketballs.

If that's just OK for all of you then, I mean, come on. That's just like, that's COVID speaking. That should just-- OK, you get the point. Any other questions on either my mental state, or emotional state, or on the double-slit experiment? If not we'll go into one last little part, one kind of CODA for double slit. And then we'll have time for questions, as well.

OK, so I'll get it together. I'll pull myself together. I'll talk now about this last part, which also started being thought about very early, even in the late 1920s, as further ways to try to probe what's going on when we think about this quantum level. So, again, this I'm going to do, especially schematically in the class session. There is a bit more explicit discussion in the optional lecture notes on the course site.

Let's go back to that same setup, the one we just ended on. Let's say we're sending one electron at a time, waiting ample time in between, firing them toward this system where the distance between slits A and B is much, much larger than the kind of characteristic size of the particles themselves.

But now we want to say these particles clearly must have gone through either slit A or slit B. That seems like a reasonable question to ponder. Which slit do they pass through on any individual experimental run? The distance between the slits is so much larger than the characteristic size that each particle must have had to have chosen to go through either slit A or slit B. So why don't we try to find out?

And this is a very clever idea. Heisenberg's lectures on this, even in those 1929 lectures-- and others have developed it since then. Let's put a bunch of test particles behind only one of the slits so now it'll be like an alarm system. If an electron happens to go through slit A, it'll smack into these test particles. We'll have collisions. We'll have scattered particles zooming out of this region of the experiment that we could then detect and measure.

And if no test particles are scattered, then we know the electron must have gone through the other slit. So we're going to bunch together, get kind of close in space, some targets behind only one of the two open slits. And that way, we have what's called a slit detector. We have some mechanism of determining event by event through which slit the particle actually passed.

And so now what happens when both slits are open, but we measure through these slit detectors through which slit each electron passed, then the distribution, even for these quantum mechanical electrons, the distribution reverts back to this thoroughly classical sum of two independent probabilities.

So we do our usual experiment. We do this 10,000 times. Sometimes we see the scattered particles from behind slit A. Sometimes we see no scattered particles. It must have gone through slit B. And then separately from that, we count up where all the electrons actually wind up at that backstop with our closely-spaced detectors.

And we get, basically, the classical distribution. It looks like the sum of two independent classical particle-like distributions. When we ask through which slit each individual particle passed, the particles give us a particle-like answer. They pass through either slit A or slit B and had correspondingly no interference pattern between those two options.

On the other hand, when we don't even try to ask through which slit did those individual particles pass, we remove that slit detector altogether. Then we get back to the results that we'd already found, that when both slits are open, we get this characteristic wave-like interference pattern. And we only get that when we've given up even the opportunity to ask this particle-like question about through which slit had individual particles passed.

So we are allowed to ask the question. We actually can ask a question. People have done it with a slightly more sophisticated version than just that cartoon. And yet, somehow, the nature of the results changes very deeply and fundamentally, based on the nature of the question we asked. So, again, you can see why that happens.

I'm being pretty schematic and kind of waving my hands. It's only a couple lines of algebra. And you might have already guessed it. If I go back to the cartoon, just to say quickly what's going to happen here, in order for this to be a reliable slit detector, this bundle of particles has to really be behind A and not behind B. That means that the spread in space can't be arbitrarily large.

If we call this vertical direction the y direction, like my arrow here shows, then the delta y over this distribution can't be too big. In fact, it has to be much smaller than the distance D. If the distribution were too big, then we could see that test particles were scattered into-- but they would no longer tell us whether the particle had gone through slit A or slit B.

We would lose the resolution in space if the bundle of particles was too far spread out in the y direction. So we have to contain or compress the distribution of test particles within a small region delta y.

But that means they must have a correspondingly large uncertainty in the momentum because of the uncertainty principle. And, therefore, there'll be a correspondingly large uncertainty in the recoil momentum of the electron after scattering of exactly the right amount-- this part is kind of amazing-- of exactly the right amount needed to perfectly wash out those very clear interference fringes to get back to two bell-shaped curves, so that very regular, very narrow peaks of the interference fringes.

So it's literally the uncertainty principle that precludes being able to both measure through which slit an individual particle passed and also retain these very clear features of the interference pattern. I think it's pretty amazing and very beautiful. And it's only a couple lines of algebra to show. So if even this most recent hand-waving effort of mine was too quick, I encourage you to look at the lecture notes. But that's the upshot.

So what we're really finding conceptually-- and this is what people began to converge around, even by the late 1920s-- was that if we ask a particle-like question such as, what was your path? Through which slit did an individual particle pass? We will get a definite answer. We will get a particle like answer. Each particle either went through slit A or went through slit B with corresponding particle-like statistics. It's all consistent.

We'll get a clear answer. And the whole rest of the problem will be consistent with particle-like questions and answers.

On the other hand, if we ask a wave-like question, how does this wave function evolve in the region between the slits and the detector, for example, then we'll get a wave-like answer, the way the probability amplitude, the Schrodinger wave function, will spread out in space, in some regions they'll be constructive interference and the likelihood to find electron will be higher.

In other regions of space you'll get destructive interference, corresponding to lower likelihood for an electron to be found. But we only get wave-like answers when we ask wave-like questions.

So let me summarize. And we'll have a few moments for some discussion and questions. So in a really quite limited span of time, really in less than a whole calendar year between spring of 1925 and spring of 1926, both Werner Heisenberg and separately, Erwin Schrodinger, introduced whole new approaches to what they were seeking, a kind of first principles theory of the quantum realm, a quantum mechanics, rather than this grab bag of ad hoc quantum conditions.

They were looking for this quantitative description of the atomic realm that had certain quantum ideas built in from the start, discrete spectral lines from excited atoms, de Broglie wave, matter waves, and so on, rather than kind of stapling these on at the end. Now these approaches looked quite different at first and even kind of not very encouraging, to be polite about it, to each other's inventors.

Heisenberg's approach really emphasized and built upon what he considered a kind of inherent discreteness. That was the main lesson of the quantum as far as Heisenberg was concerned. Schrodinger, in the meantime, built separately on de Broglie's suggestion. The real lesson is a surprising continua, that even solid matter like electrons have associated wavelike behavior. They're building on different kinds of conceptual building blocks.

And yet, by the summer of 1926, several physicists had built this mathematical map between these very, very different looking approaches. And even though there was now, arguably, one internally consistent mathematical approach, what it all meant about the world was as unclear then, often, as it remains even to this day.

So one question was, what's this wave? The wave function is now an object that can be manipulated using familiar techniques. It's a differential equation. We can understand the spectrum of allowable eigenvalues. But what is it actually telling us about the world?

And with Max Born's suggestion in the summer of 1926, physicists pretty quickly came to the idea that the wave function is not telling us about the physical extent of an object in space but actually about the likelihood for events to happen. And in particular, the absolute square of this wave-like quantity should give us the probability.

Quickly on the heels of that, less than a whole year later by spring of '27, Heisenberg had derived his now famous uncertainty principle, that there are these unavoidable trade offs in the precision with which we could ever hope to specify characteristics of the kind of quantum realm, a kind of pairwise trade off where we can make one quantity really sharp but only at the expense of making some corresponding or complementary quantity very imprecise.

And the trade off, the seesaw, again, being controlled by Planck's constant. And then these kinds of broader conceptual dualities, like this wave particle duality we've seen now a few times. But the type of answer we could ever hope to find depends on the type of question we ask.

So it's not that the electron really is a particle or a wave, that it's a kind of contextual question. Then what kind of properties do we hope to investigate? Then we're going to foreclose some answers from ever even being possible, even as we get other quite definite answers.

So Niels Bohr, partly from his intense discussions with Heisenberg, with other colleagues, he used that to enunciate a very general philosophy of the quantum realm by September of 1927, right on the heels of this, which he came to call complementarity.

So it's not an either or world. It's a both and. We are required, in this case, to draw on both ideas and actual mathematical techniques with which we characterize wave behavior, and ideas and techniques with which we characterize particle behavior. They are mutually exclusive. We literally can't use them at the same time. And yet we need both. A complete description requires both, even though they don't fit together at once.

And that, to Bohr, was the deepest meaning, the deepest lesson of quantum theory. Not everyone agreed. But that's where Bohr arrived by 1927. OK, any questions on any of that? That is so straightforward, so boringly familiar, that there's not even a single question to ask about the fundamental nature of reality. Boy, holiday weekends. No.

Well, if you'll permit me, on tomorrow's class I'm going to try to shake your quantum complacency even further. So tomorrow we'll explore quantum weirdness in even more directions. So if this stuff is humdrum and familiar, give me one more shot for tomorrow.

So Fisher asks, was Schrodinger's work more quickly accepted because it was more basic mechanics? I think that's absolutely right, Fisher. And I think there was a kind of-- people could do stuff with it. So even while the concepts were still very far from clear, what's the wave function? What is a wave of? What is it telling us? That was by no means clear. And even after the answers that we would now recognize were put forward, those didn't claim consensus right away.

And yet, in between, people could still do stuff. You could train PhD students to look at this or that specific question because the apparatus, the mathematical apparatus, was already something they'd practiced since their youngest, earliest studies. So I think that really had a lot to do with how rapidly Schrodinger's work was really welcomed and that it helped. Obviously it helped a lot when Schrodinger, and Jordan, and Dirac elucidated the kind of mathematical mapping.

So then it really became a question of convenience and not either/or. It's not like you had to vote for team Schrodinger versus team Heisenberg. Now you could say, oh, this set of techniques is familiar to me. I have lots of experience from classical mechanics or Maxwell or whatever.

And so then it became a kind of tool of choice. And these tools were much more familiar to many more people at the time. That absolutely played a role in hastening many people's attention to this stuff, good, good point.

Aiden asks, is Heisenberg's principle kind of spiritual? For some people, yes. Although that gets-- there's a lot to say about that. Are there those who think a more literal analysis would be that we simply need to discover from observation, not involve test particles?

So, Aiden, I thought you were going a different way at first. So one topic, which is-- I'll say controversial. Some people pursue it. Other people think it is full of holes. One idea would be to say, does the uncertainty principle give us a physical reason why there might be something like free will-- consciousness and the experience of free will?

You can see part of the concern, going back to the days of both Newton and, for that matter, Maxwell. If we live in a universe in which determinism holds perfectly, if events in the future are uniquely determined by events in the past, and then we turn on our sci-fi imaginations and imagine we're made of matter, atoms, and molecules.

If atoms and molecules have to obey the laws of nature then, in principle, they are subject to determinism, at least in a pre quantum mechanical way. Then aren't we all just pre-programmed? Aren't we merely machines with no more free will than comets who can't help but follow a Keplerian orbit as they orbit the sun? And believe it or not, that was a debate that got people very excited and exorcized and mad at each other for a long, long time.

And so there was some thought by some people early on, starting, really, in the 1920s and '30s, that maybe the uncertainty principle, by breaking the iron hold of determinism, leaves a kind of space within our scientific theorizing for things that seemed, or at least some might have hoped, were not fully uniquely determined, whether that's our own sense of individuality or the course of human events more broadly and so on.

Now that gets a little tricky because, unfortunately, the uncertainty principle argues, at least most physicists would say, the upside of the uncertainty principle is that everything's random. And that's not quite how my thoughts feel. Before COVID 19, my thoughts didn't always feel only random, now they do.

So appealing to the certain principle breaks the hold of determinism. But what it seems to put in its place is random mush, as opposed to something that otherwise might answer to the kind of lived experience of phenomena of something like free will. So it's not like that's a single answer that that's done.

But that opens a space for kind of theorizing that goes on to this day, sometimes on the margins of both kind of physics and also of areas like neuroscience and philosophy. But at least it opens a different way of asking legitimate, hard questions with some new conceptual inputs. Let me put it that way.

So even though I don't think people have found that answer yet, it's interesting that opens a space to ask old questions, maybe in some new ways. I wrote a book a couple of years ago called *How The Hippies Saved Physics,* which is a very obnoxious title, where people thought maybe there were other kind of unexplained or unaccounted for phenomena that quantum theory might be able to account for, like claims of ESP, or other so-called occult phenomena, mind reading, for example.

And, again, I don't think they explained that at all. But it was a way of trying to reason about strange or unexpected human-scaled phenomena using a new set of tools that really weren't going to go away, like the uncertainty principle or like things we'll talk about in tomorrow's class, things like quantum entanglement.

So I'm not telling you that's the answer. I'm telling you that opened up a space for people to ask either familiar questions from unfamiliar perspectives or sometimes ask new kinds of questions. So in that sense, what's the ultimate meaning of the uncertainty principle? I don't know. And, frankly, I don't think anyone knows. Or at least it's still a subject of really robust debate, like that quantum-to-classical transition. And I think it's fantastic that we have tools to even pose those questions now.

So I get excited by the nature of the question rather than having a favorite answer. That's just me proselytizing. So Richard asks, once general relativity was widely accepted, were people already ringing alarm bells about the conflicts between general relativity and quantum mechanics. Yes, yes, they were, including Albert Einstein himself, who had a few opinions on the matter.

And so this goes back in large part to things like the fall of determinism. And we'll come to, actually, some of these examples even in tomorrow's class. So we'll come to some of Einstein's most enduring critiques of quantum theory in the light of his thinking about things like relativity tomorrow. So partly I'll just bracket that. And I'll just simply say, yes, it's a great question. We'll see some examples tomorrow. We could talk more

Lulu asks, how's my mental and emotional state? I think you can see, I'm still agitated I love this stuff. And tomorrow I won't be able to contain myself. I'll talk about some of my own group's research in the foundations of quantum mechanics. So good, I mean, not that the research is so good. The question is so good.

OK, Aiden says, does Heisenberg's principle mean there's neither freewill nor determinism? So what we can do is recognize that now, as a question with which we, as a community, now have new tools to try to puzzle through answers. I'll put it that way.

I mean, one of the things that happens is people try to explain certain esoteric features of consciousness using quantum theory. But then it turns out that human brains are dense, hot, and wet. And a lot of these quantum effects work really well when we're near vacuum at low temperatures and super dry. So even though some things could happen, in principle, among quantum objects, having to maintain a kind of inherent quantumness is really hard in an environment like the human brain.

So, again, that launches new kinds of questions and investigations, rather than saying, oh, we've solved it. So those things become really interesting and lots of smart people ask a hard question about them around the world because we have this new injection of new conceptual things to really wonder about.

Let me pause there. I don't want to run too long. Remember we have class tomorrow, twice in a row this week. We're back in our ordinary Zoom link. You can find it through Canvas. And so tomorrow we'll sit a little bit longer with some of these weird and wacky quantum phenomena. So stay well, see you tomorrow. Talk to you soon.