[SQUEAKING] [RUSTLING] [CLICKING]

DAVID KAISER: With today's class, we're now transitioning into the last main kind of unit thematically for the course. And so the last main section is trying to follow one set of threads, a very set of ideas and developments weaving from the end of the Second World War toward more recent time periods.

And I would joke and say we're going to follow what our objectively the single most important developments in the history of human thought. That might not be how everyone reads it. There are things that I find actually really cool and exciting, so that's why I chose them. I'll just be honest, this is my favorite.

And so this last unit, we're going to be looking at some examples within-- trends within postwar physics, looking in particular at a merger that gets cobbled together between high-energy physics, nuclear and particle physics, and certain ideas about astrophysics and cosmology. So there are many, many other exciting developments then and now.

Of course, I actually don't mean these are the only important ones, but these ideas are themselves, I think, still very exciting, and they also offer a way to trace through some of the further embedding of very particular sometimes very esoteric-sounding ideas in concrete-changing institutions, social contexts, political features, and all the rest.

So we can read these developments in high-energy theory and cosmology and astrophysics as taking place in shifting contexts by real people, much as we work so hard to do with the first and the earlier sections of the class as well. So that's our we're going to launch in now for our last main unit. And again, if you have any questions, of course, please don't hesitate to reach out to me.

So I think I'll go ahead and start sharing screen, and we'll launch into this last part of our adventure. So today, we're going to focus on some developments that would begin to coalesce very soon after the Second World War into what came to be called high-energy physics. That wasn't quite what people called it at the time, but we recognize it today as heading toward things like particle physics or high-energy phenomena.

And we're going to see that this wasn't-- as you might expect, this work was not unfolding in a vacuum. In fact, it was tethered in some surprising ways to developments during the Second World War. So it really does, we see again, a pivot from that middle unit of the course as we head toward the last unit.

So our three main steps for today, we're going to actually turn the clock back a bit. We're going to go back to the early days of quantum theory, even before the Second World War, and we'll see that there was a lot of work being done in the 1920s and 30 seconds.

And then for the second and third parts of today's class, we're going to see how some of those efforts were spun in new directions, how they got a kind of unexpected series of inputs or even jolts from many physicists' experience during the Second World War. So that's our plan for today.

So if you remember from not too, too long ago in our class, we were looking at what came together in rapid order around 1925 and '26 that came to be called quantum mechanics. We saw Heisenberg's work on what came to be known as matrix mechanics, and very soon after that, Erwin Schrodinger began working on wave mechanics. And before too long, several people had shown that these two very different-looking approaches actually were mathematically relatable. So we had something called quantum mechanics in hand really by 1926, '27.

Right on the heels of those developments, many of those same physicists began right away trying to apply these new ideas and the new formalism to radiation. Not just to say the structure of an atom, like the Bohr atom as it gets rethought en route to things like Schrodinger's approach to hydrogen, but actually trying to apply some of those same conceptual ideas even to Maxwell's treatment of light, to electromagnetic radiation.

After all, as we did see a number of weeks ago, Einstein himself had suggested that light might consist of light quanta that, by this point, came to be called photons, Einstein suggested that, as we saw, all the way back in 1905.

By 1927, by early 1927, Paul Dirac, the British physicist, some of whose work we looked at recently, he actually proposed a way to try to reconcile quantum theory with Maxwellian waves, with electromagnetic fields as early as 1927. This is where many historians now date the earliest stirrings of quantum field theory.

In particular, Dirac's work came to be called Quantum Electrodynamics, or QED. It was a quantum treatment of Maxwellian electrodynamics. And you can see the first main article which really launches this was submitted already in February of 1927, quite early in the study of quantum theory more generally.

So again, as we've seen many times, you would have seen this even before well before this course, physicists, including very mathematically adept ones like Paul Dirac, they'd known for a long, long time that when we have a arbitrary, very complicated-looking curve-- let's say some function that has some very strange-looking pattern to it, we can always decompose that arbitrary curve or that funny-looking curve thanks to Fourier analysis and treat it as the sum or the superposition of very regular waves, very purely periodic waves, of varying wavelength or frequency.

And as long as we get the weights right-- 2 parts of this, 3 plus 4i parts of that, the weights could be complex numbers. But in general, if we weight these kind of basis vectors or waves of known properties and we tune the weights very carefully, then we can actually reproduce virtually any arbitrary-looking curve, and that's the foundation of Fourier analysis going back to the 18th century. That part is not new.

What Dirac wondered was, could something like that be used, even in this quantum realm, for a funny-looking wave or a field extended in space like a Maxwellian electromagnetic field? Now his first thought, as early as 1927, was that the approach won't be merely or only Fourier analysis.

After all, even Heisenberg had found, much to his initial confusion, that when trying to deal with these quantum mechanical ideas, certain quantities no longer obeyed ordinary rules, say, of multiplication, of these so-called commutators or commutation relations kept popping up where the product of two quantities would depend on the order in which they were multiplied. An x times p was not the same as p times x. And in fact, the difference between those two scaled with Planck's constant was an indication there was something inherently quantum mechanical going on.

So Dirac realized-- or suggested-- that something like that must be going on with these quantum fields as well. And so it won't just be a Fourier analysis. You have to get some of this order of operations structure to come in, but he wanted to keep as much as possible of this quite familiar Fourier approach.

And so he began introducing this notion of fields extended through space and time, and did, going all the way back to Michael Faraday, which we looked at in some of the very earliest classes this term, but having to do a little tweak. It wouldn't just be a Faraday or a Maxwell field distributed through space changing over time, it would have a little structure-- so a little added structure to take into account this kind of new quantumness, let's say.

The fact that these quantities would obey sometimes some funny commutation relations and all that. So he wanted to get as close as he could to Fourier analysis while recognizing that some things might need a little bit extra mathematical structure.

So if we look at this expression, the new parts that Dirac has added really can be summarized in this a and this thing that's called a dagger. These are clearly related to each other. They're complex conjugates of each other, they're very similar to that. And in Dirac's hands, these things weren't only numbers that were kind like an ordinary Fourier analysis, these were actually quantum operators. They did stuff or represented specific kinds of changes to the physical system.

So the one over here that is called a dagger, that could be interpreted, Dirac suggested, as creating one quantum, one photon in a very particular state, a state of given energy and momentum. And this a with a hat but with no dagger would be interpreted as taking away, as annihilating or absorbing one photon in a definite state of energy and momentum.

And so-- and then you'd have different weights in front of each of those processes. So you have standard Fourier modes, just like in ordinary 18th century mathematics. These would be ways of representing, say, a photon in a state of definite momentum. You would have this kind of accounting to say, did you add or subtract a specific quantized excitation of that field? That's what the a and a dagger show.

And then again, you have something more like Fourier weights. How many of those do you add in to get back your arbitrary field through space and time? So if we do this kind of accounting and it's clearly inspired by ordinary Fourier analysis.

Then Dirac showed we could represent the state of this field-- let's say it's the Maxwellian electromagnetic field, you can represent it as a collection of these quanta in various well-defined states. The field was nothing other than a collection of quantum particles in states of definite energy momentum.

And then if a field changed over space or over time, you could interpret that as changing your Fourier weights, as either emitting or absorbing quanta in these specific states. So it's a way of just doing accounting so far.

Not too long after that, though-- really, just weeks after Dirac submitted this first paper, Heisenberg introduced his uncertainty principle, and not long after that, spring/summer of 1927, a number of people began trying to these ideas together. Put together Dirac's accounting scheme to represent some field extended through space and time as a collection of quanta in particular states, but then to add in this very critical caveat from quantum physics of the uncertainty principle.

The uncertainty principle that we looked at together briefly in class a number of weeks ago concerned a trade-off between position and momentum along that direction in space, say delta x and delta p. It was very quickly shown by people like Heisenberg right around the same time. There were other pairs of quantities that had similar trade-offs that also obeyed an uncertainty principle relationship, one of the most important being the energy involved in a given physical process and the time over which that process unfolded.

So if these two quantities could not be precisely specified at the same time to arbitrary precision, then much like position and momentum, there would be this inevitable trade-off.

And in the context of Dirac's kind of Fourier-inspired accounting trick where the field could be decomposed as collections of quanta in various states, the way many people began to interpret the uncertainty principle in the light of Dirac's formalism was that certain quanta, certain excitations of that field could temporarily break the rules.

They could temporarily violate the conservation of energy by borrowing some energy, delta E, as long as they paid it back sufficiently quickly so that there was a trade-off between the precision with which the energy of that state could be specified at a given moment in time and the time over which these conservation-violating processes could unfold.

So again, if you go back to Dirac's accounting scheme, that suggested to many people, again, as early as 1927 and 1928, very early in this game, that the exact state of a quantum field at any given time could never be stipulated to arbitrary precision. There would always exist what came to be called quantum fluctuations.

These were associated with something called virtual particles. Particles that were in some state of energy and momentum, but they had-- they were like-- they had broken the rules temporarily. They were virtual particles by exploiting this energy uncertainty relation to borrow energy to be in a particular state briefly and then pay it back correspondingly quickly.

The first time I heard about this, I was actually in high school. I was really enamored of these popular books by the writer Isaac Asimov. I'm sure of you know Asimov's writings, including his famous fiction novels, science fiction novels. He wrote a number, I think, really very nice popular books-- nonfiction books as well.

And I remember coming across this idea in this book here, part of his series called *Understanding Physics.* And he had, I think, a really lovely metaphor for this notion of virtual processes that could temporarily violate the conservation of energy, and he described them that they're really kind of like misbehaving schoolchildren.

That when the teacher has her back, his or her back turned, children might break the rules of the classroom-- in this case, passing a note or something more extraordinary like standing on your desk and dropping your pants or whatever it might be. There could be all kinds of rulebreaking before the teacher turns back around.

And given-- depending on the nature of the violation, the students would be wise to wrap it up correspondingly quickly. If you're only passing a note, that's not such a big deal. You might take your time with it. If you're doing something more outrageous compared to the rules of the classroom, you better get it done with a correspondingly shorter delta t.

That always-- I found that actually very helpful. This notion that elementary particles might be like elementary school children temporarily violating what seemed to be the laws of nature.

OK. So why would that matter in the context of Dirac's formulation of quantized fields? Again, people began putting these pieces together already by the late 1920s. So imagine trying to understand the behavior of a single electron that's traveling through space over time between locations, say, x1 and x2. And let's say it has a definite particular value of momentum.

If these virtual processes could be happening all the time-- in fact, if they can't even be avoid it because they arise from the uncertainty principle, then there's some chance that along the way during its journey, this electron would spit out a virtual photon, a virtual quantum of light, as long as it reabsorbed it some short time later.

So it could be-- during its path from x1 to x2, there could be a so-called virtual process or a quantum fluctuation such that what we thought originally was just a single particle moving by itself might temporarily dissociate into the electron and some virtual quantum of the Maxwell field, a virtual photon, that borrows some energy and some momentum k and pays it back some short time later.

What became so curious to these physicists early on is that this photon, according to this scheme, could borrow any amount of energy. The borrowed energy, the violation of the conservation of energy as dictated by the uncertainty principle could be any amount as long as it paid it back correspondingly quickly.

So there was no-- this seemed to be no upper limit on how much energy or momentum could be borrowed, not quite stolen, by this virtual particle. So then the question became, could you calculate likely scenarios in the context of these temporarily rulebreaking virtual particles?

So Dirac and others went on to show, based on the equations of how these fields seem to evolve, that the probability amplitude-- something like-- remember, like the square root of the probability, roughly speaking-- would go like 1 over k where k was the momentum to both emit and then reabsorb one of these virtual photons.

And that makes some at least intuitive sense. That's like saying, it's less likely for a very, very high momentum state to be temporarily created and destroyed. It's more likely to have borrowed to have broken the rules more softly, let's say. So that makes some sense. It was derived of course, much more carefully than that, but you can make a little bit of sense of the nature of the relationship. Less likely for very high-energy virtual processes.

But nonetheless, as long as the virtual photon paid it back correspondingly quickly, there seemed to be no upper limit. And so therefore, to calculate the likelihood for the electron to travel from x1 to x2, you have to sum up all of these possibilities. The virtual photon could have borrowed some value k or k prime or k double prime all the way, in principle, all the way up to infinity.

So when you start summing up all of these possibilities, you actually get something called a logarithmic divergence, which is a fancy way of saying, the integral blows up. It goes like the log of k, but you're supposed to integrate k the way up to infinity as if this photon had literally borrowed an infinite amount of energy, everything up-- every possibility up to infinity.

So it looked like the likelihood for an electron to travel from point x1 to point x2 became infinite. That didn't seem to make much sense. The way people began to describe it, again, by the late 1920s and early '30s was to say the electron's self-energy diverges. The energy due to its interaction with its own electromagnetic field blew up. If the self-energy is diverging, that was similar to saying its own mass was infinite, because as we saw many, many weeks ago, according to Einstein's relation, E equals MC squared, an energy and a mass are essentially interchangeable.

So it was like saying that the quantum corrections to the mass of an electron became infinite and that did not make a lot of sense. So the very first idea, which a number of physicists began exploring, again, by the early '30s, was to simply cut off the integrals. Let's just say that these virtual photons could not actually borrow literally infinite momentum. Maybe there's some upper limit. And there's some upper-- some finite upper limit to the integral, then you should have a large, perhaps, but finite answer.

But there are a couple problems with that, which, again, were identified very early in the 1930s. If you just put in by hand some upper momentum, up to which one integrates these expressions, then these equations would no longer respect the symmetries of special relativity. You would be inserting a preferred momentum that if you just tried to describe the same process while riding very quickly on a train, you'd get totally different answers.

Even though this was a series of equations that began with Maxwell's equations, which, of course, almost by definition, Einstein showed us, must obey the symmetries of special relativity. So how would you have a self-consistent description of elementary particles that could also be consistent with relativity if you just put in by hand some upper limit to these integrals? That didn't seem to work just mathematically.

A little more deeply conceptually, what that suggested was that if you just put in a cut-off by hand at some highest but finite value of this of borrowed momentum for these virtual particles, then any two observers would basically disagree on anything they tried to calculate. By losing these symmetries of special relativity, then there was no reason for any two calculations ever to agree for any two observers. That seemed like a pretty big challenge.

Meanwhile, it wasn't only that an electron moving through space could temporarily emit and then reabsorb a virtual photon, the inverse process could happen as well, all within the scheme of Dirac's accounting system for quantum electrodynamics, or QED. So the inverse could happen in which a single photon is moving through space from, say, location x1 toward location x2-- sorry. But mid-course, it could actually temporarily, spontaneously emit and then later reabsorb a pair of virtual particles-- in this case, a virtual electron and a virtual positron, the antimatter cousin of the electron.

This would conserve electric charge, it would violate conservation of energy only temporarily just like in that virtual photon case. So if the uncertainty principle combined with Dirac's accounting scheme was to hold, then this kind of virtual process should be happening all the time as well.

So then one could ask about an electrons interaction again with itself, with its own electromagnetic field. After all, the electric and magnetic fields on this quantized view merely consisted of collections of photons. That was the whole point of Dirac's accounting scheme. How many photons in this state, how many photons in the other state? That's what the Maxwell field consisted of on this new view.

So if a single photon could spontaneously emit these electron-positron pairs, then that must be happening all the time in the vicinity of any charged particle because the electromagnetic field, that electric and magnetic field propagating outward from that charged particle could always be subject to these virtual processes, at least in principle.

So the picture that emerges-- this one looks like a daisy, some sort of flower, we have the original electron you actually asking about, and then this spontaneous set of pairs of positrons and electrons that could be popping in and out of existence all the time because of the uncertainty principle.

And this became known as vacuum polarization. Polarization, as you may remember, is just a fancy way of saying that there's a preferred direction in space. The idea was that all these virtual particles would become oriented in space because the positively-charged positrons would be attracted to the original electron, the negatively-charged virtual electrons would be repelled by the original electron.

you'd have this kind of real structure in space, an orientation, or indeed, a polarization of space. So what seemed to be empty space itself, the vacuum, might not be so empty. After all. In fact, it might have a space-dependent structure to it, and that came to be called vacuum polarization.

So one could, again, try to account for the size of this effect. What would happen if you tried to measure the charge of that electron? Well, you would no longer encounter just the electron, so this thinking went, but actually, the electron plus this cloud of these virtual charged particle pairs.

So if you're measuring the electron from over here, you're really seeing the complete effect of the electron plus this screen of the virtual pairs around it. So you could then ask, well, what was the impact of the screening from those virtual-charged particles? You could try to do the same trick as the self-energy calculation for the electron.

Count up all the ways that these virtual pairs could temporarily borrow some energy momentum, as long as they paid it back. They could, in principal, borrow an infinite amount of momentum. And so once again, the integral diverged. It diverged again logarithmically, it was soon clarified.

So it was like saying that the electron had some charge of seven in appropriate units plus infinity, just like it had a mass of 5 plus infinity. It didn't seem to make any sense at all. And so the effect of these quantum fluctuations or virtual particles seemed, once again, to give an infinite change to the simplest level or approximate equations, and it didn't seem at all clear how that could be happening in real life.

So this led to, as you might imagine, a lot of discussion. This was right on the heels of quantum theory, this is still in this period when the physicists, especially in Central Europe, were able to communicate all the time by letter, by conference, by visiting each other. They'd go to Bohr's Institute, they'd visit in Gottingen and so on.

And so this became like the hot topic, one of the pressing concerns for this early kind of quantum theory generation. Many of the leading physicists-- again, mostly in Europe at this time-- who were confronting these new challenges of quantum processes that simply couldn't be calculated, couldn't yield any finite answers, they thought this pointed to a really big next conceptual revolution.

Remember, the revolution, as they thought of it, a revolution of quantum theory, was like months old. They'd just gone through this seemingly huge head-spinning rupture of the uncertainty principle and noncommuting matrices and all these things. In their own memories, some of their own earlier years as younger physicists, they tried to work through and incorporate the large conceptual shifts associated with relativity. They think, OK, we're due for another one.

And this became a steady drumbeat interpretation by people like Niels Bohr and Werner Heisenberg. That that's OK. We've been stuck like this before. This shows that we're going to learn something really, really drastically different about how nature works to finally get around these virtual processes.

So one example that Heisenberg floated by the mid-1930s, for example, was that maybe space is actually not continuous. That's a pretty big rupture. Maybe there's some shortest possible length, some fundamental shortest length in space, that space is not a continuous fabric as described by Einstein's relativity, but in fact, maybe there's a quantumness to space itself, a discreteness.

If that were true, then there would have to exist some largest possible momentum that would go inversely with that length. And so there'd be a finite largest momentum, That means all those integrals would not actually go to infinity, they would go to the physically allowed largest maximum length, and that would give a mathematical cure to these poorly-behaved integrals, but at an enormous conceptual cost.

I mean, yet another pretty significant rethinking of space and time, the very fabric of a continuous space-time would no longer hold. So that seems like a pretty big challenge to relativity even if it did give a reason why these integrals might be finite.

Other people said, well, maybe the problem isn't with the nature of space, maybe the problem is with our understanding of the Uncertainty Principle. This whole idea of these virtual particles, these quantum fluctuations in the state of an extended quantum field, that really seemed to come directly from the uncertainty principle, that delta e delta t being non-zero.

So others said, well, maybe the fault is with quantum theory itself, and that this seemingly bedrock feature of quantum mechanics, Heisenberg's Uncertainty Principle, maybe that isn't quite right has to be either tossed out or somehow amended.

And so the mathematical challenges of getting integrals to converge, which is a well-defined, though very hard challenge, that inspired in many of these folks by the late '20s and throughout the 1930s, visions of some sort of grand rupture in how they would have to understand space, time, and matter at a very deep level.

So I'm going to pause there. Some time for questions on that. I see something in the chat. Ah, good. So Sarah asks what led to the idea that virtual particles existed? Thank you, Sarah. I should have clarified. This was a hypothesis. It seemed to be, at least to many of these folks, what looked like a straightforward corollary of the Uncertainty Principle, but you're absolutely right.

One of the first questions they had was, did we misinterpret the Uncertainty Principle? Or is there some kind of theoretical out that will enable us to keep the Uncertainty Principle but not attribute this particular kind of process to nature?

Another question was like, if they're virtual, if they're just stealing this energy and paying it back so quickly, how would we ever know? It was the kind of-- if a tree falls in a forest kind of argument. So some people said scrap the Uncertainty Principle, others said this is kind of an unobservable theoretical story.

And so if your equations blow up, then so what? That doesn't mean nature's doing this after all, and we'll come to see how that question, are these virtual particles part of nature or only an artifact of the equations, that gets picked up again after the Second World War. So there were split decisions, split opinions throughout the 1920s and '30s on that. It's a good question.

Fisher asks, in the case of a and a dagger used by Dirac, was that generalized to all systems points in space or just approximating them as a quantum harmonic oscillator? Good, Fisher. So the idea was indeed modeled on these harmonic oscillators, but the idea was that these would be, how we describe, extended objects through space and changing over time as these collections of these field oscillations, of these quanta.

And it was actually Heisenberg-- I think it was Heisenberg who, a few years later, applied the same thing even to-- not just to light like the electromagnetic field, but even to the field of an electron. So the idea that gets-- that starts coming together by the early mid-1930s, really building directly on this early work by Dirac and others, was that every kind of matter and radiation in nature was the hypothesis.

Everything. Electrons, positrons, protons, neutrons, as soon as they were known about a few years later, that they are all excitations, localized excitations of some kind of underlying quantum fields. And that to characterize the state of the field, you could equivalently just add up how many quanta in this state, how many quanta in that state, how many-- much like Fourier analysis.

And so you could change the state of the quantum electron field by emitting or absorbing electrons, you could change the state of the quantum Maxwell field by emitting or absorbing photons, and that would be like just representing a continuous function in Fourier space by manipulating the Fourier weights or dealing with the original function. That there should be a kind of equivalent accounting scheme.

But that these were-- that the particles that people began measuring in cloud chambers, in cosmic rays-- from cathode rays, that these were really localized excitations of a raw quantum field that somehow came first. The excitations were accidents of the state of the field, something like that. And that it was, again, building on Dirac's early approach to those others, prominently Heisenberg.

Said, this must happen-- what if this happens for every form of matter we've ever thought about? Electrons, positrons, and so on. So that was becoming more and more their common set of assumptions, and that's what led to things like virtual electron positron pairs, but then the effects seem to blow up. Virtual photons, but the effects seem to blow up. So that's what led to this one step forward, an infinite number of steps back, which is not very good progress. Dealing with infinities is not usually very fun.

Excellent questions. Any other questions on the virtual particles quantum field idea more generally? That was, of course, a bit schematic, but I want to get at least a gist across. If not, that's OK, I'll jump back in. Because we're going to see how some people return to this question soon after the end of the Second World War. So the questions that are already being raised get seen in a new light.

So as a reminder some of the things we looked at more recently in class, during the Second World War, researchers all over the world, and certainly in many labs in the United States, got involved in all kinds of basically military-oriented projects, and the United States most famously. That included the Radar Project, and Manhattan Project, but also dozens and dozens of other projects. And likewise, for physicists, chemists, and many kinds of engineers in many, many other parts of the world.

So at the MIT-based Rad Lab at the Allied Headquarters for Radar, we saw that researchers, physicists, many kinds of engineers together had really pushed the envelope in both producing and measuring electromagnetic radiation of a particular wavelength band.

So the area that they made the most progress in, where they focused most of their efforts was in the microwave region of the spectrum, and in particular, a centimeter wavelength, 1 to 3 centimeters because you need to do things like pick out a periscope of a German U-boat, that kind of thing. So going to short wavelength as opposed to multiple meters' length like in radial band.

The microwave spectrum was really operationally critical. We saw a bunch of examples of that during the Second World War. So after the war-- I still find this mind-boggling given how hard it is to do anything with the federal government these days as a researcher.

After the end of the war, there was all this suddenly surplus equipment at places like the Rad Lab. And many, many researchers who had worked photon at the Rad Lab during the war were basically invited to bring along their pickup trucks and load up their trucks and just take away whatever equipment they wanted, almost that informally. It was only slightly better controlled than that. It was almost a free-for-all.

There was an enormous amount of surplus equipment from places like the Rad Lab. In this case, extraordinarily sensitive electronics tuned, finely tuned to be very accurate in both producing and measuring microwave-band electromagnetic radiation, things like filling this table in our own Building 4 back from the Rad Lab days.

So that wasn't only researchers at MIT, this was researchers who were going back to their home universities throughout North America, many of them in the United States, to places like Columbia University, for example, and many other sites. So you had this very informal dispersal, I'd say very generous dispersal of equipment that was once highly, highly coveted because it was so mission critical to the Radar Project, and it went to many, many labs across the country for postwar peacetime research in the microwave band.

And so that's what helped to jumpstart two very significant sets of experiments that took place at Columbia very soon after the end of the war by Rad Lab alumni, by physicists who indeed had spent the wartime working intensively on radar. These two new experiments were called the Lamb shift, which we'll talk about first, and then something that became known as the anomalous magnetic moment.

And I just want to underscore, both of these came from people using specific skills that they had honed during the wartime Rad Lab, and in each case, literally using equipment that was surplus from the wartime projects.

Let's look at the Lamb shift. First it was named for the principal author, Willis Lamb. That's why it's called the Lamb shift. So as you may remember, going back when we looked at quantum theory a few weeks ago, even according to the Bohr model of the atom from so-called old quantum theory, let alone from the fancier and fancier treatments of the hydrogen atom for Schrodinger's equation by 1926, and that's something we didn't talk about in this class, but Paul Dirac's relativistic generalization of Schrodinger's equation, which he introduced around 1928.

According to each of these very highly quantitative treatments of the electron in a hydrogen atom, the energy of an electron in the 2S state and the 2P state should be identical. There might be a splitting if you put the atom in an external magnetic field, but just on its own, in the absence of an external field, the energy of the electron depended only on the so-called principal quantum number-- in this case, 2, not on the angular momentum that would determine distinguish an S state from a P state.

So according to every single iteration to date of quantum mechanics, say up to the 1940s, there should be literally zero difference in the energy levels of a 2S state and a 2P state for that lonesome electron in a hydrogen atom.

And instead, very soon after the war, this first author, Willis Lamb, who was, by this point, a mid-career accomplished professor who had spent the wall working on radar, and his then-graduate student, Robert Rutherford, they were able to use some of the surplus equipment, and indeed, the skills they'd really worked on throughout the war, to actually measure a very, very tiny difference in the energy levels of exactly those two states that should have had no difference at all.

In fact, it was roughly one part in a million. That would have been literally unthinkable that such a tiny shift could even, in principle, be measured before these advances from the Rad Lab with the new equipment and so on. If one didn't have this kind of exceptionally sensitive microwave-band electronics, then even if you thought there might be an energy difference of that order, no one would ever dream that it could be measured.

That's what begins to change very dramatically after the war when people like Lamb realized they were measuring exquisitely sensitive shifts in the context of honing their radar devices all the time. Moreover, this energy difference would be right in the sweet spot for which their electronics had been designed during the war.

And in fact, I highlight this part from the very first page of this now-famous article, they go on and say the great wartime advances in microwave techniques in the vicinity of 3 centimeters wavelength, by which he means radar, made possible the use of new physical tools for the study of the n equals 2, principal quantum number 2, fine structure states of the hydrogen atom.

He basically tells us, we can do this because of radar, that was true. It was just amazing to see him write it. The energy difference corresponded to be something-- corresponding to radiation with wavelength just under 3 centimeters, exactly the sweet spot they'd been focusing so much effort on during the war.

In particular, what they found when they did these experiments was the energy of the 2S state, the spherically symmetric state for the electron, was just a little bit higher, by about part in a million, compared to that of the 2P state. So the 2P energy state was consistent with these earlier 1920s calculations for the energy and the 2S state seemed to be elevated just a little bit by roughly one part in a million.

So just coming back to Sarah's question from the chat, this so-called Lamb shift, this tiny but now remarkably measurable difference in the energies between the 2S and the 2P states, that began to get a bunch of theorists, after the war, to go back to this question of virtual particles. Could this energy shift, this now measured experimental result, could that be accounted for? Could that be attributed to the real physical impact of these previously hypothetical virtual particles?

This was what looked like the first maybe direct evidence that could lead to a notion that virtual particles are part of the world and not only a pencil-and-paper hypothesis.

So Willis Lamb presented these preliminary results at a meeting I'll talk more about later today, a meeting on Shelter Island, which is a tiny little island just off the north fork of Long Island. Many of you, I'm sure, have heard about Long Island near New York City. And there are some very much smaller islands nearby, one of which is called Shelter Island.

And there was a meeting there, a private meeting of physicists in June of 1947 right around the time that they submitted these results and Lamb had been invited to present. So on the train ride back to Upstate New York from that meeting, the Cornell physicist, Hans Bethe, actually worked out a rough calculation of what came to be called the Lamb shift by trying to take literally, to take seriously this notion of virtual particles.

So we'll look at sketch level of what Bethe's new argument was like. This was not the end of the story by any means, but it really helped reorient a number of physicists to think seriously about virtual particles in a new light.

So his idea was that if these virtual particles exist, they're constantly popping in and out of existence. You couldn't not have them because the Uncertainty Principle. Then the original electron in that hydrogen atom would behave-- would be subject to something like Brownian motion. It would be constantly jostled by this bombardment by these virtual particles that are constantly borrowing and paying back this energy. It'll be nudged here, nudged there.

So the effect of that on the behavior of the original electron in the hydrogen atom, Bethe suggested, would be smearing out of the position of that electron because it's constantly being bombarded. There'd be some quantum fluctuation in its position delta r.

So if its position is now smeared out with some within some finite range, then how would that impact its potential energy? After all, remember, in the hydrogen atom, you have a positively-charged proton exerting a Coulomb attraction, an electromagnetic attraction on this negatively-charged electron.

So if the position of the electron is being constantly jostled because of the real physical impact of virtual particles, then you'd have to Taylor-expand the potential energy. It would be what you originally thought it was, plus some correction due to the jostling, some delta r displacement. And just for the ordinary rules for Taylor expansion, that would then multiply the first derivative of the potential in the radial direction. There'd be a second-order correction proportional to the second derivative of potential, and, in principle, on from there.

Bethe next argues that this linear term that goes directly with the-- directly proportional to this smearing, that'll probably average out. That should cancel out, Bethe suggests, because the electron's as likely to get kicked downward toward the proton as to get kicked further away.

So on average, that displacement should vanish. Even though it's non-zero at any given moment, its average value would likely vanish. However, as is typical for any fluctuation phenomena, in classical physics as well as quantum mechanics, Bethe next argued that the square of that displacement will not vanish typically. That there'll be some non-vanishing average value for the square of that smearing, the delta r squared term. It might be tiny, but it won't actually average out to 0.

If that's the case, then you really better look at into this term that it multiplies in your expansion, how do you incorporate the second derivative of the potential? Well again, if you use the particular potential of interest, the Coulomb potential, the second derivative will yield-- will have most impact really right at the origin, right at r equals 0. In fact, it's proportional to a delta function.

And so in a picture form, what that suggested to Bethe was that because of this unavoidable jostling of the electron due to constantly being kicked and pushed around by these virtual particles, there will be a contribution to the effective potential that is a positive-- look at the sign here, it's a positive shift, some small amount but positive addition to the energy you otherwise would have calculated, and it'll only really matter very close to the origin.

So whereas this function would, in principle, fall off all the way to minus infinity as you go toward the origin when r-- if r literally could become 0, this thing would diverge. Instead, you shift the shape of that function with this positive correction term that really would only matter very, very close to the origin, to near r equals 0.

So then he goes on further, he's working this out on the train ride back up to Schenectady before he gets back to Ithaca. It's a pretty good train ride. Next, Bethe says, well, this really should preferentially affect electrons in the 2S state rather than the 2P state. Remember, Lamb had found not just that there was a difference, but it was the 2S state whose energy was ever so slightly raised compared to the quantum mechanical prediction.

He said, oh, well that also might make sense. If the positive shift in the potential energy really only matters very near the origin, it's the electron in the 2S state-- that's this spherically symmetric shape that has any non-vanishing chance, every now and then to find itself near the origin.

That's the fancy way to say that, is to say that the quantum wave function for the electron in the 2S state has a non-vanishing value at the origin, it has some probability to be found at the origin at least some of the time, so its energy will be affected by this modified shape of the potential.

On the other hand, an electron in the 2P state has basically no chance ever to be at the origin. Its wave function literally vanishes at r equals 0, and so its energy is basically unaffected by the change. If you're only changing the shape of the potential in a region that this electron never goes near, then its energy should be unaffected, and yet there might be a small but positive shift in the S state.

That's pretty cool. It was at least trying to say, here's a plausibility story for why there might be a shift and why it might be raising the 2S state-- the energy of the 2S state.

Now keep in mind, Bethe still couldn't calculate this exactly, he still had all these integrals blowing up when he tried to really calculate the effect. In this case, it was like saying how would you calculate this part? That's the part that really comes from summing up all the borrowed energy from this quantum fluctuations.

So he did something again pretty darn clever on that fateful train ride. He said, well, if this integral is blowing up like the logarithm, let me put in by hand a maximum value. So it breaks the symmetries of special relativity, it has all the illnesses that people had identified in the 1930s, but he said he had a new motivation to try because it looked like maybe this is a measurable effect, after all.

So he chooses a perfectly reasonable maximum momentum that could have been borrowed, and he gives it equal to-- basically the equivalent to the rest energy of an electron. So he's going to integrate not to infinity, but to imagine that these virtual particles could have borrowed up to the energy of an actual electron at rest.

And he said, if it's a little different than that, the answer will only vary logarithmically. So that's a pretty good order of magnitude, physically reasonable scale to try even though mathematically, it's totally illegitimate. It breaks all the symmetries and all the rest.

When he put in that value, he found very close agreement to Lamb's measured result. Not only was there a positive shift for the 2S state, he could now estimate semi-plausibly how big that shift should be and compare it to these highly-sensitive new results that Lamb had just presented.

So that starts getting people to think pretty seriously that virtual particles maybe are actually part of the world and not just a kind of clever trick on paper. The other big result right around the same time that really cemented that new approach turned out came from the same kinds of historical processes just down the hall.

So literally down the hall from where Willis Lamb and Robert Rutherford were working at Columbia, another group of physicists was working at Columbia, also Rad Lab veterans, also using surplus equipment for microwave electronics. This was a group led by Isidor Rabi, who I think had either just one or was about to win the Nobel Prize, and then two of his younger colleagues, Nafe and Nelson.

So they were also trying to perform these very, very sensitive experimental measurements on the behavior of hydrogen atoms, very similar to what Willis Lamb was doing, because like Lamb, they now had this incredible experimental control and experience in the microwave band.

So only a few weeks before Lamb and Rutherford wrote up their results, this group, starting in mid-May of '47, had likewise performed very sensitive measurements on properties of an electron in a hydrogen atom, and again, they found a very tiny but now measurable deviation from their theoretical expectations.

This time, it involved quantities related to the spin of the electron. Remember a couple-- again, a couple classes back when we were talking about the last vestiges of the old quantum theory, people like Samuel Goudsmit and George Uhlenbeck and soon others introduced this hypothesis of quantum mechanical spin.

Maybe the electron spins like on its own axis like the Earth spins to turn day to night, and not only has an orbital spin around the proton, say, an analogy to the Earth moving around the sun. That intrinsic spin or angular momentum would yield a magnetic moment. The behavior of such a spinning charged object in an external magnetic field would depend on this quantity called the magnetic moment.

In fact, by this time, this had been named the Bohr magneton, the actual value of the magnetic moment associated with the spin of an electron. It has an electric charge, it has some unit of angular momentum, h bar over 2. And so there would be an expectation for a magnitude, how large would the magnetic moment be for an electron.

And what Rabi and his colleagues found was that if you really measure this extremely sensitively in a way they only could now just begin to do, the value they kept measuring of the magnetic moment of electron was actually a little bit larger-- in this case, about one part in a thousand instead of one part in a million.

But still, something that would have been barely measurable, if at all, before the Second World War now is within their toolkit. Now they can actually measure slight differences of a few parts in 10 to the 3. And again, it seemed to be shifted to the larger value than what the quantum theoretical prediction suggested.

So again, like Willis Lamb, Rabi presented this experimental result at that same Shelter Island conference, which I'll say more about in a moment, in June of '47.

So these mostly theoretical physicists who were attending the meeting at Shelter Island, they heard from a small number of experimentalists, including Willis Lamb and Isidor Rabi, and these were, again, like hot-off-the-presses, totally cutting-edge results, some of which hadn't even been published yet, these folks at the meeting had a sneak preview of these very high-precision new wartime-enabled new experiments.

So once again, theorists, hearing this kind of unexpected result from Isidor Rabi, wondered if this could also be a kind of measurable empirical effect of genuine virtual particles changing the behavior of that electron. To go back to this kind of flower petal or daisy picture of the polarized vacuum, you have the original electron and the virtual electron positron pairs springing up all around it due to the uncertainty principle.

Well, each of those are also objects with some electric charge and with some intrinsic spin. So maybe the system, the electron plus its cloud of virtual particles, maybe that whole system was contributing to the effective magnetic moment. That would be like having a bunch of compass needles all aligning in the Earth's external magnetic field. And so the total effect would be-- you'd have to add up the effect from all of those spinning compass needles, not just the one you began with.

And maybe that would be enough to just slightly nudge the measured value of that magnetic moment to be a little bit higher, this contribution from the cloud might indeed be a little bit higher than if you only consider the electron it's its own.

So I'll pause there. Any questions there? So Alex says, the greatest reuse pile ever. That is basically true. I just can't believe, again, how generous and relaxed the federal government was. The federal government might not often merit those two adjectives these days. It was generous and relaxed in letting people like literally just truck back to their home labs tons and tons of surplus equipment.

And Tiffany writes only to me-- I don't if you mentioned it to everyone, Tiffany, but you say, [INAUDIBLE] benefit from this as well. Do you want to say more, Tiffany? Very directly, yeah. In fact again, much like with the Columbia groups-- thank you, Tiffany. It wasn't just the equipment, it was young people like Ray Weiss, who, at the time, was quite young, as you know.

So Ray-- our very own Ray Weiss, I will go on record and say our beloved Ray Weiss, I adore him, and also our Nobel Laureate, he got his start at MIT as an undergraduate working in the then-brand new Research Lab for Electronics, which exactly, as Tiffany says, was an intentional carryover of the wartime Rad Lab to a postwar footing.

And Ray is basically the equivalent of a UROP student from very early on, began working literally with surplus equipment and learning how to use-- how to conduct very precise microwave-band electronic measurements. That's a great example, Tiffany. Yeah, good.

So thank goodness. Thank goodness the black holes still collided and we had government surplus stuff to measure it a little while later. Yeah. Good. Any other questions on the Lamb shift, the anomalous magnetic moment, the impact of the Rad Lab on tools and skills and personnel? I find that really-- I just love that-- that unexpected continuity. And we'll see another example of that in the last part of class today.

If there are no other questions, again, I'll charge on, but please don't be shy about asking. Let's see, Fisher has a question. How did Lamb to look for this? Good. Very good.

I don't think that Lamb was intentionally trying to ask about virtual particles, per se. I think what Lamb realized was that there were all these-- it was actually called the fine structure, all these very minute shifts in the energy states of electrons and hydrogen atoms. Remember, that had been recognized even in the 1890s, although obviously not nearly the same level of precision, of course.

But that's what gave rise to things like the Zeeman effect, the anomalous Zeeman splitting. If you put a gas of hydrogen atoms in an external electric field or an external magnetic field, the spectral lines that come out can show these very minute shifts, and that was read back as saying there's a shift in the energy states of the electrons.

And so part of what was of interest for people like Willis Lamb was to ask, could there be measurable effects from, say, the spin of the nucleus? Could the could the fact that the nucleus has some moving parts affect the measured energy states of, say, the electrons from the spectrum and so on?

So those were things that were just never going to be measurable using the techniques from the 19-teens or '20s, but he thought, again, the article-- the title of the article itself was about the fine structure. He was interested in, what's the best, most quantitatively precise way with which we can characterize the energy states of electrons in hydrogen?

And so he didn't go in asking about virtual particles, per se, but he said, I have a pretty cool new set of tools to measure in that energy band, and he and his student, Rutherford, then relatively quickly were able to find this, indeed, unexpected energy shift.

Good. OK. I'm going to go on to the last part for today, but again, of course, please keep the questions coming, I'd be delighted to talk more. OK, so that middle part for today's class was all about some kind of experimental continuities, the fact that it mattered that people were asking these questions after the Second World War probably rather than before. And

So for the last part today, we're going to ask a similar question, but now about some theoretical responses, which, again, will-- I think it's quite amazing-- will find also seem to depend in a pretty critical way on the intervening experience of the Second World War.

So I mentioned a few times that Shelter Island meeting, and I'll just say a bit more about that meeting itself. It was actually none other than Robert Oppenheimer who organized it. In fact, as we'll see in the next class, he organized three in a row, three annual meetings of which this was the first. The June 1947 meeting about one week in duration was the first of a planned series of three.

And Oppenheimer was very explicit. He really thought it was time to get the physicists of thinking again about open questions at the cutting edge of theoretical physics, including these open questions in quantum field theory which had really ground to a halt at the time that he himself was a student and a postdoc going back to the '20s and '30s. He thought, now that the wartime efforts were behind them, it was time to literally jumpstart the work.

He also had very, let's say, patriotic-- we might say nationalistic-- goals in mind. That he wanted to really help further the US-based community in theoretical physics. So he invited several leading Europeans over for the meeting, some of whom were already in North America.

But his real goal was to get the younger generation of US-based theorists in a room together-- in fact, in a lovely bed and breakfast called the Ram's Head Inn. They literally lived together for a week and did nothing but talk about quantum physics for a week being paid for by the National Academy of Sciences.

So here's one of the famous photographs. This is actually Willis Lamb standing here. Lamb had actually been trained as a theorist, but had converted over to an experimentalist. Here's John Wheeler. I love this, he's like reading the funny pages, not even paying attention. I love that photo.

Here is Abraham Pais who had actually fled the Nazis as a young person and relocated in Princeton. Here's Richard Feynman, MIT class of 1935 or so, 1934. Here's Herman Feshbach, who then made his career at MIT, and then here's Julian Schwinger. So here's a photo from the bed-and-breakfast on Shelter Island literally during that June '47 meeting.

So nearly all the participants actually already knew each other even though some of them were still quite young. Feynman was 29 years old at the time, Schwinger was also about the same age. So it wasn't that these were very senior figures who'd been around each other for decades, they had been around each other on the wartime projects. They knew each other because of some of these very intense experiences from very recent times even though there were still-- many of them quite young in their careers.

And as we'll see-- and this is work that I really owe to my own mentor, Sam Schweber, who wrote this amazing book on-- a little piece of which I'll describe now, that the reaction from some of these folks soon after the war to these very stubborn problems of infinities in quantum electrodynamics, their response was really quite different than what we saw from people like Niels Bohr and Werner Heisenberg.

Very few of these physicists who came back to the question of virtual particles and infinities and all that after their wartime experiences, very few of them talked about a grand, sweeping conceptual revolution. Very few of them said, we're going to sweep away everything we know about relativity and quantum theory. We'll have even grander conceptual leaps.

Instead, as Sam has shown really quite powerfully in this landmark study, most of these wartime project veterans said, well, we have a bunch of equations, we now have some measurements from people like Lamb and Rabi and soon others, what can we do to get the numbers out? That's the phrase that Sam associates with this generation.

How do you find some clever workaround to get some numbers out to compare with real empirical measurements? So rather than getting stuck with fancy-sounding conceptual first principles, you have some equations, you have some reason for confidence in them, get to work and get some numbers out and compare with what the other groups are measuring.

And really, I think an amazing example of that comes in from Julian Schwinger. Schwinger actually had spent the war at MIT. He was at the MIT Rad Lab. He then went all the way back to Harvard after the war was over. He was actually, for many years, the youngest tenured professor in any field at Harvard. He was tenured at age 29 and had spent the war, as I say, at the Rad Lab.

He was one of these invited participants at Shelter Island. And as we saw some weeks ago, he had been immersed in these kinds of engineering effective styles of calculation during his work at the Rad Lab. And he himself years later credited the engineers with really changing his own thinking about some complicated physical systems.

So as we saw, if you're working with these kinds of waveguides, rather than spheres and perfect cylinders, then the usual symmetry arguments that physicists love and that are often so powerful for us, those are simply not going to cut it.

So rather than trying to calculate from first principles, if you're looking at some really, frankly, ugly piece of real-world equipment or some complicated, highly non-symmetric piece of real-world equipment, don't sit down and start using spherical symmetry or cylindrical symmetry. Learn from the engineers to work in terms of effective circuits, or really more generally, input-output.

So as we saw last time, instead of worrying about the rules for how to resistors add if they're in parallel or series and all that, just, say, stick a lead on here, stick a lead on here, there's some effective resistance, and get on with your life. As they say, there's a war on, buddy. So don't get stuck on this first principles calculation.

So when he then came to Shelter Island, really just two years after the end of the war-- not even two whole years-- after the end of the war, and he heard directly from people like Willis Lamb and Isidor Rabi about these new microwave-band measurements, he began thinking about virtual particles. By his own admission, he began trying to apply this Rad Lab engineering-style input-output calculation to what had previously seemed really incalculable.

So let's take, for example, that question of the anomalous magnetic moment where this collection of the electron plus its cloud of virtual particles seem to have a different value of its ratio of charge and spin to its mass compared to what one would expect in the absence of those virtual particles.

So Schwinger realized, we could never turn off the uncertainty principle. So you would never actually have just the electron its own. So why would you ever set up your equations as if you have a pristine, pure single electron and then try to fold in all the messiness of the world? That's not how we handle waveguides. You don't start with spherical symmetry and then cut back to add in more realism. Just never separate them in the first place. Treat it more like an input-output relationship.

So his main idea was to never try to calculate the effect of this electron and then separately add in the effects of all these virtual ones. Just manipulate your equations from the start so it's always a combination that you're dealing with. Again, that would be like the effective resistance on that circuit.

So you can rewrite your equations before you sit down and try to calculate particular things, so you're always dealing with the sum of the so-called bare mass or bare electric charge plus the corrections or the impact of those virtual quantum processes. So instead of trying to write this down and then calculate this from scratch, rewrite your equations, you always have the input-output, the actual quantity that would ever be subject to measurement.

So if you do that, Schwinger then goes on to show, quite astonishingly, then you never need to calculate these things on their own because they never appear on their own. They always appear in combination with these other constants, the so-called bare mass or the bare charge.

And as we'll see, I want to-- I just want to remind us or give a foreshadowing, it turns out, it wasn't only Julian Schwinger who pieced this all together, it just boggles my mind, but totally independently, and in fact, four years earlier, really, in the middle of this very horrible, bloody war, Sin-itiro Tomonaga, a very young Japanese theoretical physicist working in Tokyo, had come up with the exact same series of steps.

And he also, it turned out, was immersed in the Japanese radar project. And he also, after the war, credited his switch in thinking to this input-output effective quantities approach to his work on radar.

So these two independent approaches to a longstanding challenge, and it turns out, Tomonaga really actually got there first for many of these steps, so that was not known at the time in the United States because the nations were at war and there wasn't regular mail service to deliver journals and all the rest. So I'll come back to some of Tomonaga's work in the next class, but I do want to mention it.

This was jaw-droppingly original when Schwinger did it in the late '40s to his US-based colleagues, and only later did they learn that it had been done actually even earlier under much more difficult conditions.

OK. So what was the rest of Schwinger's argument, then? If you never separate the kind of corrections, so-called corrections from quantum theory from those bare quantities, then you can, all of a sudden, find finite sensible answers after all. So he goes on to show, that just as the experts had found in the 1920s and '30s, if you actually try to just calculate the effect of all these virtual particles on their own, you really do find these logarithmic divergent integrals.

And this is-- you have a little more information about this in the reading for today by physicist Robert Mills, that so-called tutorial on the infinities in QED. This is the kind of thing that Mills writes about. So Schwinger confirmed that, yes, indeed, the form of the equations, we can write them kind of abstractly across many, many applications, whether it's the corrections due to the bare mass or the corrections to the charge, they take this form in general.

And they are generally logarithmically divergent where y here is basically like the borrowed energy or momentum of those virtual processes. So yes, you get log of infinity, which is still infinity-- sorry.

But instead, if you don't just calculate that on its own, you always do this grouping first. So you always deal with the effective quantities, then, in fact, some remarkable cancellations can occur. So this is what he began working on in the weeks and months after hearing from Lamb and Rabi at Shelter Island.

So Schwinger first rewrites all his equations in terms of these effective quantities-- and this could, again, be charge or mass, it works-- you have to do it for both. Let's say it's charge in this instance, you have some bare charge which is never separated from the screening, you can never turn off the Uncertainty Principle.

In this case, the screening would be a minus sign because you're actually screening off part of the original charge that comes in an opposite sign. The effective charge is a little weaker than the bare charge would have been because of this cloud, he reasons.

So now, instead of calculating this part on its own due to the virtual particles, calculate the combination together because they never appear separated from each other. So all of a sudden, now, you have this as your integral to evaluate rather than this on its own.

And now, it's just a few more steps of algebra, which even the youngest tenured professor at Harvard was able to do. So now you can just combine the denominators, now you have a new numerator, just a little more algebra. All of a sudden, what's happened, what matters is you've increased the power of momentum in the denominator.

Instead of integrating over one power of momentum-- that's what gives you that logarithm, now you're integrating over two powers, and that's going to be a convergent integral. And in fact, you can do that integral, it becomes a logarithm of a perfectly finite answer.

So this became known as renormalization. That's the fancy word that was applied to this based on this early demonstration by people like Julian Schwinger. You first arrange, always have the so-called bare and quantum corrections together because when you evaluate them together as one joined quantity, you get these exact cancelations.

It turns out, the bare charge of the electron really is infinite in this scheme. It's just that you can never interact with an electron on its own. So that infinite initial charge is partly screened by the infinite effect of those virtual particles and not arbitrary infinities-- in fact, particular forms of infinity that you can show exactly cancel.

As long as we can write them in this grouped form, the infinity here is exactly canceled by the infinity here, and it leaves a perfectly unambiguous finite result. So this avoids that problem of cutting off the integrals at some arbitrary level.

This can now be made, as Schwinger show very elegantly, this approach can be made to respect all the symmetries of special relativity. Different observers, now in principle, should agree. You extract finite results. Basically the trick is by never trying to calculate this apart from that. That's really the upshot of that maneuver that Schwinger introduces.

Then it goes on in the next six months to show not only is this feasible in principle, that this type of logarithmically divergent integral could be cured if you never treat it on its own, but he then applies it to the very specific measurement that Rabi and Nafe and Nelson had reported on at Shelter Island, that anomalous magnetic moment.

And for the first time in the history of the universe-- he got here even quicker than Tomonaga, this part, Schwinger is able not just to find a finite answer, but to find an answer that agrees to remarkable accuracy with that hot-off-the-presses experimental measurement that the Columbia group had just reported.

So not only does he get an answer that the correction from incorporating the virtual particles goes by actually a fairly simple-looking expression involving just constants of nature-- it's the charge electron squared divided by h bar and c with this critical factor of 2 pi, evaluate that numerically, the shift by which the effective magnetic moment should have risen compared to the original quantum mechanical prediction is about one part in 10 to the 3.

In fact, he could calculate out to multiple significant digits and compare that, and it's unbelievably spot-on within the experimental error of that early measurement from the Columbia group. That finally convinces people, by December of 1947, that these virtual particles might really be part of the world after all and that maybe not all hope is lost.

If you focus, as a very pragmatic wartime researcher had had to do, if you focus on getting the numbers out and focus on what relationships should appear on their own or only in combination, Schwinger shows-- and others were quickly able to generalize, that you might be able to calculate the effects of virtual particles after all.

So let me summarize, I'll have time for some more questions. So beginning right after the introduction of quantum mechanics, several physicists, including the names we already know, Dirac and Heisenberg and Jordan and Pauli and others, began developing a quantum field theory trying to treat extended objects through space and time as collections of field excitations obeying these special quantum mechanical rules.

And so you could represent any field as a collection of quanta in particular states with the very important caveat that the uncertainty principle at least suggested that these virtual processes could temporarily break the rules as long as they paid the energy back correspondingly quickly.

However, that was a compelling cartoon story. It led to a total lack of ability to quantify those behaviors throughout the 1930, and that led some people to claim we need another sweeping conceptual evolution that the whole edifice will change yet again because these quantum virtual processes yield infinite results.

And yet after the Second World War, a new generation of both experimentalists and theorists returned to this now decades-long challenge, and they came at it with different skills, sometimes literally different equipment, different theoretical approaches, and a different kind of mindset for many of them, and were able to find a way to actually yield finite and, in fact, quite sensible numerical values from these very abstract-sounding quantum virtual processes.

So I'll stop there. I'm happy to say if people have questions for that. Any questions or comments or concerns? I should also say, that Robert Mills piece is a little more complicated than I would have liked. It's a pretty handy reading, but it still assumes a lot of formalism. So if it was a hard reading, don't worry about it.

What I wanted you to get out of it is at the level that we talked about at in class today. I'd be glad if people have questions, I'd be delighted to talk more about more details, but if that article was a little off-putting or seemed too hard, don't worry. If you've got this basic lesson, that that's what we want to get out of the Mills piece, at least that much for now.

OK. If there's no questions, I won't force you to stay, but I invite you to come to office hours, drop me an email anytime. And we'll pick up this story on Wednesday when we look at other folks who returned to the question of virtual particles and quantum fluctuations. So see you Wednesday. Stay well.