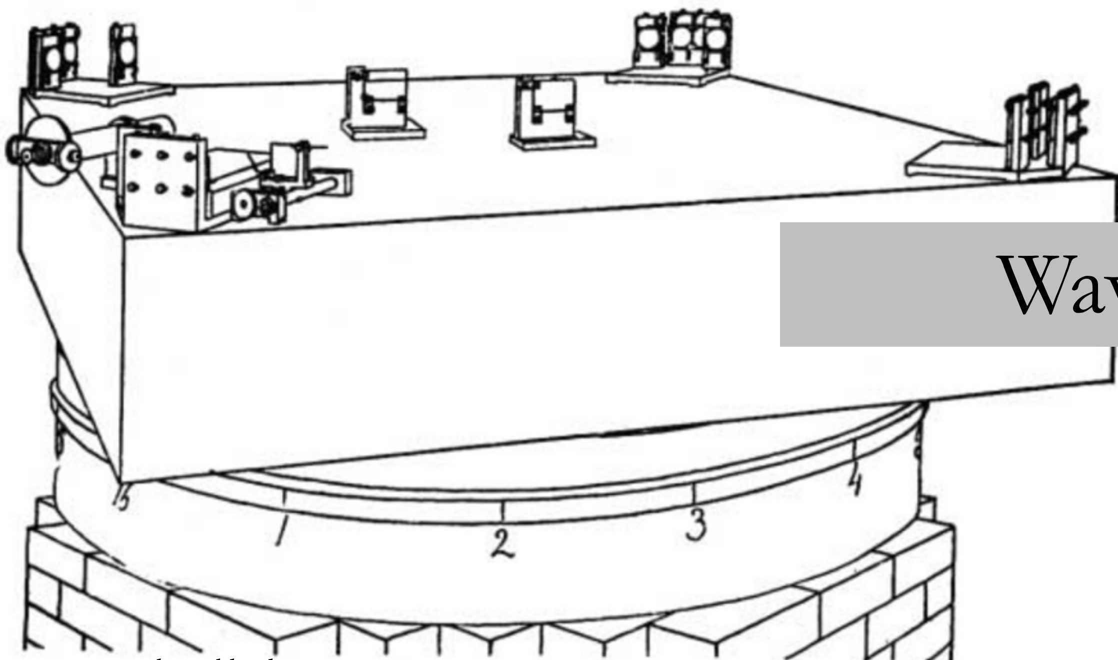


Waves in the Ether

8.225 / STS.042, Physics in the 20th Century
Professor David Kaiser, 16 September 2020

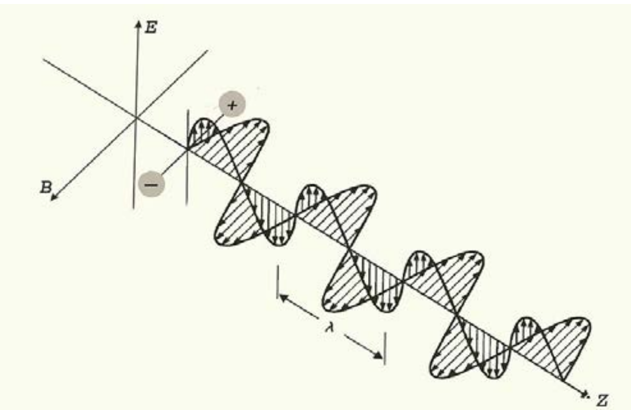
1. Waves for Maxwell and Lorentz

2. The Michelson-Morley Experiment

3. Lorentz Contraction

* See *Lecture Notes* on "Electrodynamics for Maxwell and Lorentz" for more detailed derivations and discussion.

Maxwell's "Transverse Undulations"



In 1865, Maxwell concluded that light was nothing other than "transverse undulations" of **E** and **B** fields propagating in the ether.

Why? Recall that $\mathbf{D} = \varepsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$ (ε is the dielectric constant, μ is the magnetic permeability). In the ether, $\varepsilon \to \varepsilon_0$, $\mu \to \mu_0$. In a source-free region of space, $\rho = J = 0$.

$$\boxed{\left[\nabla^2 - \varepsilon_0\mu_0\frac{\partial^2}{\partial t^2}\right]\mathbf{E} = 0\,, \quad \left[\nabla^2 - \varepsilon_0\mu_0\frac{\partial^2}{\partial t^2}\right]\mathbf{B} = 0}$$

wave equations!

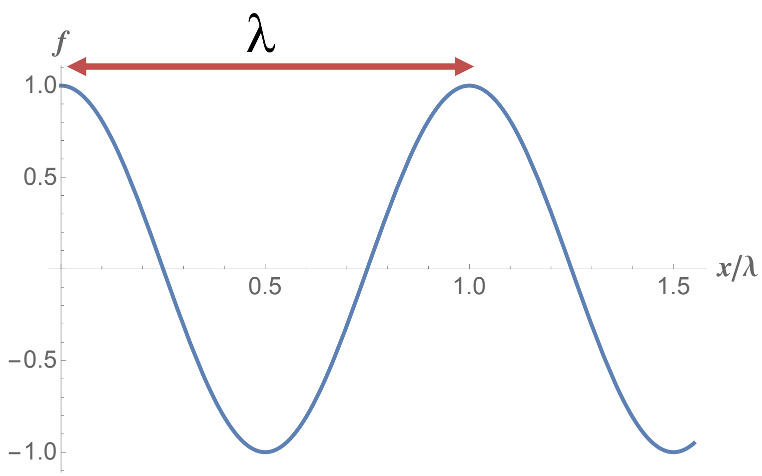
Wave Equation

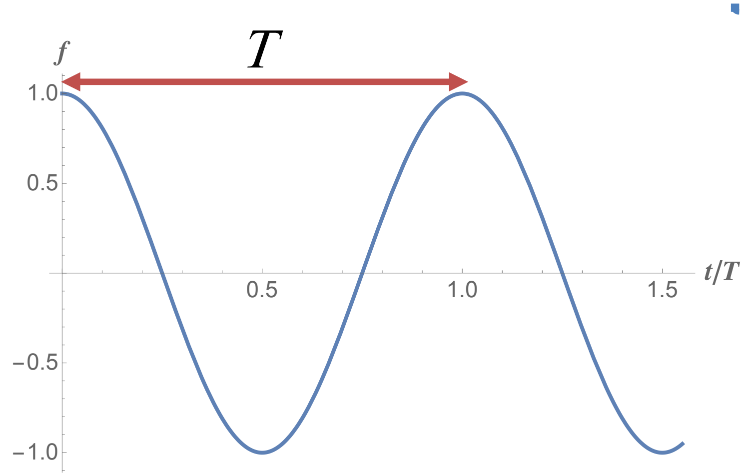
Every good Cambridge Wrangler knew the general form of a wave equation:

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right] f(t, \mathbf{x}) = 0$$

where v is the speed of the traveling wave. Consider motion in one spatial dimension, $\mathbf{x} \to x$:

$$f(t, x) = A \sin(kx + \omega t) + B \cos(kx + \omega t) \qquad k \equiv \frac{2\pi}{\lambda}\,, \ \omega \equiv \frac{2\pi}{T}$$





$$v = \frac{\lambda}{T} = \frac{\omega}{k}$$

Wave Equation

Every good Cambridge Wrangler knew the general form of a wave equation:

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right] f(t,\mathbf{x}) = 0$$

where v is the speed of the traveling wave. Maxwell had found

$$\boxed{\left[\nabla^2 - \varepsilon_0\mu_0\frac{\partial^2}{\partial t^2}\right]\mathbf{E} = 0\,,\ \left[\nabla^2 - \varepsilon_0\mu_0\frac{\partial^2}{\partial t^2}\right]\mathbf{B} = 0}$$

When he substituted the best-known values for ε_0 and μ_0, he found $\varepsilon_0\mu_0 \simeq \frac{1}{c^2}$ where $c = 3 \times 10^5$ km/s, the speed of light (known from astronomical measurements). So he concluded that light was *nothing other than* waves of **E** and **B**, traveling through the luminiferous ether at a speed $v = 1/\sqrt{\varepsilon_0\mu_0} = c$.

Lorentz: Generalize for Moving Bodies

Beginning in the 1880s, the Dutch mathematical physicist *Hendrik Lorentz* tried to generalize Maxwell's work: what if the emitter or receiver of light was *moving*?

Andries van Eertvelt, "Dutch ships sailing off a rocky shore," ca. 1610

Galilean coordinate transformation:

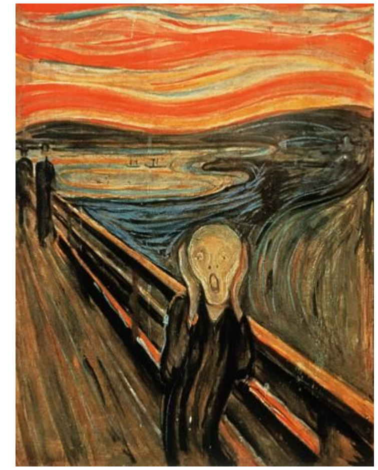
$$\boxed{\begin{aligned} x' &= x + vt\,, \\ y' &= y\,, \\ z' &= z\,, \\ t' &= t \end{aligned}}$$

But then Maxwell's equations yielded $\left[\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\left(\frac{\partial}{\partial t'} - v\frac{\partial}{\partial x'}\right)^2\right]\mathbf{E}'(t', x') = 0$, and similarly for $\mathbf{B}'$.

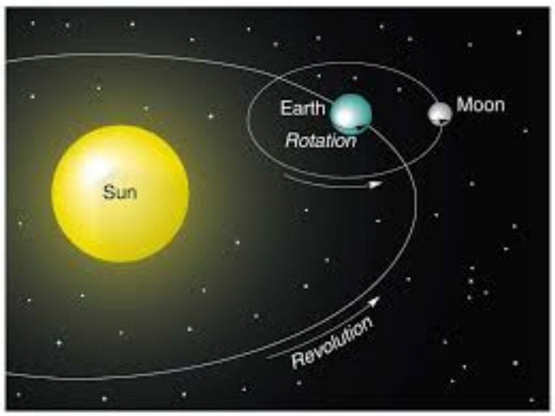
Lorentz: Generalize for Moving Bodies

If $\left[\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\left(\frac{\partial}{\partial t'} - v\frac{\partial}{\partial x'}\right)^2\right]\mathbf{E}'(t', x') = 0$, then the solutions

$$\mathbf{E}'(t', x') \neq A\cos(kx + \omega t) + B\sin(kx + \omega t)$$



Edvard Munch, "The Scream," 1893
Image is in the public domain.



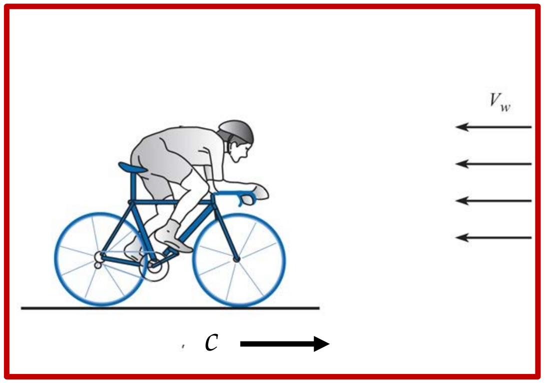
Lorentz was concerned: the *Earth* is moving through the ether, and yet *we* measure light to behave like sines and cosines all the time...

His response (part 1, 1895): introduce a *new time variable*, "local time," $t' = t'(x, t, v)$. Then he could save the form of Maxwell's wave equation, even for bodies in motion.

Lorentz always considered this a "mathematical trick." Local time was "fictitious," and genuine time t always referred to the ether rest frame.

Questions?

The Michelson-Morley Experiment

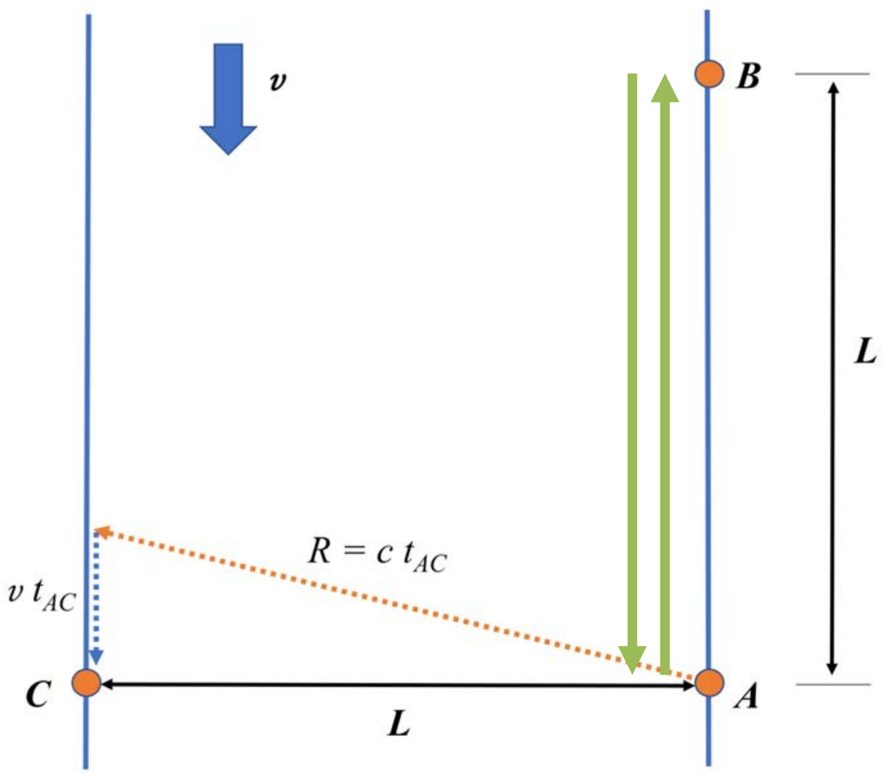


Like all his colleagues, Lorentz knew that light propagated in the ether. A natural question to ask: Could one detect the Earth's *motion through the ether*, based on the behavior of light on Earth? The effect should be like a bicyclist feeling a head-wind when moving through the atmosphere.

Beginning in the early 1880s, the American experimental physicist *Albert Michelson* sought to measure such an effect. He designed and built a new type of device — an *interferometer* — to try to measure effects of the Earth's motion through the ether on the propagation of light.

Michelson's work was some of the first work by physicists in the US that leading physicists in Europe considered worth paying attention to. He became the first US-based physicist to win the Nobel Prize (1907).

The Michelson-Morley Experiment



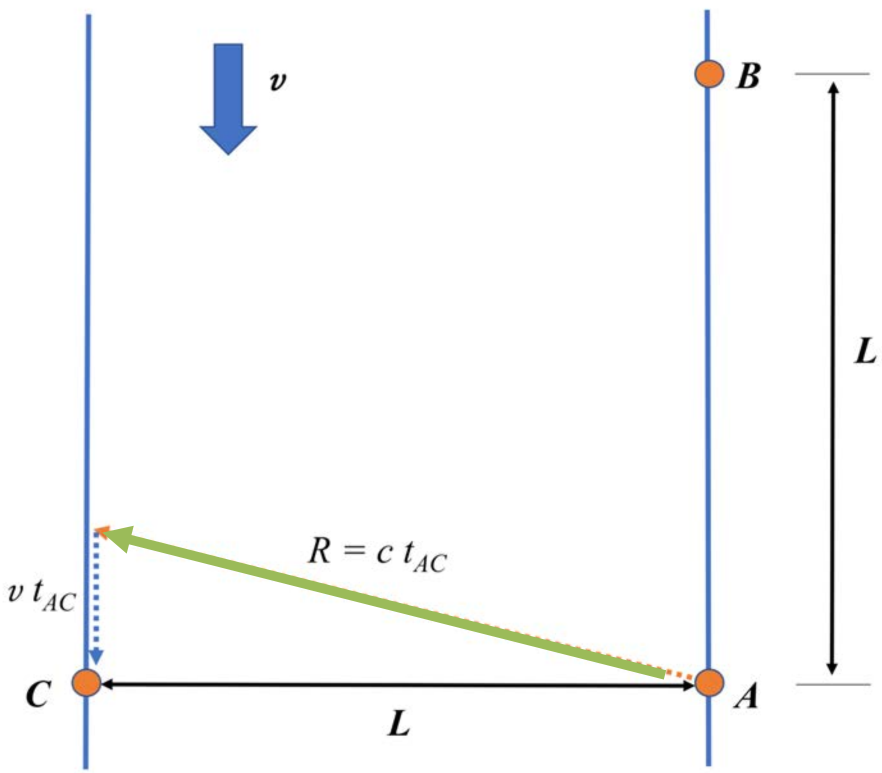
Michelson's insight was to use the *interference of light* as a very sensitive test.

Analogy: two swimmers in a river. Each must begin at A, swim a distance L and return to A. They can only swim at speed c with respect to the water. Meanwhile, the river flows with a current of speed v.

Swimmer 1: $t_{AB} = \dfrac{L}{(c-v)}$ $\qquad t_{BA} = \dfrac{L}{(c+v)}$

$$t_{ABA} = \frac{L}{(c-v)} + \frac{L}{(c+v)} = \frac{2L}{c}\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]}$$

The Michelson-Morley Experiment



Michelson's insight was to use the *interference of light* as a very sensitive test.

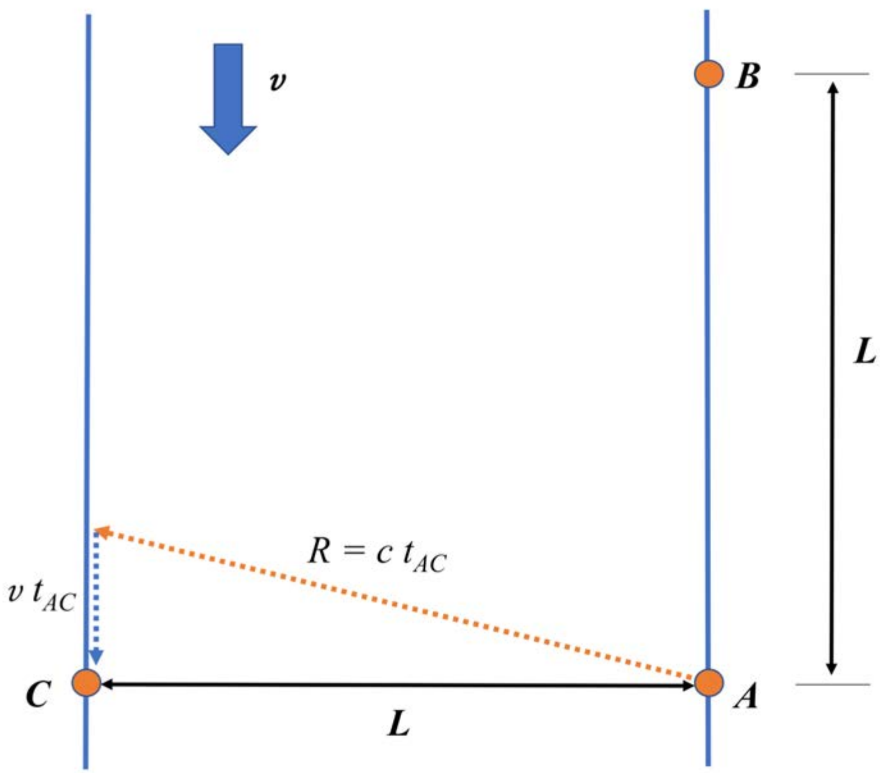
Analogy: two swimmers in a river. Each must begin at A, swim a distance L and return to A. They can only swim at speed c with respect to the water. Meanwhile, the river flows with a current of speed v.

Swimmer 2:

$$R^2 = (vt_{AC})^2 + L^2 \longrightarrow (ct_{AC})^2 = (vt_{AC})^2 + L^2$$

$$t_{ACA} = \frac{2L}{c}\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

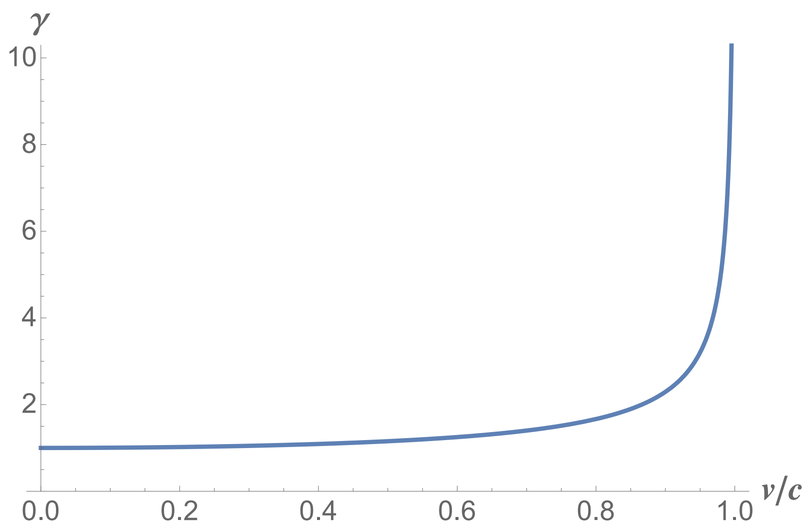
The Michelson-Morley Experiment



Who wins?

$$t_{ABA} = \frac{2L}{c}\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]} \qquad t_{ACA} = \frac{2L}{c}\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

The Michelson-Morley Experiment



Who wins?

$$t_{ABA} = \frac{2L}{c}\frac{1}{\left[1-\left(\frac{v}{c}\right)^2\right]} \qquad t_{ACA} = \frac{2L}{c}\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

Define $\gamma \equiv \dfrac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$ (Note $\gamma \geq 1$ for $0 \leq v \leq c$.)

$$t_{ABA} = \frac{2L}{c}\gamma^2 \qquad t_{ACA} = \frac{2L}{c}\gamma$$

When there is a current in the river ($v \neq 0$), Swimmer 1 takes *longer* to complete her lap than does Swimmer 2.

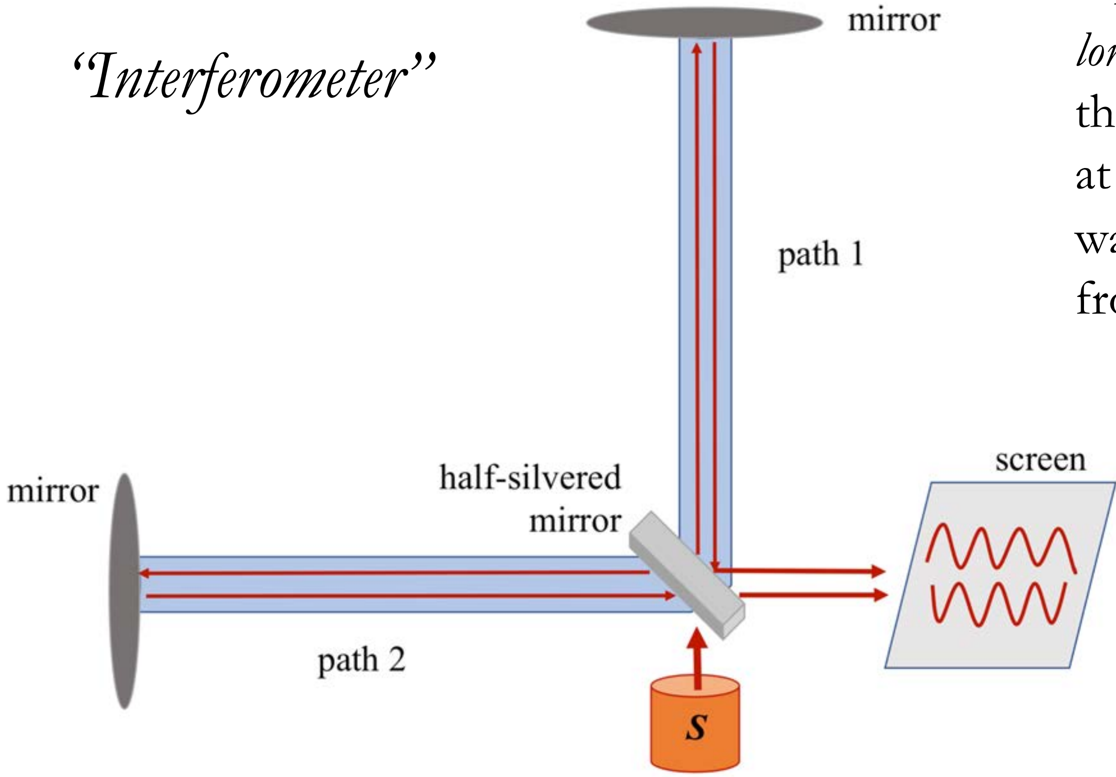
$$\Delta t = t_{ABA} - t_{ACA} = \left(\frac{2L}{c}\right)\gamma(\gamma - 1)$$

$$= \left(\frac{L}{c}\right)\left(\frac{v}{c}\right)^{2} + \mathcal{O}\left[\left(\frac{v}{c}\right)^4\right]$$

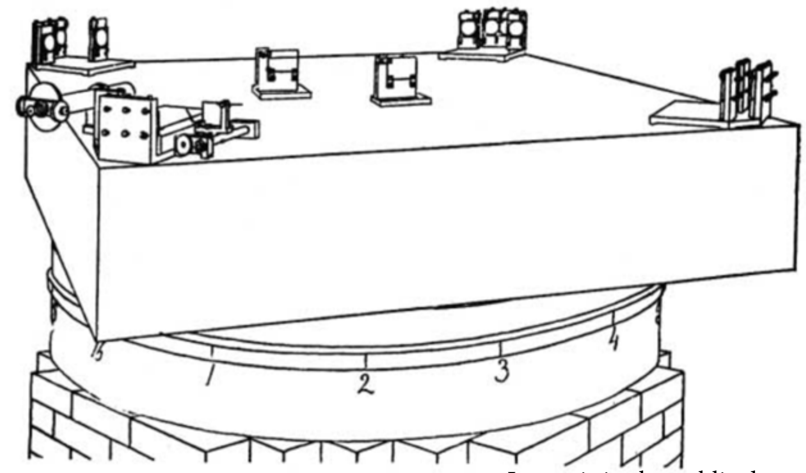
"second order effect"

The Michelson-Morley Experiment

"Interferometer"

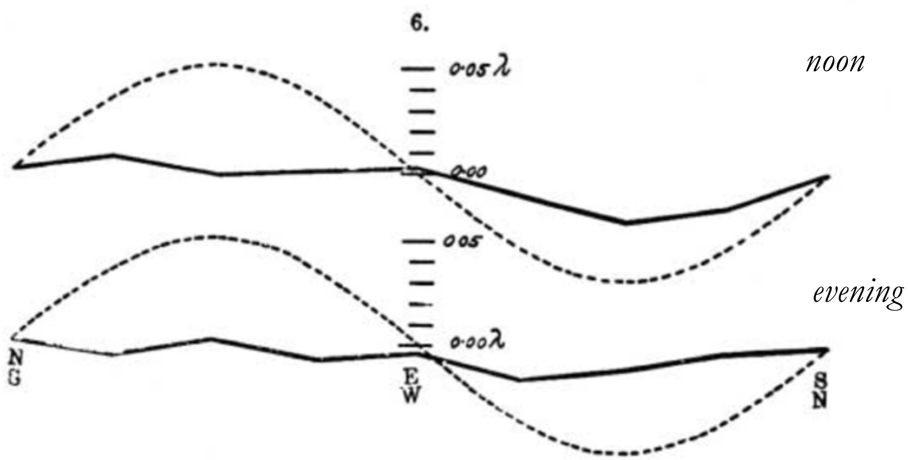


If the light waves that travel path 1 take *longer* to complete a lap than the light waves that travel path 2, then the waves will arrive at the screen *out of phase*: crests from one wave will no longer line up with crests from the other. *Interference*!



1887 experiment: L = 11 meters; entire apparatus floating in a vat of mercury to dampen vibrations. (*Don't try this at home!*)

The Michelson-Morley Experiment



From Michelson and Morley (1887)

Dotted curves show *one-eighth* the expected magnitude of the effect; solid lines show the measured shift in interference fringes.

Results consistent with $\Delta t = 0$. To the end of his life (1927), Michelson considered his experiment a failure.

Sensitive to $(v/c)^2 \sim 10^{-8}$!

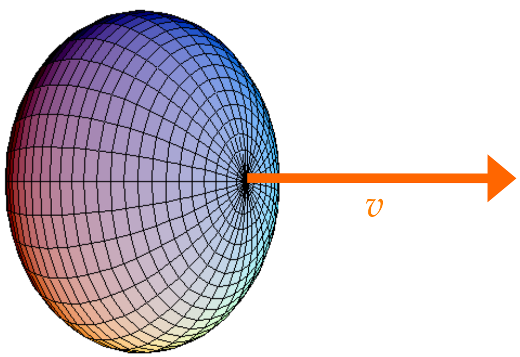
And yet ... “null result”:

“It seems fair to conclude from the figure that if there is any displacement due to the relative motion of the earth and the luminiferous ether, this cannot be much greater than 0.01 of the distance between the fringes. [...] The actual displacement was certainly less than the twentieth part [of the expected value] and probably less than the fortieth part.” (pp. 340-341)

Questions?

Lorentz Contraction

Lorentz followed Michelson's work closely. To account for the null result, Lorentz argued that physicists had neglected a *physical contraction* of the apparatus along its direction of motion through the ether.



$$\boxed{L \rightarrow L' = \frac{L}{\gamma}}$$

The molecules in the arm of the interferometer would be *squeezed* due to their motion through a physical, resistive medium, much like a beach ball dragged under water.

That would shorten the *length* of the arm that was heading directly into the "ether wind": *length contraction*!

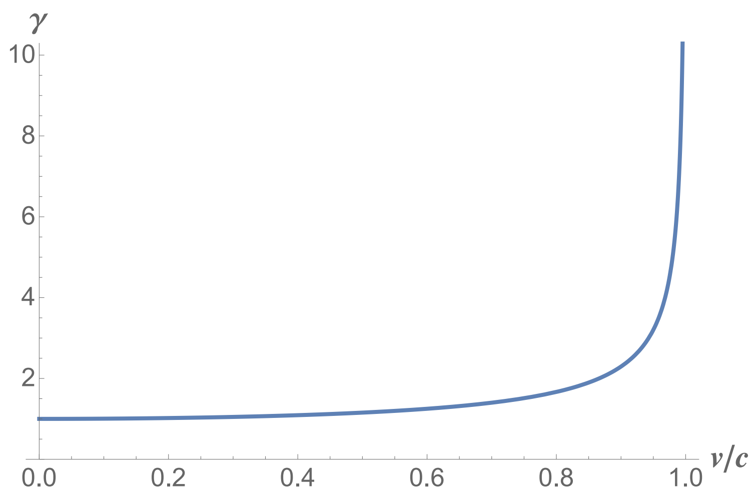
In that case, Swimmer 1 would have *less distance to cover* for her lap, and her lap-time would become

$$t'_{ABA} = \frac{2L'}{c}\gamma^2 = \frac{2}{c}\left(\frac{L}{\gamma}\right)\gamma^2 = \frac{2L}{c}\gamma = t_{ACA}$$

Then $\Delta t = 0$: the race *should* be a tie, after all. No wonder Michelson and Morley measured no shift in the interference fringes.

Lorentz Transformation

Lorentz combined his two responses to the electrodynamics of moving bodies — “local time” and length contraction — to derive a *new* set of coordinate transformations for bodies moving with respect to the ether (*replacing* the Galilean transformation):



Galilean transformation

$$x' = x + vt\,,$$
$$y' = y\,,$$
$$z' = z\,,$$
$$t' = t$$

Lorentz transformation

$$x' = \gamma\,(x + vt)\;,$$
$$y' = y\,,$$
$$z' = z\,,$$
$$t' = \gamma\left(t + \frac{vx}{c^2}\right)$$

Note the factors of $\gamma \equiv \dfrac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = 1 + \dfrac{1}{2}\left(\dfrac{v}{c}\right)^2 + \mathcal{O}\left[\left(\dfrac{v}{c}\right)^4\right]$: imperceptible change for $v << c$.

Wave Equation Restored

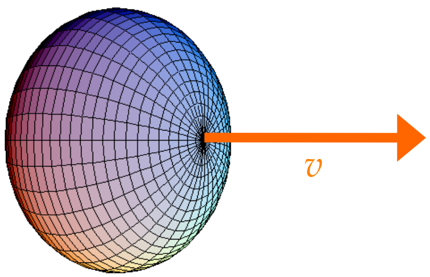
When Lorentz used his new coordinate transformation to reconsider Maxwell's wave equation for **E** and **B** fields, when either the emitter or receiver were moving with respect to the ether, he found the original form restored:

$$\boxed{\left[\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}\right]\mathbf{E}'(t', x') = 0}$$

and likewise for $\mathbf{B}'$. Even on a moving Earth, light should travel at speed *c* and its **E** and **B** fields should oscillate like sines and cosines.

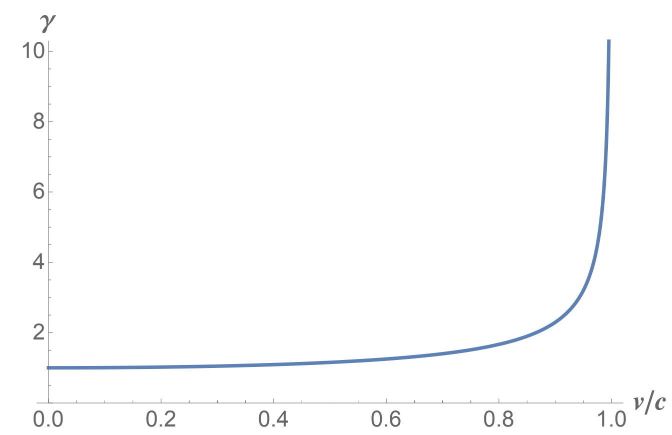
Lorentz Summary

Lorentz addressed two puzzles about the electrodynamics of moving bodies: *mathematical* ("local time") and *experimental* (null results from the Michelson-Morley experiment).



He argued that since the ether was a physical, elastic medium, it would exert a *force* on objects moving through it. That force would yield a *physical contraction* of objects' lengths along their directions of motion through the ether.

The effects scaled with γ, and hence would remain small for $v << c$.



General strategy: begin with *dynamics* (the study of forces) in order to account for *kinematics* (motion of objects).

(An unknown patent clerk in Switzerland had other ideas …)

STS.042J / 8.225J Einstein, Oppenheimer, Feynman: Physics in the 20th Century
Fall 2020