Rethinking Light

\[ E = f(\lambda). \]
\[ T = 1650 \text{°abs.} \]

8.225 / STS.042, Physics in the 20th Century
Professor David Kaiser, 30 September 2020
“Old Quantum Theory” (~ 1900 – 1924*)

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* dates are approximate! As we will see, physicists debated and revisited each of these topics over a span of years.

* # See Lecture Notes on “Blackbody radiation and Compton scattering”

day’s topic
Germany became a united country in 1871, and accelerated a program of rapid industrialization. In 1887, at the urging of scientists and engineers, the government founded the Physikalisch-Technische Reichsanstalt* (PTR) to foster research at the intersections of basic science, applied research, and industrial development. (Akin to the U.S. Bureau of Standards, now the National Institute of Standards and Technology [NIST].)

Early, pressing challenge: evaluate competing proposals for large-scale electric street lighting.

*Imperial Physical-Technical Institute
Blackbody Radiation

When an object is heated to a high enough temperature, it will *glow*. The color of the emitted light shifts with the temperature to which the object has been heated.

While working on calibrations for various electric lighting proposals, researchers at the PTR noted a "universal glow": the pattern of light emitted from various objects when they were heated to sufficiently high temperatures seemed to be *independent* of the composition of those materials.

They postulated that for an ideal "blackbody" — an object that absorbed all light that fell upon it, reflecting none — there might exist a single, universal *spectrum*, or pattern of the intensity of light emitted at various wavelengths.
Blackbody Radiation

The universal blackbody spectrum seemed interesting for at least two reasons: it could be useful for *industrial calibrations and standardization*, and it might offer clues about *fundamental interactions* between light and matter. So PTR researchers collected more and more data on emissions from (near) blackbodies throughout the 1880s and 1890s.

![Graph of emitted energy density vs. frequency](image)

**Lummer and Pringsheim, blackbody spectrum, 1890s**

Image is in the public domain.

\[ u(\nu, T) \]: energy density, how much energy was radiated per volume per frequency \( \nu \).

As the temperature rises, the intensity of the emitted light rises and the frequency of the peak emission rises.

\[ T_1 < T_2 < T_3 \]

Recall: \( \nu = \frac{c}{\lambda} \)
Blackbody Radiation

As the empirical pattern of the blackbody spectrum was becoming more clear, how to explain that pattern was becoming more puzzling.

Max Planck was following the developments at the PTR closely. He became the new Ordinarius professor of theoretical physics in Berlin in 1892, and was an expert in statistical mechanics.

In thermal equilibrium, each degree of freedom should have average energy $kT$ (“equipartition theorem”).

Combined with Maxwell’s treatment of electromagnetic radiation, that yielded $u(v, T) \sim kT \nu^2$: the energy density should rise without limit as the frequency of emitted light increased!

$u(v, T)$: energy density, how much energy was radiated per volume per frequency $\nu$. 

Lummer and Pringsheim, blackbody spectrum, 1890s Image is in the public domain.
Blackbody Radiation

Given his proximity to the PTR, Planck had access to better, more up-to-date data on blackbody spectra than most of his colleagues. He knew not only that the \( u(\nu,T) \) curve must fall with increasing \( \nu \); he had empirical evidence about how sharply the curve actually fell. So he began to tinker.

On December 14, 1900, he presented a paper to the German Physical Society:

\[
u(\nu, T) \sim \frac{\nu^3}{[e^{h\nu/kT} - 1]} \]

\( h \): a new, universal constant, now known as “Planck’s constant.”

\[\text{For } \nu \ll kT, \quad u(\nu, T) \sim kT \nu^2\]

\[\text{For } \nu \gg kT, \quad u(\nu, T) \sim \nu^3 e^{-h\nu/kT}\]
The Quantum Scale

In his 1900 treatment of blackbody radiation, Planck introduced a new universal constant, $h$. It set the scale for departures from physical descriptions based solely on Newton’s or Maxwell’s equations.

$$h = 6.261 \times 10^{-27} \text{ erg-seconds}$$

On the scale of human-sized objects, $h$ is incredibly small:

Drop a grape from 5 cm: $E \Delta t \sim 10^{28} h$

Fly buzzing your head: $|L| \sim 10^{28} h$

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The Origin of Quantum Theory?

Scientists typically say that Planck launched a new “quantum revolution” with his work in 1900. In modern derivations, we arrive at Planck’s expression for \( u(\nu, T) \) by stipulating that the energy exchanged between matter and radiation must be quantized, in units \( E = h\nu \), rather than treated as continuous. Is that what Planck thought he was doing?

Kuhn: In Planck’s 1900 derivation, he fixed the total energy of the system to be an integer number of units of \( h\nu \),

\[
E_{\text{total}} = \sum_i E_i = Nh\nu
\]

but he did not require \( E_i = h\nu \). In fact, in Planck’s description, the energies for sub-systems fell “in the range \( (E_i, E_i + \Delta E_i) \).” Planck used bins of size \( \varepsilon = h\nu \) for his accounting, but then counted how many oscillators of continuous energy fell within bins \( [0, \varepsilon], [\varepsilon, 2\varepsilon], \) and so on.

In his 1906 lectures, Planck still spoke of continuous (rather than quantized) energy exchange.

The Origin of Quantum Theory?

Scientists typically say that Planck launched a new “quantum revolution” with his work in 1900. In modern derivations, we arrive at Planck’s expression for $u(\nu,T)$ by stipulating that the energy exchanged in units $E = h\nu$, rather simply an act of desperation. ... [Introducing bins $\varepsilon = h\nu$] was purely to Kuhn. In Planck’s letter to a colleague, 1931: “What I did can be described as a formal assumption and I really did not give it much thought...”

be an integer number of units of $h\nu$,

$$E_{\text{total}} = \sum_i E_i = N h\nu$$

but he did not require $E_i = h\nu$. In fact, in Planck’s description, the energies for sub-systems fell “in the range $(E_i, E_i + \Delta E_i)$.” Planck used bins of size $\varepsilon = h\nu$ for his accounting, but then counted how many oscillators of continuous energy fell within bins $[0, \varepsilon]$, $[\varepsilon, 2\varepsilon]$, and so on.

In his 1906 lectures, Planck still spoke of continuous (rather than quantized) energy exchange.

(The debate continues: see M. Nauenberg, American Journal of Physics 84 (2016): 709-720.)
Questions?
Photoelectric Effect

In 1900, Planck had been concerned with the interaction between light and matter, not the propagation of light on its own. In 1905, Einstein began to think about the nature of light itself.

March 1905: light quanta
May 1905: Brownian motion
June 1905: special relativity
September 1905: $E = mc^2$

In particular, he was trying to make sense of recent experimental results (1902) from Philipp Lenard on the ejection of electrons from a metal plate, when high-energy radiation shines on it.
Photoelectric Effect

In his experiments, Lenard applied a tunable voltage $V$ across conducting plates. When one of the plates was irradiated with ultraviolet light (UV), electrons were ejected and traveled to the other plate, completing a circuit (hence current was measured in the ammeter $A$). By adjusting the voltage, Lenard could measure how much energy the electrons had upon being ejected from the plate (“stopping voltage”), and compare that with the frequency of the radiation that had shone onto the plate.

Lenard’s data indicated that there was some threshold frequency, below which no electrons were ejected. Above that frequency, the electrons’ energy rose linearly with the frequency of incident light, but was independent of the intensity of the light.

![Diagram showing photoelectric effect](image)
Photoelectric Effect

Why were Lenard’s results puzzling?

According to Maxwell’s work, the energy carried by a light wave is proportional to its intensity:

\[ E \sim I \sim |E|^2 + |B|^2 \]

So why couldn’t Lenard’s device eject electrons with high-intensity light of low frequency? If light acted like Maxwellian waves, there should be no threshold frequency \( \nu_0 \) in the photoelectric experiments.

Likewise, according to Maxwell’s work, the energy imparted to the electrons that were ejected should have been proportional to the light wave’s intensity, with no particular dependence on the light’s frequency.
In what he considered the “most radical” of his papers of 1905, Einstein offered a “heuristic” explanation of the photoelectric effect: *What if light itself were quantized?*

“It seems to me that the observations about [...] the production of cathode rays by ultraviolet light [...] are more readily understood if one assumes that the energy of light is *discontinuously distributed* in space. [As suggested here,] the energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a *finite number of energy quanta* which are localized at points in space, which move without dividing, and which can only be produced and absorbed as *complete units.*”

* heuristic means “suggestive,” worth considering, not definitive.
where \( \Phi \) (the “work function” of the metal) characterized how tightly electrons were bound to their atoms in the plate.

If each light quantum carried energy

\[
E_{\text{light quantum}} = h\nu
\]

then following a collision with an electron in the metal plate, the electron would acquire energy

\[
E_{\text{electron}} = h\nu - \Phi
\]

\( \nu \) was very close to Planck’s new constant, \( h \).

The threshold frequency was then given by \( \nu_0 = \Phi/h \): only when each incoming light quantum had sufficient energy would electrons overcome the work function and be ejected.
Einstein suggested: the light shining on the metal plate was not like an extended, continuous wave washing up on shore, but like a shower of marbles, each of which could collide with electrons in the metal.

Einstein also re-derived Planck’s blackbody formula for $u(\nu, T)$, assuming the radiation consisted of a gas of individual particles with energies $E = h\nu$.

So: everyone was convinced, right? Enormous skepticism about the light-quantum hypothesis for ~15 years.
Questions?
More physicists began to take Einstein’s light-quantum hypothesis seriously after experiments by Arthur Compton at the University of Chicago in 1922.

Compton scattered high-energy X rays off of electrons in graphite and measured the change in wavelength of the X rays after scattering.

\[ \Delta \lambda = \lambda' - \lambda \propto (1 - \cos \theta) \]

Compton could only make sense of his results by thinking in terms of two-body scattering among particles: light-quantum and electron.
Compton Scattering

Before collision

\[ p_{\text{light}} = \frac{E}{c} \quad \text{(Maxwell)} \]
\[ E_{\text{photon}} = h \nu = \frac{hc}{\lambda} \quad \text{(Einstein)} \]
\[ p_{\text{photon}} = \frac{h \nu}{c} = \frac{h}{\lambda} \]

\[ p_{\text{electron}} = 0 \quad \text{(at rest)} \]
\[ E_{\text{electron}} = mc^2 \quad \text{(Einstein)} \]

After collision

\[ p'_{\text{photon}} = \frac{h}{\lambda'} \]
\[ E'_{\text{photon}} = \frac{hc}{\lambda'} \]
\[ p'_{\text{electron}} = \gamma mv \]
\[ E'_{\text{electron}} = \gamma mc^2 \]

\( \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \)
Compton Scattering

Before collision

\[ p_{\text{light}} = \frac{E}{c} \] (Maxwell)
\[ p_{\text{electron}} = 0 \] (at rest)
\[ E_{\text{photon}} = h \nu = \frac{hc}{\lambda} \] (Einstein)
\[ E_{\text{electron}} = mc^2 \] (Einstein)

After collision

\[ p'_{\text{photon}} = \frac{h}{\lambda'} \]
\[ p'_{\text{electron}} = \gammamv \]
\[ E'_{\text{photon}} = \frac{hc}{\lambda'} \]
\[ E'_{\text{electron}} = \gamma mc^2 \]

Conserve energy:
\[ E_{\text{photon}} + E_{\text{electron}} = E'_{\text{photon}} + E'_{\text{electron}} \]
\[ \frac{hc}{\lambda} + mc^2 = \frac{hc}{\lambda'} + \gamma mc^2 \]

Conserve momentum:
\[ p_x : \quad \frac{h}{\lambda} + 0 = \frac{h}{\lambda'} \cos \theta + \gamma mv \cos \varphi \]
\[ p_y : \quad 0 + 0 = \frac{h}{\lambda'} \sin \theta - \gamma mv \sin \varphi \]
Compton Scattering

Compton treated light like *particles*, yet measured changes in *wavelength*: a *wave-particle duality!*

**Conserve energy:**
\[
\frac{E_{\text{photon}} + E_{\text{electron}}}{\lambda} = \frac{E'_{\text{photon}} + E'_{\text{electron}}}{\lambda'} + mc^2
\]

**Conserve momentum:**
\[
\begin{align*}
px & : \quad \frac{h}{\lambda} + 0 = \frac{h}{\lambda'} \cos \theta + \gamma mv \cos \varphi \\
py & : \quad 0 + 0 = \frac{h}{\lambda'} \sin \theta - \gamma mv \sin \varphi
\end{align*}
\]

By treating the problem just like (ordinary) two-body scattering among particles, Compton now had three equations and three unknowns: \( \lambda', \theta, \varphi \).

\[
\Delta \lambda = \lambda' - \lambda = \left( \frac{h}{mc} \right) (1 - \cos \theta)
\]
Rethinking Light: Summary

A new priority upon industrialization led to investment in institutions like the PTR in the 1880s. Researchers studied *blackbody radiation* both to aid with industrial standardization and for clues about the universal behavior of light and matter. *Max Planck* found a way to characterize the energy of the emitted radiation by *avoiding* the usual calculation and inserting a new fundamental constant \( h \).

*Albert Einstein* took Planck’s work more seriously than even Planck did, and offered a “heuristic” suggestion that light was *quantized*. Then he could account for the photoelectric effect. Most physicists remained deeply skeptical.

*Arthur Compton* studied the scattering of X rays off of electrons. He could only make sense of the *change in wavelength* by invoking Einstein’s light-quantum hypothesis, which suggested a *wave-particle duality*. 