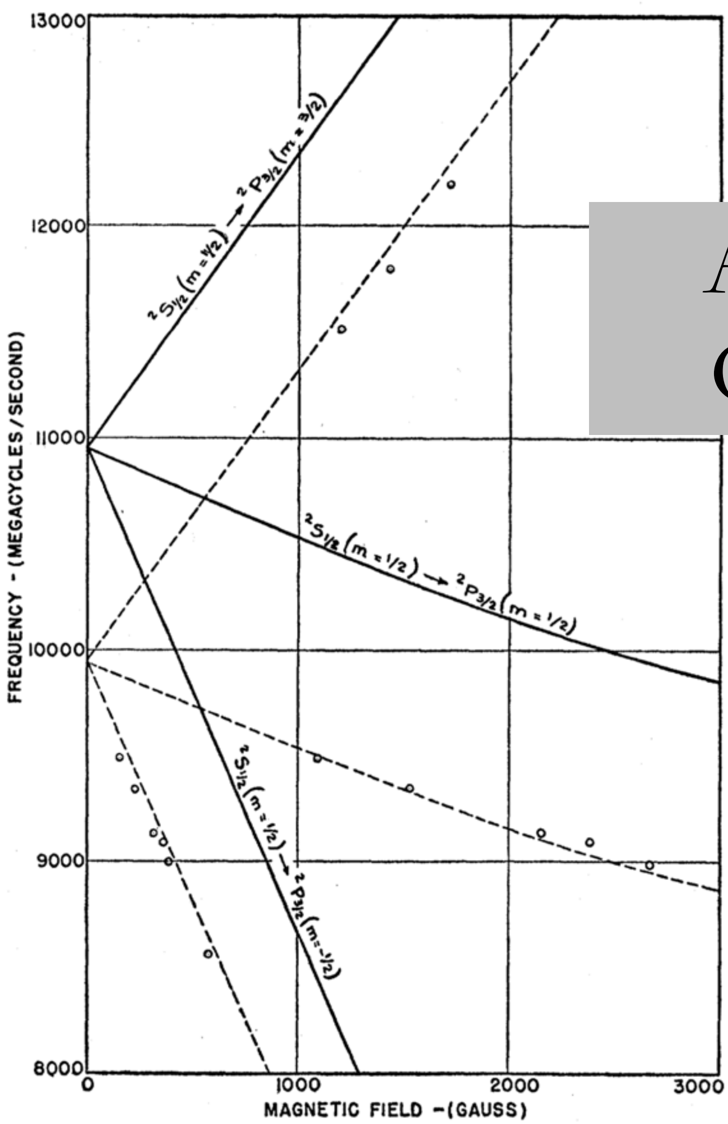


A Conservative Revolution: QED and Renormalization

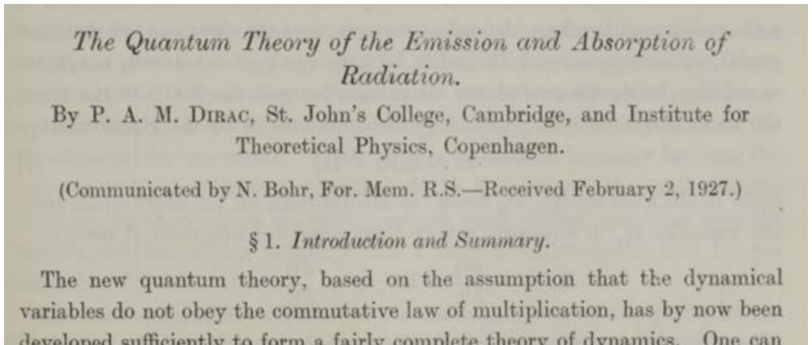
8.225 / STS.042, Physics in the 20th Century
Professor David Kaiser, 16 November 2020

1. Quantum Fields and Infinities

2. Quantum Electrodynamics after the War: New Experiments

3. Julian Schwinger and Renormalization

Quantum Field Theory



The Quantum Theory of the Emission and Absorption of Radiation.

By P. A. M. Dirac, St. John's College, Cambridge, and Institute for Theoretical Physics, Copenhagen.

(Communicated by N. Bohr, For. Mem. R.S.—Received February 2, 1927.)

§ 1. *Introduction and Summary.*

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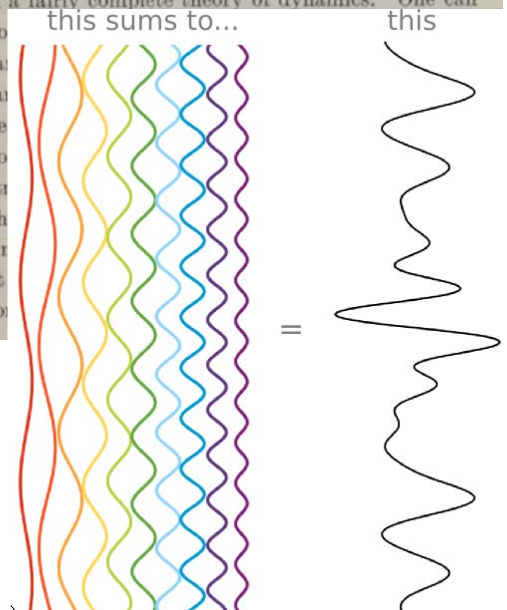

Fourier analysis (from Evan Bianco, AgileScientific)

Nearly as soon as physicists had cobbled together quantum mechanics in 1925-26, several began trying to apply the new formalism to *radiation*. After all, Einstein had suggested that light consists of *quanta* back in 1905. *Paul Dirac* proposed a quantized description of Maxwellian electromagnetic fields in 1927, thereby helping to launch *quantum field theory*.

Using *Fourier analysis*, we may represent an arbitrary curve as a *superposition* of waves with specific wavenumbers $|\mathbf{k}| = 2\pi/\lambda$:

$$F(t,\mathbf{x}) = \int_0^\infty \frac{d^3k}{(2\pi)^{3/2}} \tilde{F}_{\mathbf{k}}(t)\, e^{i\mathbf{k}\cdot\mathbf{x}}$$

Dirac: generalize $[\hat{\mathbf{x}}, \hat{\mathbf{p}}] \equiv \hat{\mathbf{x}}\hat{\mathbf{p}} - \hat{\mathbf{p}}\hat{\mathbf{x}} = i\hbar$ for *fields* to

$$[\hat{\phi}(t,\mathbf{x}), \hat{\Pi}(t,\mathbf{y})] = i\hbar\, \delta^{(3)}(\mathbf{x}-\mathbf{y})$$

$$\hat{\phi}(t,\mathbf{x}) = \int_0^\infty \frac{d^3p}{(2\pi)^{3/2}} \left\{ f_{\mathbf{p}}(t)\hat{a}_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} + f^*_{\mathbf{p}}(t)\hat{a}^\dagger_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}} \right\}$$

Quantum Field Theory

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§ 1. *Introduction and Summary.*

The new quantum theory, based on the assumption that the dynamical variables do not obey the commutative law of multiplication, has by now been developed sufficiently to form a fairly complete theory of dynamics. One can treat mathematically the problem of any dynamical system composed of a number of particles with instantaneous forces acting between them, provided it is describable by a Hamiltonian function, and one can interpret the mathematics physically by a quite definite general method. On the other hand, hardly anything has been done up to the present on quantum electrodynamics. The questions of the correct treatment of a system in which the forces are propagated with the velocity of light instead of instantaneously, of the production of an electromagnetic field by a moving electron, and of the reaction of this field on the electron have not yet been touched. In addition, there is a serious difficulty in making the theory satisfy all the requirements of the restricted

Paul Dirac's first publication on a quantum-theoretical treatment of the electromagnetic field, 1927.

$$\hat{\phi}(t,\mathbf{x}) = \int_0^\infty \frac{d^3p}{(2\pi)^{3/2}}\left\{f_{\mathbf{p}}(t)\hat{a}_{\mathbf{p}}e^{i\mathbf{p}\cdot\mathbf{x}} + f^*_{\mathbf{p}}(t)\hat{a}^\dagger_{\mathbf{p}}e^{-i\mathbf{p}\cdot\mathbf{x}}\right\}$$

$\hat{a}^\dagger_{\mathbf{p}}$: creates one quantum in the state $(E_p, \mathbf{p})$

$\hat{a}_{\mathbf{p}}$: destroys one quantum in the state $(E_p, \mathbf{p})$

Then the state of the field $\hat{\phi}(t,\mathbf{x})$ can be represented as a collection of *quanta* in various states (E_p, **p**). Changing the field corresponds to *emitting* or *absorbing* quanta in specific states.

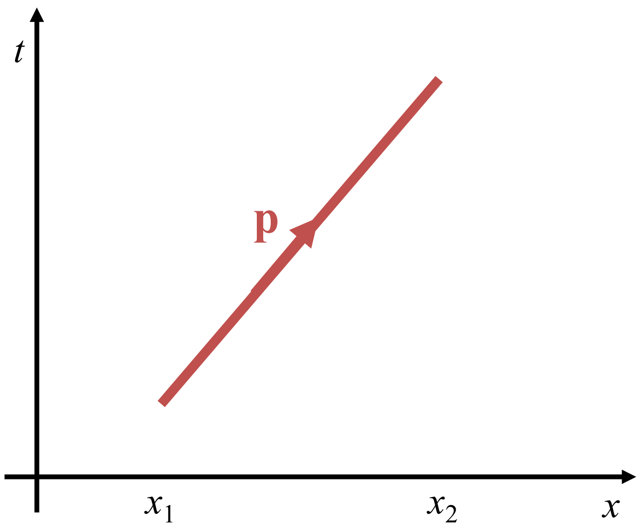
But the uncertainty principle allows for "virtual" processes that *temporarily* violate the conservation of energy:

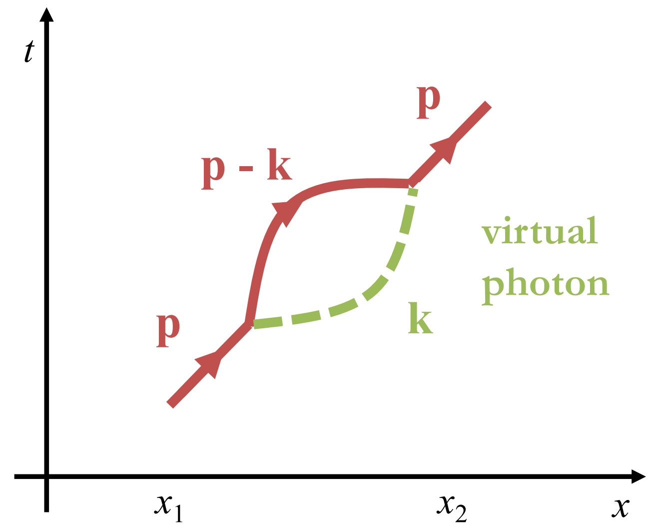
$$\Delta E\,\Delta t \geq \frac{\hbar}{2}.$$

Hence the *exact* state of the field at any given time can never be stipulated; there will always exist *quantum fluctuations*.

Virtual Particles: Self-Energy

Consider an electron traveling from location x_1 to x_2 with some momentum $\mathbf{p}$:



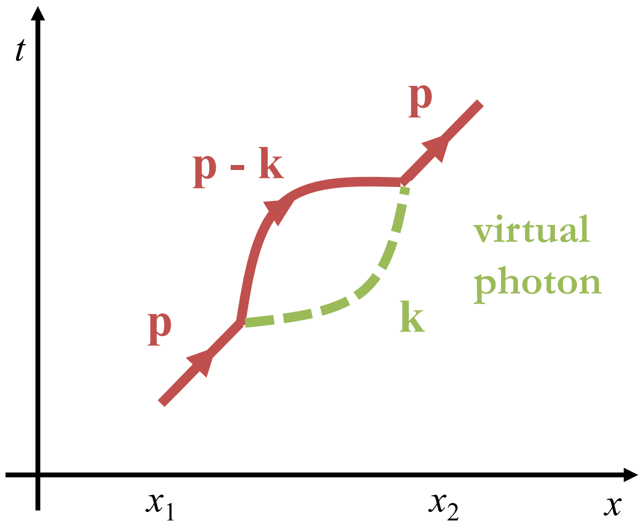


The virtual photon could "borrow" *any* amount of energy $\Delta E = c\hbar k$, with $k = |\mathbf{k}|$ and $0 \leq k \leq \infty$, as long as $\Delta t \sim \hbar/\Delta E$ was correspondingly brief. The **probability amplitude** $\sim 1/k$ to emit and re-absorb a virtual photon of momentum $\mathbf{k}$ (higher-momentum states less likely). But since the virtual photon could have *any* value $|\mathbf{k}|$, one must sum up all the possibilities:

$$A \sim \int \frac{dk}{k} = \ln k \longrightarrow \infty$$

The electron's "self-energy" ***diverges***!

Virtual Particles: Self-Energy



$$A \sim \int \frac{dk}{k} = \ln k \longrightarrow \infty$$

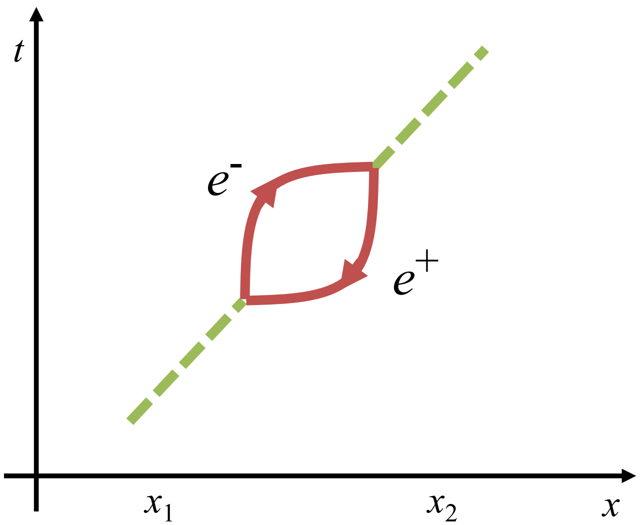
A first idea, which several physicists explored during the 1930s, was simply to *cut off* the integrals, rather than integrate up to $|\mathbf{k}| \rightarrow \infty$.

But inserting a cut-off k_{max} meant that the equations would no longer respect the symmetries of special relativity — even though this was *supposed* to describe (a quantized version of) Maxwell's equations, which *do* obey the relativistic symmetries.

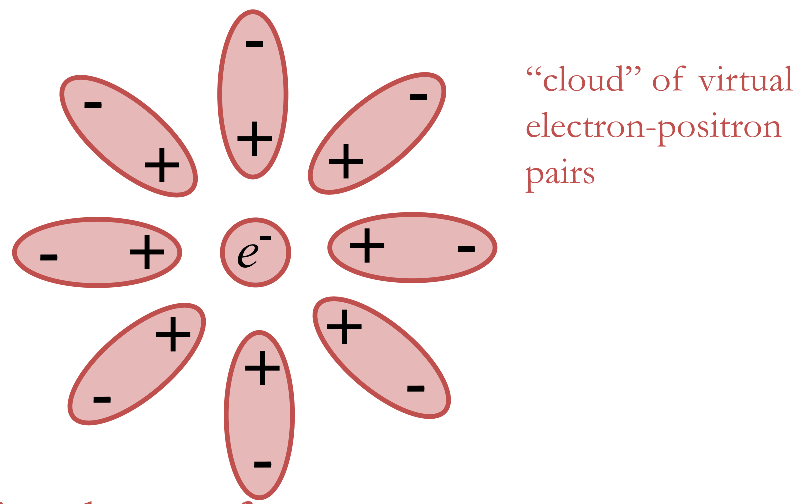
A practical consequence: if one simply inserted k_{max} by hand, then the results of any calculation would remain *ambiguous*: different inertial observers would no longer agree on observable quantities.

Virtual Particles: Vaccum Polarization

Similar virtual processes affected *electric charge*. A photon could spontaneously emit and later reabsorb a virtual electron-positron pair.



Hence one could ask about an electron's interaction with *its own electromagnetic field*. After all, the **E** and **B** fields consist of photons, any of which could spontaneously undergo these virtual processes. In the vicinity of an electron, *empty space itself* would become *polarized*.

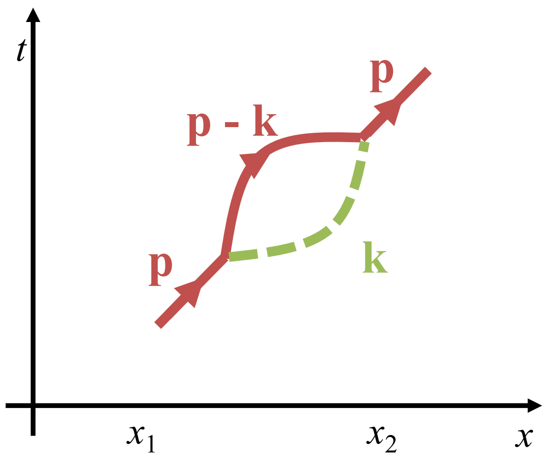


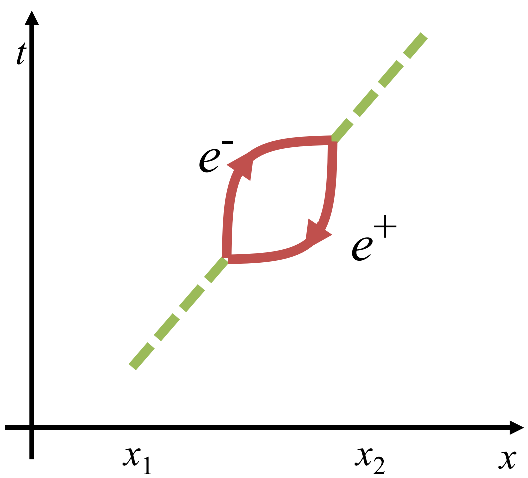
The original charge would be *screened*. By how much? Each virtual pair could borrow up to $|\mathbf{k}| \to \infty$, so

$$\delta e \sim \int \frac{dk}{k} = \ln k \longrightarrow \infty$$

The effective charge of an electron ***diverges***!

Virtual Particles: Interwar Responses





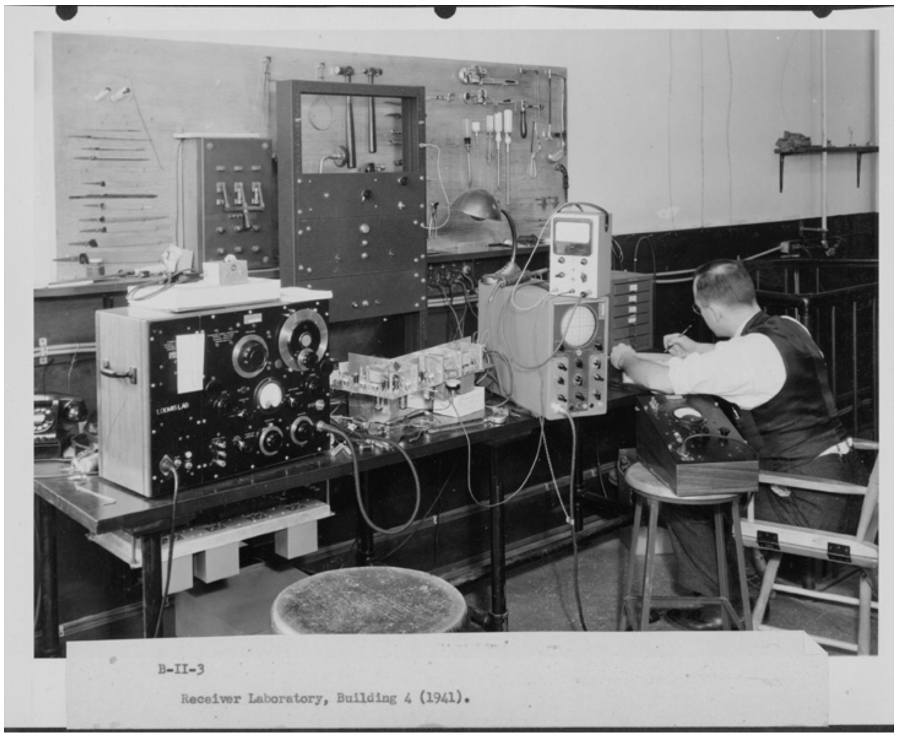
Several leading physicists in Europe, such as *Niels Bohr* and *Werner Heisenberg*, argued that the infinities in quantum electrodynamics pointed to the need for (yet another) radical conceptual revolution, akin to relativity and quantum mechanics.

Heisenberg: maybe there exists a *fundamental (shortest) length,* λ_0, and hence some largest possible momentum $\hbar k_{max} = h/\lambda_0 < \infty$. That would remove the infinities *but* require a radical rethinking of space and time, beyond relativity.

Others: maybe the idea of *virtual particles* is at fault, in which case the *uncertainty prinicple* at the heart of quantum mechanics would need to be revised.

Questions?

QED after the War: New Experiments



MIT Rad Lab "Receiver Laboratory," Building 4, 1941

During the Second World War, researchers at the MIT-based *Radiation Laboratory* had developed advanced capabilities to produce and measure centimeter-wavelength (microwave) electromagnetic radiation, for the radar project.

After the war, several Rad Lab researchers were able to take *surplus equipment* back to their universities, which enabled them to conduct new experiments with highly sensitive electronics.

Two of these experiments took place at Columbia University in the late 1940s by Rad Lab alumni, using surplus Rad Lab equipment: the *Lamb shift* and the *anomalous magnetic moment*.

The Lamb Shift

PHYSICAL REVIEW VOLUME 72, NUMBER 3 AUGUST 1, 1947

Fine Structure of the Hydrogen Atom by a Microwave Method* **

WILLIS E. LAMB, JR. AND ROBERT C. RETHERFORD
Columbia Radiation Laboratory, Department of Physics, Columbia University, New York, New York

(Received June 18, 1947)

THE spectrum of the simplest atom, hydrogen, has a fine structure[1] which according to the Dirac wave equation for an electron moving in a Coulomb field is due to the combined effects of relativistic variation of mass with velocity and spin-orbit coupling. It has been considered one of the great triumphs of Dirac's theory that it gave the "right" fine structure of the energy levels. However, the experimental attempts to obtain a really detailed confirmation through a study of the Balmer lines have been frustrated by the large Doppler effect of the lines in comparison to the small splitting of the lower or $n=2$ states. The various spectroscopic workers have alternated between finding confirmation[2] of the theory and discrepancies[3] of as much as eight percent. More accurate information would clearly provide a delicate test of the form of the correct relativistic wave equation, as well as information on the possibility of line shifts due to coupling of the atom with the radiation field and clues to the nature of any non-Coulombic interaction between the elementary particles: electron and proton.

The calculated separation between the levels $2^2P_{\frac{1}{2}}$ and $2^2P_{3/2}$ is 0.365 cm^{-1} and corresponds to a wave-length of 2.74 cm. The great wartime advances in microwave techniques in the vicinity of three centimeters wave-length make possible the use of new physical tools for a study of the $n=2$ fine structure states of the hydrogen atom.

population and the high background absorption due to electrons. Instead, we have found a method depending on a novel property of the $2^2S_{\frac{1}{2}}$ level. According to the Dirac theory, this state exactly coincides in energy with the $2^2P_{\frac{1}{2}}$ state which is the lower of the two P states. The S state in the absence of external electric fields is metastable. The radiative transition to the ground state $1^2S_{\frac{1}{2}}$ is forbidden by the selection rule $\Delta L = \pm 1$. Calculations of Breit and Teller[4] have shown that the most probable decay mechanism is double quantum emission with a lifetime of 1/7 second. This is to be contrasted with a lifetime of only 1.6×10^{-9} second for the non-metastable 2^2P states. The metastability is very much reduced in the presence of external electric fields[5] owing to Stark effect mixing of the S and P levels with resultant rapid decay of the combined state. If for any reason, the $2^2S_{\frac{1}{2}}$ level does not exactly coincide with the $2^2P_{\frac{1}{2}}$ level, the vulnerability of the state to external fields will be reduced. Such a removal of the accidental degeneracy may arise from any defect in the theory or may be brought about by the Zeeman splitting of the levels in an external magnetic field.

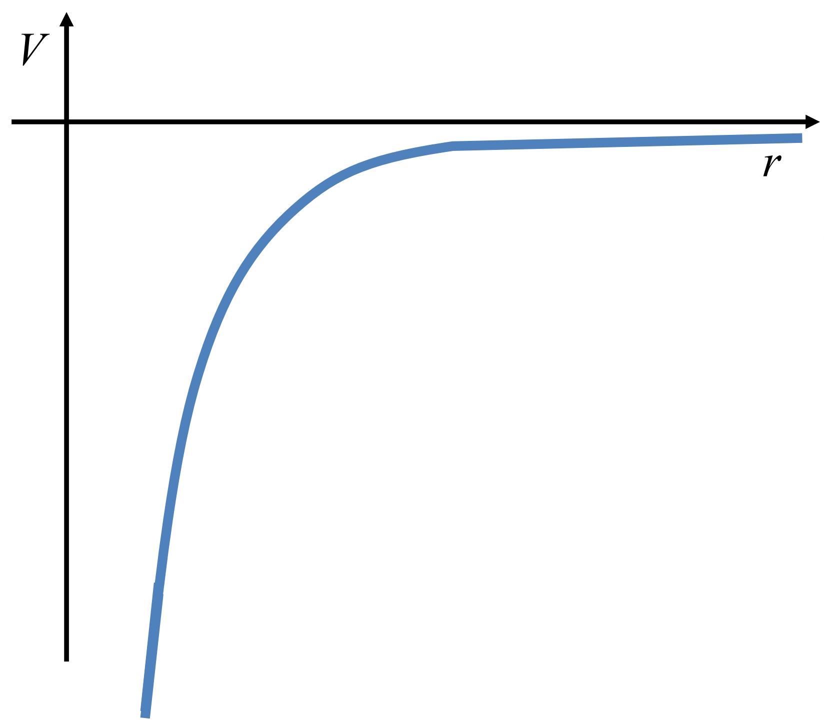
In brief, the experimental arrangement used is the following: Molecular hydrogen is thermally dissociated in a tungsten oven, and a jet of atoms emerges from a slit to be cross-bombarded by an electron stream. About one part in a hundred

According to the Bohr model, Schrödinger's equation, and even Dirac's relativistic equation for the electron, the energy of an electron in the 2*S* state of hydrogen should be identical to the energy of the 2*P* state: $E_n \sim 1/n^2$.

Instead, *Willis Lamb* and his student *Robert Retherford* were able to use Rad Lab equipment and techniques to measure a *tiny* difference between the energies of the 2*S* and 2*P* states in 1947: $\Delta E/E \sim 10^{-6}$. In particular, the energy of the 2*S* state was slightly *greater* than predicted by ordinary quantum mechanics.

This led several theorists to wonder: perhaps the energy shift was due to effects from *virtual particles*. Lamb presented the preliminary results at a private physics meeting on *Shelter Island* in June 1947. On his train ride back to upstate New York, *Hans Bethe* worked out a rough calculation of the effect.

The Lamb Shift

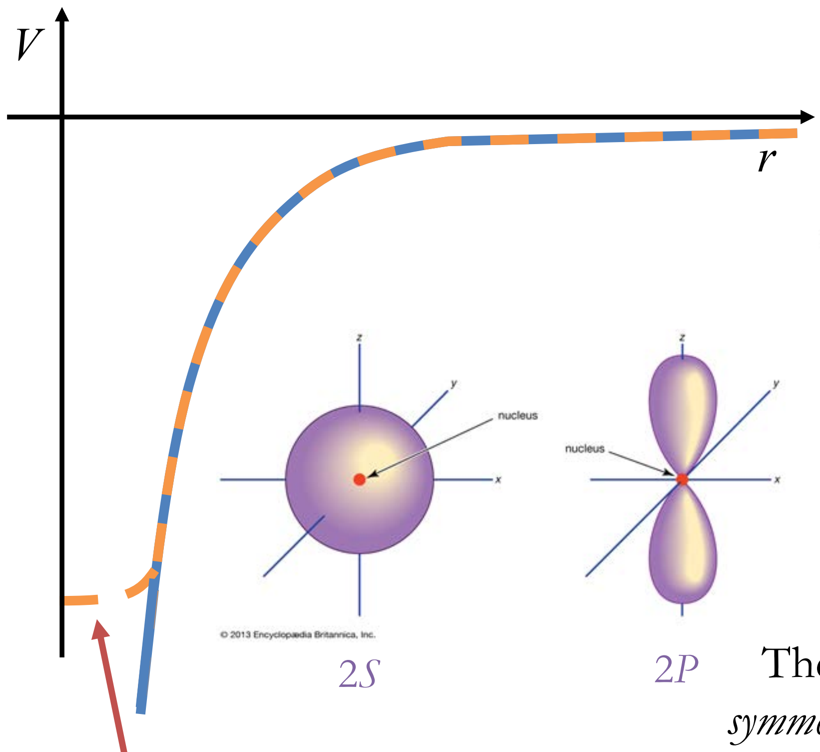


Bethe's basic idea was that virtual particles would constantly bombard the electron, smearing out its average position within some range $\delta\mathbf{r}$. The potential energy $V(r)$ would therefore become *perturbed*:

$$V(r+\delta r) = V(r) + \delta\mathbf{r}\cdot\nabla V + \frac{1}{6}\langle(\delta\mathbf{r})^2\rangle\nabla^2 V + \ldots$$

The random perturbations would average out, so $\langle\delta\mathbf{r}\rangle = 0$, even though $\langle(\delta\mathbf{r})^2\rangle \neq 0$.

The Lamb Shift



The potential near the origin is *truncated* compared to the "bare" potential

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$$V(r) = -\frac{e^2}{r} \longrightarrow \nabla^2 V(r) = +4\pi e^2 \delta^{(3)}(\mathbf{r})$$

The wavefunction for an electron in the $2S$ state is *spherically symmetric* and *nonvanishing* at the origin, $\psi(\mathbf{r}=0) \neq 0$, so its energy is *lifted* by the modified behavior of the effective potential, whereas the wavefunction for the $2P$ state *vanishes* at the origin, $\psi(\mathbf{r}=0) = 0$, so its energy is unaffected by the change.

The Lamb Shift

Bethe's basic idea was that virtual particles would constantly bombard the electron, smearing out its ... osition within some ra... would

To evaluate the *size* of the effects from virtual particles, Bethe put in a cut-off for the divergent integrals by hand, $\hbar k_{\max} = m_e c$, and found close agreement with Lamb's result.

$$\ldots + \delta \mathbf{r} \cdot \nabla V + \frac{1}{6}\langle (\delta \mathbf{r})^2 \rangle \nabla^2 V + \ldots$$

The random perturbations would average out, so $\langle \delta \mathbf{r} \rangle = 0$, even though $\langle (\delta \mathbf{r})^2 \rangle \neq 0$.

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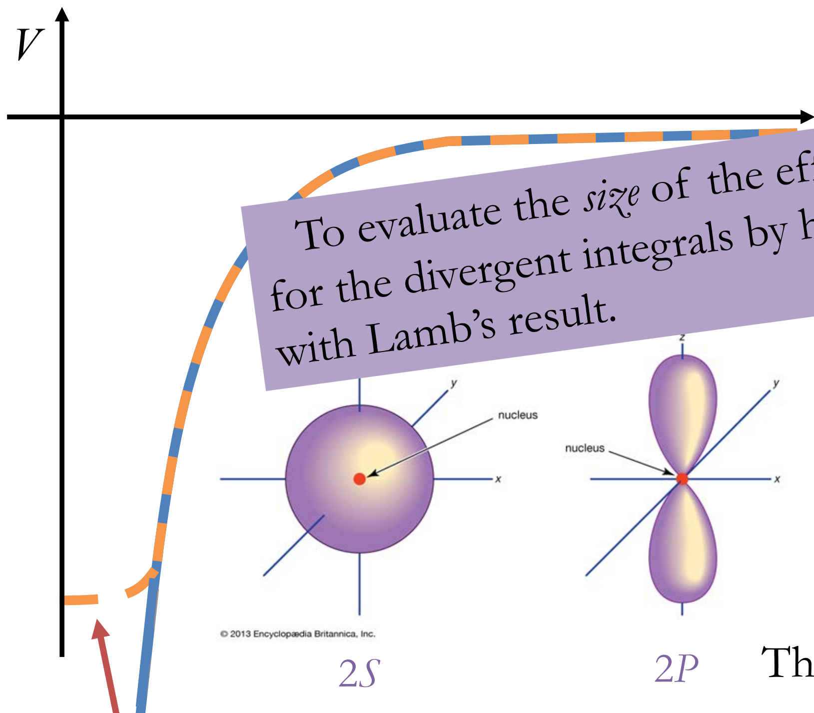


The wavefunction for an electron in the $2S$ state is *spherically symmetric* and *nonvanishing* at the origin, $\psi(\mathbf{r} = 0) \neq 0$, so its energy is *lifted* by the modified behavior of the effective potential, whereas the wavefunction for the $2P$ state *vanishes* at the origin, $\psi(\mathbf{r} = 0) = 0$, so its energy is unaffected by the change.

The potential near the origin is *truncated* compared to the "bare" potential

Anomalous Magnetic Moment

The Hyperfine Structure of Atomic Hydrogen and Deuterium†

J. E. NAFE, E. B. NELSON, AND I. I. RABI
Columbia University, New York, New York
May 19, 1947

THE hyperfine structure separation, ν_H and ν_D, of atomic hydrogen and deuterium were measured directly by means of the atomic beam magnetic resonance method.[1-3] For each atom two resonance lines were measured, each at the same value of the magnetic field, and the ν_H and ν_D were evaluated entirely from differences in the frequencies. Neither the value of the magnetic field nor the g values of the atomic and nuclear systems enter into the final result.

In H, where the value of the nuclear spin $I=1/2$ and the atomic $J=1/2$, the π-transitions $(1, 1)\leftrightarrow(0, 0)$ and $(1, 0)\leftrightarrow(1, -1)$ were measured at the same value of the magnet current. The difference between these two frequencies gives ν_H directly (see Eqs. 9–12 of reference 3). For D, where $I=1$ and $J=1/2$, the line $(3/2, 1/2)\leftrightarrow(1/2, -1/2)$, $(3/2, -1/2)\leftrightarrow(1/2, 1/2)$, an unresolved doublet, and the line $(3/2, 3/2)\leftrightarrow(1/2, 1/2)$ were measured in quite weak fields of the order of one gauss. The first line gives ν_D almost directly, and the difference in frequency of the two lines gives a small correction of less than 0.01 percent.

Just down the hallway from Lamb and Retherford, *I. I. Rabi* and colleagues were *also* using surplus Rad Lab equipment to perform sensitive measurements of electrons in hydrogen atoms. In spring 1947, they measured the electron's *magnetic moment* and found a tiny but measurable deviation from theoretical predictions.

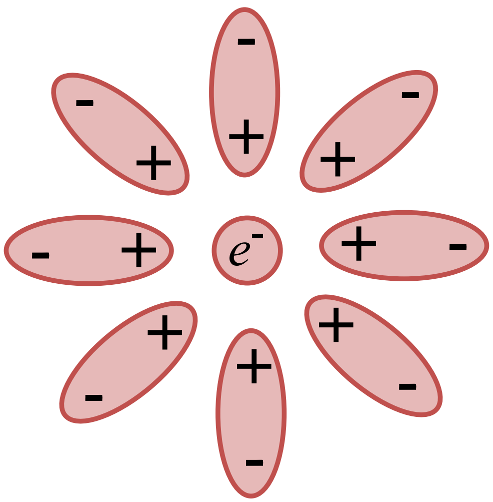
Recall the quantum-mechanical prediction: an electron has charge *e* and intrinsic angular momentum (spin) **S**, so its magnetic moment should be:

$$\mu_B = \frac{e\hbar}{2m_e}$$

"Bohr magneton"

But using the sensitive microwave electronics, Nafe, Nelson, and Rabi measured a *slightly larger* value: $\Delta\mu / \mu_B \sim 10^{-3}$.

Anomalous Magnetic Moment



"cloud" of virtual electron-positron pairs

Like Lamb, Rabi presented his group's results at the *Shelter Island* conference in June 1947. Again, theorists wondered if the measured discrepancy might be due to the *physical effects* of *virtual particles.*

Just like compas needles aligning in the Earth's magnetic field, perhaps the spins of each of the *virtual particles* surrounding a given electron would become aligned in an external magnetic field. Then the contributions from the *virtual cloud* would yield some larger *effective* spin for the system, which could shift

$$\mu_{\text{eff}} \to \mu_B + \Delta\mu$$

Questions?

Theorists' Responses after the War

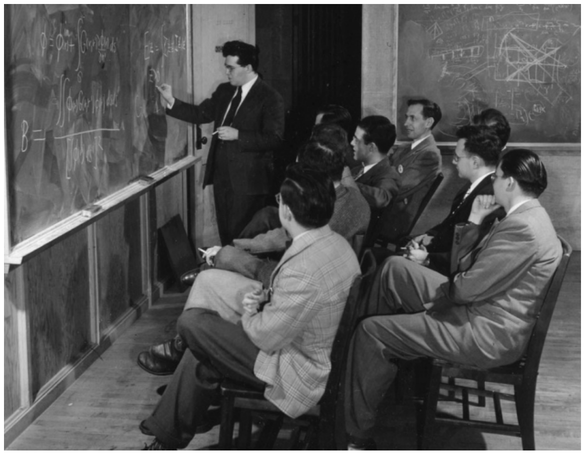
Physicists discuss QED at the June 1947 Shelter Island conference. Standing, left to right: Willis Lamb, John Wheeler. Seated, left to right: Abraham Pais, Richard Feynman, Herman Feshbach, and Julian Schwinger.

J. Robert Oppenheimer organized the informal, private workshop on Shelter Island for 24 physicists in June 1947, expressly to help jump-start research in theoretical physics after the disruptions of the war. The National Academy of Sciences paid for the meeting, whose stated goal was "to gather a group consisting of the younger men, who would understand each other's jargon."

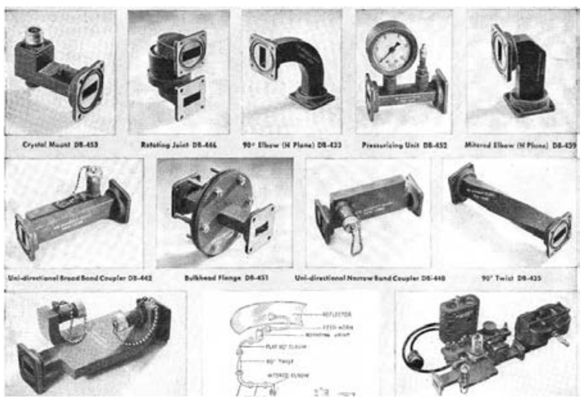
Nearly all of the participants had worked on wartime projects like radar and the Manhattan Project and knew each other.

Unlike theorists' responses in the 1930s to the infinities of QED, few of the US-based theorists after the war talked of grand conceptual revolutions. Rather, like their efforts during the war, many aimed to find practical means of "getting the numbers out" to compare with measurements, rather than getting stuck with formal, first-principles derivations.

Julian Schwinger and Renormalization

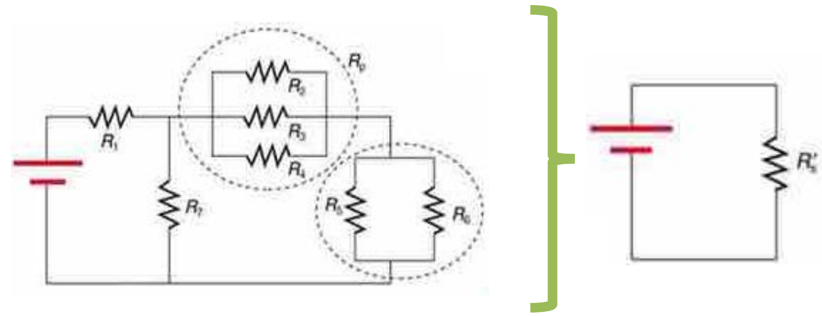


Julian Schwinger at the MIT Rad Lab, 1940s



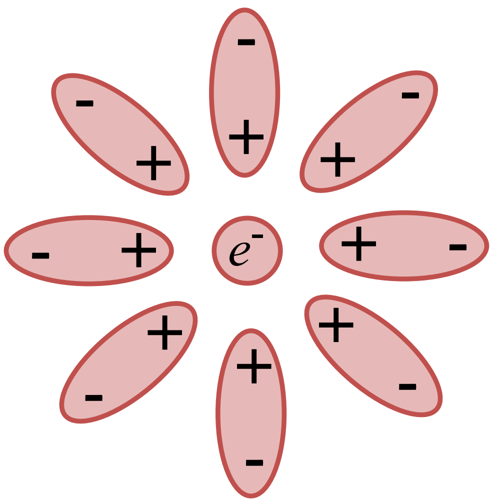
Julian Schwinger was among the Shelter Island participants. He had worked at the Rad Lab during the war, and was (at the time) the youngest person ever to get tenure at Harvard (at age 29!).

At the Rad Lab, he had learned from the engineers to focus on *effective circuits* and input-output relationships.



Upon hearing from *Lamb* and *Rabi* at Shelter Island about their latest measurements, *Schwinger* began thinking about *virtual particles* in a similar way.

Julian Schwinger and Renormalization



"cloud" of virtual electron-positron pairs

Schwinger reasoned that one could never turn off the uncertainty principle, and hence one would never encounter an electron *without* its cloud of virtual particles.

Rather than manipulate equations involving an electron's "bare" mass and charge (m_0 and e_0) and then try to calculate the effects of virtual particles, one could *rewrite* all the equations in terms of *effective* quantities, which were the only quantities that could ever be measured:

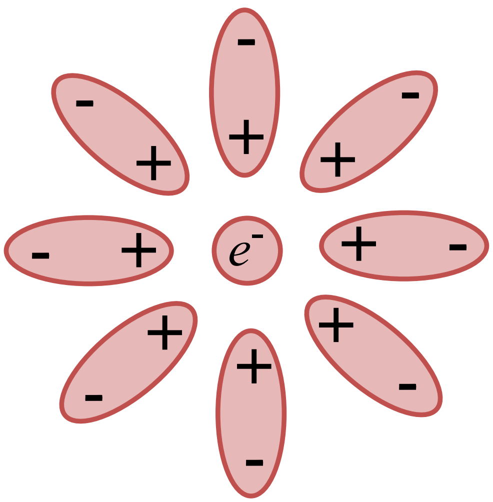
$$m_{\text{eff}} = m_0 + m_{\text{virtual}}$$

$$e_{\text{eff}} = e_0 + e_{\text{virtual}}$$

Then theorists would never need to calculate the (divergent) quantities m_{virtual} or e_{virtual} on their own.

As we will see in the next class, *Sin-itiro Tomonaga* had independently come to the *same conclusion* in Tokyo in 1943, while he was *also* immersed in wartime radar research.

Julian Schwinger and Renormalization



“cloud” of virtual electron-positron pairs

Schwinger confirmed that if one *tried* to calculate effects of virtual particles on their own, one routinely encountered integrals of the form

$$f_{\text{virtual}}(x) = \int_1^\infty dy \frac{1}{(y+x)} = \ln(y+x)\Big|_1^\infty \to \infty$$

If instead one *grouped* the “bare” and “virtual” contributions together into a single *effective* quantity, the integrals took the form

$$f_{\text{eff}}(x) = f(0) - f(x)$$

“

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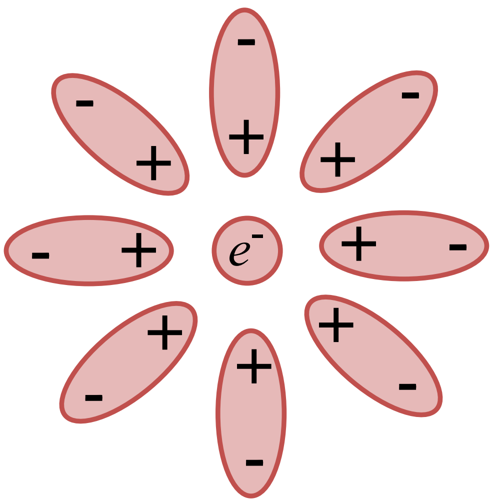
Julian Schwinger and Renormalization



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If instead one *grouped* the "bare" and "virtual" contributions together into a single *effective* quantity, the integrals took the form

$$\begin{aligned}
f_{\text{eff}}(x) &= f(0) - f(x) \\
&= \int_1^\infty dy \left[\frac{1}{y} - \frac{1}{(y+x)}\right] \\
&= \int_1^\infty dy \frac{1}{y(y+x)} [y + x - y] \\
&= x \int_1^\infty dy \frac{1}{y(y+x)} \\
&= \ln\left(\frac{y}{y+x}\right)\Big|_1^\infty = \ln(1+x) \to \boxed{\textit{finite}!}
\end{aligned}$$

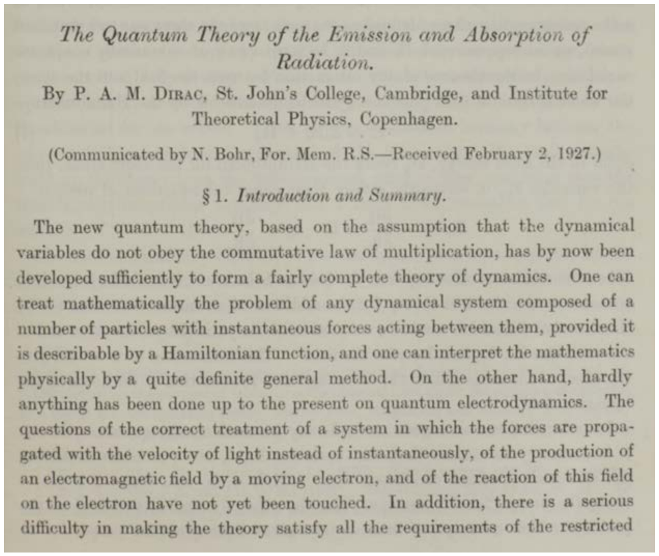
"Renormalization": arrange for *infinite constants* to *cancel*, yielding *finite* results.

Late in 1947, Schwinger applied this approach to the electron's magnetic moment:

$$\left(\frac{\Delta\mu}{\mu_B}\right)_{\text{theory}} = \frac{e^2}{2\pi\hbar c} = 0.001162$$

$$\left(\frac{\Delta\mu}{\mu_B}\right)_{\text{exp}} = 0.00118 \pm 0.00003$$

QED Summary


The Quantum Theory of the Emission and Absorption of Radiation.

By P. A. M. Dirac, St. John's College, Cambridge, and Institute for Theoretical Physics, Copenhagen.

(Communicated by N. Bohr, For. Mem. R.S.—Received February 2, 1927.)

§ 1. *Introduction and Summary.*

The new quantum theory, based on the assumption that the dynamical variables do not obey the commutative law of multiplication, has by now been developed sufficiently to form a fairly complete theory of dynamics. One can treat mathematically the problem of any dynamical system composed of a number of particles with instantaneous forces acting between them, provided it is describable by a Hamiltonian function, and one can interpret the mathematics physically by a quite definite general method. On the other hand, hardly anything has been done up to the present on quantum electrodynamics. The questions of the correct treatment of a system in which the forces are propagated with the velocity of light instead of instantaneously, of the production of an electromagnetic field by a moving electron, and of the reaction of this field on the electron have not yet been touched. In addition, there is a serious difficulty in making the theory satisfy all the requirements of the restricted

Several physicists began developing *quantum field theory* as early as 1927, right on the heels of quantum mechanics. Theorists like *Paul Dirac* (and later *Werner Heisenberg*) developed a formalism in which a quantum field could be represented as a collection of *quanta* in particular states (E_p, $\mathbf{p}$). But the *uncertainty principle* suggested that *virtual particles* could temporarily "borrow" energy, via $\Delta E\,\Delta t \geq \hbar/2$.

Processes involving virtual particles proved *impossible* to quantify during the 1930s. Corrections to quantities like an electron's *mass* and *charge* appeared to *diverge*.

Soon after the end of the Second World War, a new generation of physicists returned to QED and its infinities. Their work built directly upon wartime surplus equipment as well as calculating techniques learned during the wartime projects. They focused on *effective* quantities, rather than separating "bare" values from "quantum corrections."

$$\left(\frac{\Delta\mu}{\mu_B}\right)_{\text{theory}} = \frac{e^2}{2\pi\hbar c} = 0.001162$$

$$\left(\frac{\Delta\mu}{\mu_B}\right)_{\text{exp}} = 0.00118 \pm 0.00003$$