Mathematically, the four-quadrant diagram (Lectures, and Dipasquale and Wheaton) represents a solution to a system of four simultaneous equations that describe the operation of the property (space) and asset (capital) markets. An example of four plausible equations would be the following (these are slightly different that in the text):

Office space demand: \[ \text{Demand} = E \times 5000 \times R^{-1} \], where \( E \) is employment in millions, and \( R \) is rent in annual dollars per sq. ft.

Asset price \( (P) = R/i \), where \( i \) is the long term cap rate for real estate.

Construction rate \( (C/S) = 0.0001 \times P \), where \( C \) is the level of construction in millions of sq. ft., and \( P \) is asset price.

Stock of space \( (S) = C/\delta \) (i.e., long run depreciation is \( \delta \)% again in millions of square ft.

For the U.S. office market, long term average values for the exogenous parameters might be:

\( E = 10 \) (millions)

\( i = 10\% \) (.10)

\( \delta = .02 \)

1) Solve for the equilibrium solution to all endogenous variables given the above values for \( E, \delta \), and \( i \).

2) Use the above equations, solve for how each of the following three exogenous changes affects conditions in the capital-property markets and lead to a new equilibrium state. Solve for the new values. You should describe how and why rents, asset prices, construction, and the stock of space change in each scenario. (Note that each of the scenarios builds upon the previous ones - i.e. in C, assume that A and B have already occurred).

A Global capital starts to flow into real estate. As a consequence cap rates are driven down from .10 all the way to .05. What happens to prices and rents in the short run – if the stock and construction do not change. That is, calculate new values for \( \{P, R\} \) given old values for \( S, C \).

B In the long run construction and the stock adjust. What happens to the price of real estate and space per worker in the long-run (i.e. after the stock has fully adjusted).

C A credit crisis now ensues. Cap rates rise back to 10% and office employment (heretofore fixed) now contracts by 10% from 10m to 9m. What happens now in the short run – where the stock is fixed at the level in B? What happens in the long run if the credit constraints continue with construction and the stock now adjusting?