
NOTE: We made a mistake on the solution sheet for HW #5. On part d of problem 5 on HW #5 we omitted the work due to friction. If you submitted the answer $\frac{1}{2}mv_0^2 \frac{x_2-x_1}{x_1} + \frac{1}{2}mv_1^2$ it should have been graded correct, and you can recover any lost credit by resubmitting it to your grader with a note to regrade this problem.

1) 7.42

Please refer to the figure 7.30 p.277.

Let’s denote the compression length as $\Delta x$, the velocity after leaving the spring as $v_0$ and the maximum height it goes on the incline as $h$.

Using energy conservation law:

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 + mgy_1 + W_{other} = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2 + mgy_2$$

There is no friction present in this problem and $W_{other} = 0$. I will measure the gravitation potential energy with respect to the height of the dashed horizontal bar indicated in the figure 7.30.

a) Comparing the energy initially (when spring compressed):

$$v_1 = 0 \quad x_1 = \Delta x \quad y_1 = 0$$

with the point it leaves the spring:

$$v_2 = v_0 \quad x_2 = 0 \quad y_2 = 0$$

$$0 + \frac{1}{2}k\Delta x^2 + 0 = \frac{1}{2}mv_0^2 + 0 + 0$$
\[ v_0 = \sqrt{\frac{k}{m}\Delta x} \]  (5)

**b)** For this part we should compare the energy of particle at its highest point on the incline:

\[ v_3 = 0 \quad x_3 = 0 \quad y_3 = h \]  (6)

with the point that it has left the spring:

\[ v_2 = v_0 \quad x_2 = 0 \quad y_2 = 0 \]  (7)

(We could equally well elect to use the variables subscripted as 1 as the initial energy point)

\[ \frac{1}{2}mv_0^2 + 0 + 0 = 0 + 0 + mgh \]  (8)

Using (5):

\[ h = \frac{k(\Delta x)^2}{2mg} \]  (9)

The distance \( L \) it travels up the incline is just:

\[ L = \frac{h}{\sin \theta} \]  (10)

\[ \therefore L = \frac{k(\Delta x)^2}{2mg\sin \theta} \]  (11)

Plugging in the numbers given in the problem you’ll get:

\[ v_0 = 3.11 \text{ m/s} \]  (12)

\[ L = 0.821 \text{ m} \]  (13)
2) 7.58

Let’s denote the 0.500 kg rat as \( L \) and 0.200 kg one as \( S \).
No non-conservative force is present in this problem and we can safely use:

\[
K_{s1} + U_{s1} + K_{L1} + U_{L1} = K_{s2} + U_{s2} + K_{L2} + U_{L2} \quad (14)
\]

\[
\frac{1}{2} m_S v_{s1}^2 + m_S g y_{s1} + \frac{1}{2} m_L v_{L1}^2 + m_L g y_{L1} = \frac{1}{2} m_S v_{s2}^2 + m_S g y_{s2} + \frac{1}{2} m_L v_{L2}^2 + m_L g y_{L2} \quad (15)
\]

Point 1 refers to the initial condition when it’s released from rest.
Point 2 refers to the vertical situation. I will measure the gravitation potential energy with respect to the height of the initial horizontal bar.

\[
v_{s1} = v_{L1} = 0 \quad y_{s1} = y_{L1} = 0 \quad (16)
\]

If the animals are equidistant from the center, they have the same speed \( V \).

\[
v_{s2} = v_{L2} = V \quad y_{s2} = +\frac{l_0}{2} \quad y_{L2} = -\frac{l_0}{2} \quad (17)
\]

Where \( l_0 \) is the length of the wooden rod.

\[
\therefore \quad 0 + 0 + 0 + 0 = \frac{1}{2} (m_S + m_L) V^2 + (m_S - m_L) g \frac{l_0}{2} \quad (18)
\]

\[
V = \sqrt{\frac{m_L - m_S}{(m_L + m_S)(gl_0)}} \quad (19)
\]

Plugging in the given numbers you get:

\[
V = 1.83 \text{ m/s} \quad (20)
\]
3) 7.72

Following the hint (to express $k$ in terms of $m$ & $d$), when the fish is lowered slowly, it stops in equilibrium:

$$\sum F_y = kd - mg = 0 \Rightarrow k = \frac{mg}{d} \quad (21)$$

Let’s denote the maximum distance it reaches when it falls as $d_{max}$. Again here no nonconservative force is present and we have:

$$\frac{1}{2}mv_1^2 + \frac{1}{2}ky_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + \frac{1}{2}ky_2^2 + mgy_2 \quad (22)$$

**NOTE:** With a spring is easiest to pick $y=0$ at its unstretched length - otherwise the energy is $\frac{1}{2}k(y - y_0)^2$. We also take $y>0$ to be up.

Point 1 refers to the point it *starts* falling from rest and pint 2 as the point it *stops* moving after it stretches the spring the amount $d_{max}$. As a matter of convention I’ll pick the $y_1$ we have released our mass as the 0.

$$v_1 = 0 \quad y_1 = 0 \quad (23)$$

$$v_2 = 0 \quad y_2 = -d_{max} \quad (24)$$

$$0 + 0 + 0 = 0 + \frac{1}{2}kd_{max}^2 + mg(-d_{max}) \quad (25)$$

$$\therefore \quad \frac{1}{2}kd_{max}^2 - mg)d_{max} = 0 \quad (26)$$

There are two solutions:

*The trivial one:* 

$$d_{max_1} = 0 \quad (27)$$

which results from the fact that the energy of the starting point is equal to itself!
The other solution:
\[ \frac{1}{2} k d_{\text{max}} - mg = 0 \] (28)

\[ d_{\text{max}} = d_{\text{max}} = \frac{2mg}{k} \] (29)

Replacing k from equation (21) we get:
\[ d_{\text{max}} = \frac{2mg}{mg} \] (30)

\[ \therefore d_{\text{max}} = 2d \] (31)

This is not surprising - the fish oscillates symmetrically about its equilibrium position.

4) Bead Slides Around and Up Wire

Please refer to the figure in the problem set.

There is no non-conservative forces, so mechanical energy is conserved. The thing to remember is that for a circular motion acceleration is radial and is equal in magnitude to \( \frac{v^2}{R} \).

Our first job is to obtain v at different points and look carefully what radial at that point means and use the component of Newton’s law \( \sum F = ma \) in the radial direction. As for convention I choose the horizontal line shown at the bottom of the loop in the figure as the 0 of potential energy.
\[ \frac{1}{2} mv_a^2 + mgy_a = E_a = E_b = \frac{1}{2} mv_b^2 + mgy_b \] (32)

\[ v_a = 0 \quad y_a = 2R \] (33)

\[ y_b = R \] (34)

\[ 0 + mg(2R) = \frac{1}{2} mv_b^2 + mgR \] (35)

\[ \therefore \quad v_b = \sqrt{2gR} \] (36)

Radial at point \( b \) is the \( x \) direction:

\[ \sum F_{bx} = N_b = m \frac{v_b^2}{R} = m \frac{2gR}{R} = 2mg \] (37)

(This is twice the weight)

b)

\[ \frac{1}{2} mv_a^2 + mgy_a = E_a = E_c = \frac{1}{2} mv_c^2 + mgy_c \] (38)

\[ v_a = 0 \quad y_a = 2R \] (39)

\[ y_c = 0 \] (40)

\[ 0 + mg(2R) = \frac{1}{2} mv_c^2 + 0 \] (41)

\[ \therefore \quad v_c = 2\sqrt{gR} \] (42)

Radial at point \( c \) means the \( y \) direction:

\[ \sum F_{cy} = N_c - mg = m \frac{v_c^2}{R} = m \frac{4gR}{R} = 4mg \Rightarrow N_c = 5mg \] (43)
c) At point d which the mass reverse its direction: \( v_d = 0 \).

\[
\frac{1}{2}mv_a^2 + mgy_a = \frac{1}{2}mv_d^2 + mgy_d \quad (44)
\]

\[
v_a = 0 \quad y_a = 2R \quad v_d = 0 \quad y_d = H \quad (45)
\]

\[
0 + mg(2R) = 0 + mgH \quad (46)
\]

\[
\therefore \quad H = 2R \quad (47)
\]

*This is not surprising- in the absence of dissipation it rises to the same height from which it was released.*

You could have chosen any two points you want between which to apply energy conservation since energy is conserved along the entire path. Let’s do this part by comparing points d and c:

\[
\frac{1}{2}mv_c^2 + mgy_c = \frac{1}{2}mv_d^2 + mgy_d \quad (48)
\]

\[
v_c = 2\sqrt{gR} \quad y_c = 0 \quad v_d = 0 \quad y_d = H \quad (49)
\]

\[
\frac{1}{2}m(4gR) + 0 = 0 + mgH \quad (50)
\]

which gives again

\[
H = 2R. \quad (51)
\]