Problem 1: Collision (15 Points)

Two masses $M$ and $3M$ collide on a horizontal frictionless surface as shown. Before the collision the mass $M$ has a velocity $V_1$ in the y-direction. The mass $3M$ has a velocity $\frac{5}{12}V_0$ making an angle $\theta$ to the x-axis as shown. Assume $\sin \theta = 3/5$ and $\cos \theta = 4/5$. After the collision the mass $3M$ is at rest. The mass $M$ moves along the x-axis with the velocity $V'_1$. Neglect gravity. Give all your answers to parts b), c), and d) in terms of $M$ and $V_0$. Be careful do not confuse your symbols.

a) What are the x and y-components of the net linear momentum before the collision in terms of $M$, $V_1$, $V_0$ and $\theta$?

b) What is the speed $V_1$ of the mass $M$ before the collision?

c) What is the speed $V'_1$ of the mass $M$ after the collision?

d) What is the velocity of the center-of-mass?

\[ p_{x}^{tot} = \begin{align*} \frac{3}{4}M \cdot \frac{5}{12}V_0 \cos \theta \\ \frac{M}{V_0} \end{align*} \]

\[ p_{y}^{tot} = \begin{align*} MV_1 - \frac{3}{4}M \cdot \frac{5}{12}V_0 \sin \theta \\ \frac{M}{V_0} \end{align*} \]

\[ b) \quad p_{y}^{tot} = 0 \quad \Rightarrow \quad V_1 = \frac{3}{4}V_0 \]

c) \quad V'_1 = p_{x}^{tot} \quad \Rightarrow \quad V'_1 = V_0 

d) \quad V_x^c = \frac{p_x^{tot}}{3M+M} = \frac{1}{4}V_0, \quad V_y^c = \frac{p_y^{tot}}{4M} = 0 \]
Problem 2: Rotational Dynamics (15 points)

A spool of wire of mass M and radius R is unwound along a horizontal surface under a constant force $\vec{F}$. Assume the spool is a uniform solid cylinder that does not slip. The coefficient of static friction is $\mu_s$. Assume that the radius of the spool does not decrease significantly while the spool is rolling. Give all you answers in terms of $F$, $M$, $R$, $\mu_s$, $g$ and $L$.

a) State the moment of inertia, $I$, of the cylinder about its central axis.

b) What is the force of friction, $\vec{F}$ (magnitude AND direction) acting on the spool? Show the direction of $\vec{F}$ on the diagram.

c) What is the acceleration of the center-of-mass?

d) What is the angular acceleration?

e) What is the total kinetic energy of the spool when it has rolled through a distance $L$?

\[ I = \frac{1}{2} MR^2 \]

\[ T = I \alpha \quad \text{about } A \]

\[ T = 2FR \quad I = \frac{1}{2} MR^2 + MR^2 = \frac{3}{2} MR^2 \]

\[ \alpha = \frac{F}{3M} \]

b) Note $\alpha > \frac{F}{M}$, so $\vec{f}$ and $\vec{F}$ point to the same direction. $\vec{f} + \vec{F} = Ma = \frac{4}{3}F \Rightarrow \vec{f} = \frac{1}{3} \vec{F}$

c) When the center moves a distance $L$, B points moves a distance $2L$ (as the string is pulled by a distance $2L$). So $K = 2LF$
Problem 3: Rotational Collision (15 points)

A uniform cylindrical shell (hoop) sits on one of its flat sides on a frictionless surface. The hoop has mass \( M \), radius \( R \) and height \( H \). A bullet of mass \( \mu \) moving horizontally with velocity \( V_0 \) strikes the hoop with impact parameter \( R \) at mid-height (\( H/2 \) from the surface). After the collision the bullet continues with velocity \( V_0/2 \) in its original direction. Ignore any hole the bullet creates.

Give all your answers in terms of \( M, R, H, V_0, \) and \( \mu \).

a) What was the angular momentum, \( \vec{L} \), of the system about the center of the hoop before the collision?

b) What is the linear velocity, \( \vec{V} \), of the center of the hoop after the collision?

c) What is the angular velocity, \( \vec{\omega} \), of the hoop after the collision?

\[ a) \quad \vec{L} = \vec{M} \cdot \vec{V} \cdot R \]

pointing up (out of paper)
relative to \( \vec{A} \)

\[ b) \quad \vec{M} \cdot \vec{V} + \mu \frac{V_0}{2} = \frac{V_0}{2} \]

\[ \Rightarrow \quad V_x = -\frac{\mu V_0}{2M} \]

\[ V_y = 0 \]

\[ c) \quad \vec{I} \cdot \vec{\omega} + \mu \frac{V_0}{2} \cdot R = \mu V_0 \cdot R \]

\[ \vec{\omega} = \frac{\mu V_0 \cdot R}{2I} = \frac{\mu \cdot V_0}{2m \cdot R} \]
Problem 4: Multiple Choice (15 Points)

a)

The only force acting on a 2.0-kg object moving along the x axis is shown. If the velocity \( v_x \) is \(-2.0 \text{ m/s} \) at \( t = 0 \), what is the velocity at \( t = 4.0 \text{ s} \)?

\[
\int dt \ F = 4 + 2 - 8 = -2 \text{ N} \cdot \text{s}
\]

\[
P(t=4) = -4 - 2 = -6
\]

\[
\mathbf{v} = \frac{P}{m} = -3 \frac{m}{\text{s}}
\]

a. \(-2.0 \text{ m/s}\)
b. \(-4.0 \text{ m/s}\)
c. \(-3.0 \text{ m/s}\)
d. \(+1.0 \text{ m/s}\)
e. \(+5.0 \text{ m/s}\)

b)

A car of mass \( m_1 \) traveling at velocity \( \mathbf{v} \) passes a car of mass \( m_2 \) parked at the side of the road. The momentum of the system of two cars is

a. 0.
b. \( m_1 \mathbf{v} \).
c. \((m_1 - m_2)\mathbf{v}\).
d. \( \frac{m_1 \mathbf{v}}{m_1 + m_2} \).
e. \((m_1 + m_2)\mathbf{v}\).
Two forces of magnitude 50 N, as shown in the figure below, act on a cylinder of radius 4 m and mass 6.25 kg. The cylinder, which is initially at rest, sits on a frictionless surface. After 1 second, the velocity and angular velocity of the cylinder in m/s and rad/s are respectively

\[ F = 50 \text{ N} \]

\[ \begin{align*}
\text{a.} & \quad v = 0; \quad \omega = 0. \\
\text{b.} & \quad v = 0; \quad \omega = 4. \\
\text{c.} & \quad v = 0; \quad \omega = 8. \\
\text{d.} & \quad v = 8; \quad \omega = 8. \\
\text{e.} & \quad v = 16; \quad \omega = 8. \\
\end{align*} \]

\[ I = \frac{1}{2} m R^2 = 5.0 \]

\[ T = 50 \times 4 = 200 \]

\[ \alpha = 4 \quad \omega = 4/\text{s} \]

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d)

Five objects of mass \( m \) move at velocity \( v \) at a distance \( r \) from an axis of rotation perpendicular to the page through point A, as shown below. The one that has zero angular momentum about that axis is

\[ \begin{align*}
\text{(a)} & \quad A \\
\text{(b)} & \quad \text{A} \\
\text{(c)} & \quad \text{A} \\
\text{(d)} & \quad \text{A} \\
\text{(e)} & \quad \text{A} \\
\end{align*} \]
A square of side $\frac{L}{2}$ is removed from one corner of a square sandwich that has sides of length $L$. The center of mass of the remainder of the sandwich moves from $C$ to $C'$. The displacement of the $x$ coordinate of the center of mass (from $C$ to $C'$) is

\[ x = \frac{L}{12} \]

\[ \Rightarrow x = \frac{L}{L^2} = \frac{1}{L} \]

\[ 3 \cdot x = 1 \cdot \frac{L}{4} \]

\[ a. \quad \frac{L}{12} \]

\[ b. \quad \frac{\sqrt{2}}{12}L \]

\[ c. \quad \frac{1}{6}L \]

\[ d. \quad \frac{1}{8}L \]

\[ e. \quad \frac{\sqrt{2}}{8}L \]