PROFESSOR: So as a technical aside, let me go through and sketch the derivation of the structure of a turbulent jet, in particular the conical shape that we have when the flow is turbulent.

So in order to study the mean flow profile we began with the Navier-Stokes equations which describe the momentum conservation and mass conservation or continuity of an incomprehensible so-called Newtonian fluid.

So this is a complicated set of equations.

In particular, we have this nonlinear term here, which is the inertial term.

And we've already said that we had our high Reynolds number and turbulence results because the inertia is very strong compared to the viscous term which is here.

So that's the divergence of the viscous or the viscous forces on the fluid.

So these two terms we know are important.

They have to balance and the inertia is particularly strong and it is what leads to the very complicated flows that we see.

So you can solve these equations numerically on a computer and generate simulations that look a lot like experiments on turbulent jets.

What I'd like to do here is just to derive by simple scaling arguments what sort of the structure of the solutions could look like.

So these two terms, as we just indicated, are the ones that are most likely to balance in the time average flow.

So let's consider a time averaged steady flow which has a velocity component of v_z that depends on r and z.

So it's basically something like this, which is basically expanding, but has a certain sort of localization of the flow in the middle.

And it's smooth because we're averaging over all the complexity of the jets.

So the jet looks something like this with all kinds of vortices and eddies that are getting bigger as it goes as you're entraining more and more air from the outside.

So we're going to look at the time average flow.

And we're also going to, importantly, assume that we have an eddy viscosity.

So the kinematic viscosity, nu in the equations as I've written them here, represents the diffusion momentum.

If a parcel of fluid is moving with a certain momentum, it has a chance of passing that momentum to the neighboring fluid and moving it along with it.

And that is accomplished through viscous stresses.

So the eddy viscosity basically assumes that that diffusion process from momentum happens at the scale of the largest eddy in the flow.

And so we've talked about the assumption of eddy diffusivity.

But for eddy viscosity, what I'll write is the eddy viscosity is a typical velocity which is v_z times a length scale which is delta.

So what I'm saying here with this is that if I go out to a certain position z and ask myself, what is the sort of width of the jet at site z, then there's all kinds of eddies but the largest eddy is kind of at that scale.

And so if I write down an eddy viscosity it's going to be these sort of average velocity there are times that scale.

So that's going to be the eddy velocity.

And I'm going to replace-- so when I do my time averaging, I'm going to replace the microscopic viscosity of the fluid, kinematic viscosity, with the eddy viscosity.

So that's an important modification.

And so if I do that.

If I do this time averaging and look at the eddy viscosity, then I take these two terms and balance them, I'm going to get v_z bar, so that's my average v_z.

I'm looking at the z component of momentum here of that first Navier-Stokes equation.

And I get v_z dot derivative of v_z with respect to z plus v_r.

And there's also an r component of velocity.

So there is also some velocity in fact which is coming in from the sides.

But I'm just going be interested in this term here v_z dr.

And I'm going to balance this against the eddy viscosity, the eddy-- or nu eddy I should say, sorry, nu eddy is eddy viscosity-- times and then the Laplacian in is 1 r d/dr r dv_z dr. So that's just the Laplacian in its cylindrical coordinates.

And now I'm going to make the assumption that this ve scales as v bar z times delta.

And so now I'm going to do a scaling analysis on this equation.

And so what we see is we have v_z over z times-- and then at least for that first so-- I should say these two terms will be of comparable size because of incompressibility-- the second equation.

I won't go through the details of that.

And we'll just do a scaling argument balancing these two terms.

So if I look at v_z divided by z times v_z so that's a scaling of those two terms, I can balance that against v_z delta times-- and the scale for r is delta.

So I have 1/delta for the 1/r, 1/delta for the derivative times delta * (1/delta) * v_z.

So there's a lot there.

But notice the v_z's all cancel.

And we're left with a bunch of deltas here.

And how many, because of the eddy viscosity, we are left with-- all of this is just 1 over delta.

This is 1 over z.

And so we find here the delta scales as z.

So in other words, we have a conical shape.

So the boundary of their thickness is a constant times z.

And what we write is that delta is equal to alpha z specifically.

And we define the turbulent entrainment coefficient alpha that way.

And then once we've done that, we've already shown that from the momentum flux that v_z scales as the square root of k/rho_air times 1 over delta.

So this basically now gives me the scaling of the problem.

In fact, there is a similarity solution for the shape this profile that one could solve for.

And it has the form that, for example, the v_z is square root of-- because delta is proportional to z.

So it's the square root of k over rho a z times some function of r over alpha z.

And then there's a similar expression for the other velocity component.

And the function F looks very much as I've sketched here.

It's essentially a Gaussian type profile or a bell curve that kind of localizes the velocity across this distance delta.

The second thing that we're interested in is the mean concentration.

And that would be a concentration of, let's say, virions contained in infectious aerosol droplets.

So there's a mean concentration profile in the jet assuming that we're injecting a fluid of a constant concentration at the source of the jet.

So again, we can do some scaling arguments here.

So if we ask ourselves, what is the mass flow rate through a slice or actually the volumetric flow rate, use me, that is what is called a Q and it'll just be an average.

This will be the average velocity times the cross-sectional area at a given position.

So this is scaling like-- so area scaled is like delta squared.

And then the velocity scales in this way is 1 over delta.

So this ends up scaling as k over rho a times just delta.

So the volumetric flow rate is increasing with r and that's a sign that we are actually in training fluid as I indicated.

This is not just the fluid we're injecting but it's moving forward and it's sucking more fluid in.

And all that fluid is kind of becoming part of the turbulent jet as it grows.

Now if we ask-- so this is our flow rate, volumetric flow rate, but we can also ask ourselves, what is the flux of concentration of virions per unit volume.

Well, that would be the average concentration times the average flow rate because flow rate is volume per time and concentration is number per volume.

So this is a total number per time.

And this we will assume should be a constant because as you can see from this picture, if we're injecting a bunch of concentration of let's say droplets here, they will spread out in the turbulent flow but they don't really have a good mechanism to get out of the turbulent flow.

The turbulent flow is sucking fluid into the plume and so the particles are just kind of well mixed in that plume and we could assume they have a roughly constant concentration.

And so, if that's the case, and in fact, this constant would be lambda Q if we're thinking of, for example, infect-- c is the concentration infection quanta.

Then lambda Q is the rate of admission of infection quanta from the mouth.

We've already talked about that quantity.

And this is now telling me how the concentration infection quanta decays with time.

And so we find if we substitute now is that the concentration infection quanta at a position z scales as so I have to divide by Q so I get the inverse of this.

So I get square root of rho a over k.

And then I have lambda Q over alpha z.

So this tells me that if I plot as a function of distance from the mouth in the direction of the jet, the concentration of infection quanta that are carried by virions in aerosol droplets, then somewhere here I have, let's say, at z equals 0, is the mouth where I'm exhaling.

And the concentration there is actually cQ.

In fact that is something we've talked about before, which is that's the key disease parameter-- the concentration of infection quanta in the exhaled breath of an infected person.

So we know at the mouth, that's what we start with.

And what the turbulent theory is telling us is how that concentration is decaying with time and is decaying like 1 over z.

And so that tells us sort of our relative risk of infection in different positions relative to being mouth to mouth with the infected person.