In order of fast growth as $x$ gets large:

- $\log x$ (logarithmic)
- $x^2, x^3$ (polynomial)
- $2^x, e^x, 10^x$ (exponential)
- $x!, x^x$ (factorial)

Choose $x = 1000 = 10^3$ so that $\log x = 3$. It is OK to use $x! \approx \frac{x^x}{e^x}$.

Logarithms are exponents! $\log 10^9 = 9$. $\log \log x$ is VERY slow.

Logarithms $3$ $6$ $9$ $300$ $434$ $1000$ $2566$ $3000$

Polynomial growth $\ll$ Exponential growth $\ll$ Factorial growth.

Decay to zero for NEGATIVE powers and exponents:

\[
\frac{1}{x^2} = x^{-2} \text{ decays much more slowly than the exponential } \frac{1}{e^x} = e^{-x}
\]

Polylogarithmic scale shows $x = 1, 10, 100$ equally spaced. NO ZERO!

**Question** If $x = 1, 2, 4, 8$ are plotted, what would you see?

**Answer** THEY ARE EQUALLY SPACED TOO!

**Log-log graphs** (log scale up and also across)

If $y = Ax^n$, how to see $A$ and $n$ on the graph?

Plot $\log y$ versus $\log x$ to get a straight line:

\[
\log y = \log A + n \log x
\]

Slope on a log-log graph is the exponent $n$.

For $y = Ab^x$ use **semilog** ($x$ versus $\log y$ is now a line): $\log y = \log A + x \log b$. 
Growth Rates and Log Graphs

**New type of question**  
How quickly does \( \frac{\Delta f}{\Delta x} \) approach \( \frac{df}{dx} \) as \( \Delta x \to 0 \)?

The error \( E = \frac{\Delta f}{\Delta x} - \frac{df}{dx} \) will be \( E \approx A(\Delta x)^n \)  
What is \( n \)?

Usual one-sided \( \frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} \) only has \( n = 1 \)

Centered difference \( \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} \) has \( n = 2 \)

**Centered is much better than one-sided**  
\( E \approx (\Delta x)^2 \) vs \( E \approx \Delta x \)

[IDEA FOR \( f(x) = e^x \)]  
One-sided \( E \) vs centered \( E \)

[PROJECT at \( x = 0 \)]  
Graph \( \log E \) vs log \( \Delta x \)  
Should see slope 1 or 2

**Practice Questions**

1. Does \( x^{100} \) grow faster or slower than \( e^x \) as \( x \) gets large?
2. Does 100 \( \ln x \) grow faster or slower than \( x \) as \( x \) gets large?
3. Put these in increasing order for large \( n \):
   \[ \frac{1}{n}, \quad n \log n, \quad n^{1.1}, \quad \frac{10^n}{n!} \]
4. Put these in increasing order for large \( x \):
   \[ 2^{-x}, \quad e^{-x}, \quad \frac{1}{x^2}, \quad \frac{1}{x^{10}} \]

5. Describe the log-log graph of \( y = 10x^5 \) (graph \( \log y \) vs \( \log x \))
   
   Why don’t we see \( y = 0 \) at \( x = 0 \) on this graph?
   
   What is the slope of the straight line on the log-log graph?
   
   The line crosses the vertical axis when \( x = \) _____ and \( y = \) _____
   
   Then \( \log x = 0 \) and \( \log y = \) _____
   
   The line crosses the horizontal axis when \( x = \) _____ and \( y = 1 \)
   
   Then \( \log x = \) _____ and \( \log y = 0 \)

6. Draw the semilog graph (a line) of \( y = 10e^x \) (graph \( \log y \) versus \( x \))

7. That line cross the \( x = 0 \) axis at which \( \log y \)? What is the slope?