Unit 4: Polar Coordinates I

1. Lecture 2.030

A. For example:

\[ (r, \theta) = (r_1, \theta_1) \rightarrow (r_2, \theta_2) \]

**Complication**

\[ r_1 = r_2 \quad \text{and} \quad \theta_1 = \theta_2 \]

B. Nomenclature

If the polar equation for \( C \) is \( r = \rho(\theta) \), we don't mean:

\[ \frac{dr}{d\theta} \]

\[ \text{does not give the slope of} \ C \]

C. Area

\[ A = \frac{1}{2} \int (r^2 \sin \theta) \, d\theta \]

**provided \( r \) is a continuous function of \( \theta \)**

\[ A = \frac{1}{2} \int (r^2 \sin \theta) \, d\theta \]

Same answer - different expressions
2.4.1(L)

Describe the curve C if its polar equation is \( r = \cos \theta, \, 0 < \theta < \pi \).

2.4.2

The curve C is given by the polar equation

\[
\frac{1}{r^2} = 4 \cos^2 \theta + 9 \sin^2 \theta.
\]

Sketch C by converting its polar equation into the equivalent Cartesian form.

2.4.3(L)

a. Plot the curve C if its polar equation is \( r = \sec \theta, \, -\frac{\pi}{2} < \theta < \frac{\pi}{2} \).

b. Plot the curve whose polar equation is \( r = \theta \), and then write the equation of this curve in Cartesian coordinates.

2.4.4(L)

a. (1) The curve C is given by the polar equation \( r = f(\theta) \). What can we conclude about the symmetry of C if we know that whenever \((r_o, \theta_o)\) belongs to C so also does \((-r_o, -\theta_o)\)\?

(2) With C as above, what can we conclude about its symmetry if we know instead that whenever \((r_o, \theta_o)\) is on C so is \((-r_o, \pi - \theta_o)\)\?

b. Use the result of (a) together with the information contained in \( \frac{dr}{d\theta} \) to sketch the curve whose polar equation is \( r = \sin 2\theta \).

c. What is the Cartesian equation of the curve in (b)?
2.4.5(L)

a. Let $C_1$ and $C_2$ be defined by the polar equations $r = \cos \theta + 1$ and $r = \cos \theta - 1$, respectively. Show that $C_1$ and $C_2$ have no simultaneous points of intersection.

b. With $C_1$ and $C_2$ as in part (a), sketch these two curves.

c. Explain why the results of (a) and (b) are not contradictory.

2.4.6(L)

The curve $C_1$ is defined by the polar equation $r = \cos 2\theta$, while $C_2$ is defined by $r = 1 + \cos \theta$. Find all points at which $C_1$ and $C_2$ intersect.

2.4.7(L)

Let $C$ denote the curve whose polar equation is $r = \sin \frac{\theta}{2}$, $0^\circ \leq \theta \leq 720^\circ$. If $P$ denotes the point $(\frac{1}{2}, 240^\circ)$, does $P$ belong to $C$? Explain.

2.4.8

Find all points of intersection of the curves $C_1$ and $C_2$ if the polar equation for $C_1$ is $r = 1 + \cos \theta$ and the polar equation for $C_2$ is $r = 1 + \sin \theta$. 

Resource: Calculus Revisited: Multivariable Calculus
Prof. Herbert Gross

The following may not correspond to a particular course on MIT OpenCourseWare, but has been provided by the author as an individual learning resource.

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.