1. Read Supplementary Notes, Chapter 6, Section F.

2. Exercises:

4.4.1

Define \( f: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) by \( f(x,y) = (u,v) \) where \( u = 6x + 5y \) and \( v = x + y \).

a. In terms of the determinant of the matrix of coefficients, show how we may conclude that \( f \) exists.

b. Letting \( A = \begin{bmatrix} 6 & 5 \\ 1 & 1 \end{bmatrix} \), compute \( A^{-1} \) and then describe the mapping \( f^{-1}: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) explicitly.

c. In particular, compute \( f^{-1}(16,3) \).

d. Compute \( f(L) \) where \( L \) is the line \( y = 2x \) [i.e., \( L = \{ (x,y): y = 2x \} \)].

4.4.2

Define \( f: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) by \( f(x,y) = (u,v) \) where \( u = x + 4y \), \( v = 3x + 12y \).

a. Using determinants, show that \( f \) does not exist.

b. Describe the set \( f(E^2) \).

c. Assuming that we view \( f \) geometrically, find the locus of all points \( (x,y) \) in the \( xy \)-plane such that \( f(x,y) = (8,24) \).

d. Use (c) to show a geometric construction for finding the point \( (x,y) \) on the line \( 2x + 9y = 17 \) for which \( f(x,y) = (8,24) \).

e. Show that no other point on \( 2x + 9y = 17 \) can be mapped into \( (8,24) \) by \( f \).

4.4.3

Define \( f: \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) by \( f(x,y,z) = (u,v,w) \) where

\[
\begin{align*}
    u &= x + y + z \\
    v &= 2x + 3y + 2z \\
    w &= x + 3y + z
\end{align*}
\]

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4.4.3 continued

a. Show that \( f(E^3) \) is contained in \( \{(u,v,w):3u - 2v + w = 0\} \).
b. Interpret part (a) geometrically.
c. Describe the set \( S \) of all elements of \( E^3 \) for which \( f(x,y,z) = (0,0,0) \).
d. The points \((0,0,0)\) and \((1,1,-1)\) lie in the plane defined by \( f(E^3) \) [i.e., in \( w = -3u + 2v \)]. Describe the locus of all points \((x,y,z)\) such that \( f(x,y,z) \) is on the line \( L \) determined by \((0,0,0)\) and \((1,1,-1)\).

4.4.4

Given the system of equations

\[
\begin{align*}
x_1 + 2x_2 + x_4 &= b_1 \\
2x_1 + 5x_2 + 3x_3 + 4x_4 &= b_2 \\
3x_1 + 5x_2 + 2x_3 + x_4 &= b_3 \\
3x_1 + 4x_2 + x_3 - x_4 &= b_4
\end{align*}
\]

a. Use the augmented-matrix technique to determine the constraints under which the above equations have a solution.

b. In particular, show that if the constraints are met, \( x_3 \) and \( x_4 \) may be chosen at random, after which \( x_1 \) and \( x_2 \) are uniquely determined.

c. Let \( f:E^4 \rightarrow E^4 \) be defined by \( f(x_1,x_2,x_3,x_4) = (b_1,b_2,b_3,b_4) \), where \( b_1, b_2, b_3, \) and \( b_4 \) are as above.

(i) Show that there is no \( x \in E^4 \) such that \( f(x) = (1,1,1,1) \).

(ii) Find all \( x \in E^4 \) such that \( f(x) = (1,1,4,5) \).

4.4.5

Find the constraints under which the system

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4.4.5 continued

\[
\begin{align*}
    x_1 + x_2 + x_3 + 2x_4 + x_5 &= b_1 \\
    2x_1 + 3x_2 + 2x_3 + 3x_4 + 3x_5 &= b_2 \\
    3x_1 + 3x_2 + 4x_3 + 5x_4 + 2x_5 &= b_3 \\
    x_1 + 3x_2 - x_3 + 2x_4 + 5x_5 &= b_4 \\
    -2x_1 + x_2 - 6x_3 - 3x_4 + 5x_5 &= b_5
\end{align*}
\]

(1)

has solutions. In particular, discuss the function \( f: \mathbb{E}^5 \rightarrow \mathbb{E}^5 \) defined by \( f(x) = f(x_1, x_2, x_3, x_4, x_5) = (b_1, b_2, b_3, b_4, b_5) \), where \( b_1, b_2, b_3, b_4, \) and \( b_5 \) are as defined in (1).
Resource: Calculus Revisited: Multivariable Calculus
Prof. Herbert Gross

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