Example of impulse invariance.
Pole-zero patterns and frequency response corresponding to the example of viewgraph a.

Illustration of effect of frequency warping inherent in the bilinear transformation.

Illustration of effect of bilinear transformation on a piecewise constant frequency response characteristic.
2. Correction

The first equation on board number 1 indicates that the mapping corresponding to replacing differentials by differences is $s \mapsto \frac{z+1}{T}$. This should be corrected to $s \mapsto \frac{z-1}{T}$.

3. Comments

In the previous lecture we introduced two techniques for transforming analog filter designs to digital filter designs. The first of these, corresponding to transforming differentials to differences is not a very useful technique for the reasons discussed. The second, that of
impulse invariance is a useful technique, although it introduces aliasing which must be accounted for.

In this lecture we begin with an illustration of impulse invariance. An important observation in this example is that the zeros of the analog transfer function don't map to the z-plane in the same way that the poles do. As we stressed in lecture 14, the impulse invariant design procedure corresponds to a mapping of the poles and residues rather than a mapping of the poles and zeros. We also observe in this example the effect of aliasing.

The second method for IIR filter design discussed in this lecture is the use of the bilinear transformation. This transformation has the advantage that it maps the entire $j\omega$ axis in the s-plane into one revolution around the unit circle in the z-plane. Aliasing is therefore eliminated but in contrast to impulse invariance a nonlinear distortion in the frequency axis is introduced. This nonlinear distortion can be tolerated when the filter to be designed is piece-wise constant but usually cannot be for more general filter characteristics. In the next lecture a number of these points will be illustrated through a design example.

Impulse invariance and the bilinear transformation represent the two major analytical techniques for designing digital filters through the transformation of analog designs. There are also a number of algorithmic design procedures for digital filters. We conclude this lecture with a brief introduction to two, one of which requires the solution of a set of nonlinear equations and the second of which leads to a set of linear equations.

4. Reading

Text: Review section 7.1.1 (page 407) to example 7.3 and read section 7.1.2. (page 415) to example 7.4. Also read section 7.3 (page 438).

5. Problems

Problem 15.1

In Figure P15.1-1 is shown the frequency response of a digital filter. (a) Determine and sketch the analog frequency response characteristics which, without aliasing will map to this digital frequency response when the impulse invariant transformation is applied.
(b) Sketch the analog frequency response that will map to this
digital frequency response when the bilinear transformation is applied.

![Diagram of frequency response]

Figure P15.1-1

**Problem 15.2**

Repeat problem 15.1 for the digital frequency response characteristic in Figure P15.2-1.

![Diagram of frequency response]

Figure P15.2-1

**Problem 15.3**

A digital lowpass filter is to be designed using the bilinear transformation. The specifications are that:

(i) \((.99) < |H(e^{j\omega})| \leq 1\) for \(0 \leq \omega < \frac{\pi}{6}\)

(ii) \(|H(e^{j\omega})| < .001\) for \(\frac{\pi}{6} \leq \omega < \pi\).

Determine the specifications on an analog lowpass filter which will yield the desired digital filter when the bilinear transformation is applied with \(T = 1\).
Problem 15.4*
An analog highpass filter can be obtained from an analog lowpass filter by replacing $s$ by $1/s$ in the transfer function: i.e., if $G_a(s)$ is the transfer function for a lowpass filter, then $H_a(s)$ is the transfer function for a highpass filter if

$$H_a(s) = G_a \left( \frac{1}{s} \right).$$

Also, a digital filter can be obtained by mapping an analog filter by means of the bilinear transformation

$$s = \frac{z-1}{z+1}. \quad [\text{Assume for convenience that } T = 2 \text{ in Eq. (5.22) of the text.}]$$

Figure P15.4-1

This mapping preserves the character of the magnitude characteristic, although the frequency scale is distorted. The network in Figure P 15.4-1 depicts a lowpass filter with cutoff frequency $\omega_L = \pi/2$.

The constants $A, B, C,$ and $D$ are real. Determine how to modify the coefficients in the network to obtain a highpass filter with cutoff frequency $\omega_H = \pi/2$.

Problem 15.5*
We wish to approximate a desired unit-sample response using the least-squares inverse design procedure as outlined in section 5.3.3. of text. The desired unit sample response $h_d(n)$ is given by

$$h_d(n) = \begin{cases} 
1 & 0 \leq n < 10 \\
0 & \text{otherwise}
\end{cases}$$

15.6
corresponding to a 9th order FIR filter. This is to be approximated by a second order IIR filter of the form

\[ H(z) = \frac{b_0}{1 - a_1 z^{-1} - a_2 z^{-2}}. \]

Determine the coefficients \(b_0\), \(a_1\) and \(a_2\).