4-2 Electronic Feedback Systems

Blackboard 4.1

Stability (or lack thereof)
Bounded input → Bounded output
\[ \sum_{n=0}^{\infty} |y_n(t)| < \infty \]
\[ \sum_{n=0}^{\infty} |y_n| = \infty \]

\[ V_i \rightarrow a(s) \rightarrow a(s) \rightarrow V_o \]

\[ a(s) = \frac{a_0}{s^2 + s + 1}, \quad a_2 > 1 \]
\[ a(s) = \frac{1}{s^2 + s + 1}, \quad a_2 > 1 \]
\[ a(s) = \frac{a_0}{(s^2 + s + 1)(s^2 + s + 1)} \]

\[ \text{poles } S = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i \]

With \( a_0 = \frac{1}{4} \),
\[ \text{Root Locus } \]
\[ \frac{C_a C_b S^2 + (C_a + C_b)S + 1}{a_0 + a_0} = 0 \]

\[ S_1, S_2, S_3 = \]

\[ \frac{1}{2} \left( C_a C_b + \frac{1}{C_a C_b} \right) \]

\[ 1 + \frac{a_0}{(C_a S + 1)(C_b S + 1)} = 0 \]

Blackboard 4.2

Root Locus
\[ a(s) f(s) = \frac{a(s)}{a(s)} \]
\[ A(s) = \frac{1 + a_0 g(s)}{a_0 + a_0 g(s)} \]

poles \( 1 + a_0 g(s) = 0 \)

Let \( a(s) f(s) = \)
\[ \frac{a_0}{(C_a S + 1)(C_b S + 1)} \]

Characteristic equation:
\[ 1 + \frac{a_0}{(C_a S + 1)(C_b S + 1)} = 0 \]

\[ C_a C_b S^2 + (C_a + C_b)S + 1 = 0 \]

\[ S_1, S_2, S_3 = \]

\[ \frac{1}{2} \left( C_a C_b + \frac{1}{C_a C_b} \right) \]

4-2
This lecture provides our introduction to the stability of feedback systems. We will see again and again that the effective design of feedback systems hinges on the successful resolution of the compromise between desensitivity and speed of response on one hand and stability on the other.

Examples of first-, second-, and third-order systems illustrate how the difficulty of achieving a given degree of stability increases dramatically as the order of the system increases.
 Corrections

I left out $dt$ in both of the integrals used in the definition of stability on Blackboard 4.1. These integrals should read

$$\int_{-\infty}^{\infty} |v(t)| \, dt < \infty \quad \text{and} \quad \int_{-\infty}^{\infty} |v(t)| \, dt < \infty$$

I also left out a $t$ in the test generator signal on the same blackboard. The relationship should be

$$K \sin \frac{\sqrt{3}}{\tau} t = V_t$$

I may have left the wrong impression concerning evaluation of stability by means of loop-transmission frequency response. If the loop transmission is exactly +1 at some frequency, there is certainly a closed-loop pair of poles on the imaginary axis at that frequency. The point is that closed-loop pole locations (and particularly the important question concerning the number of closed-loop poles in the right-half of the $s$ plane) can generally not be resolved with loop-transmission information at isolated frequencies. This quantity must be known at all frequencies to answer the stability question.

Reading

Textbook: Chapter 4 through page 120.

Problems

Problem 4.1 (P4.1)

Problem 4.2 (P4.2)