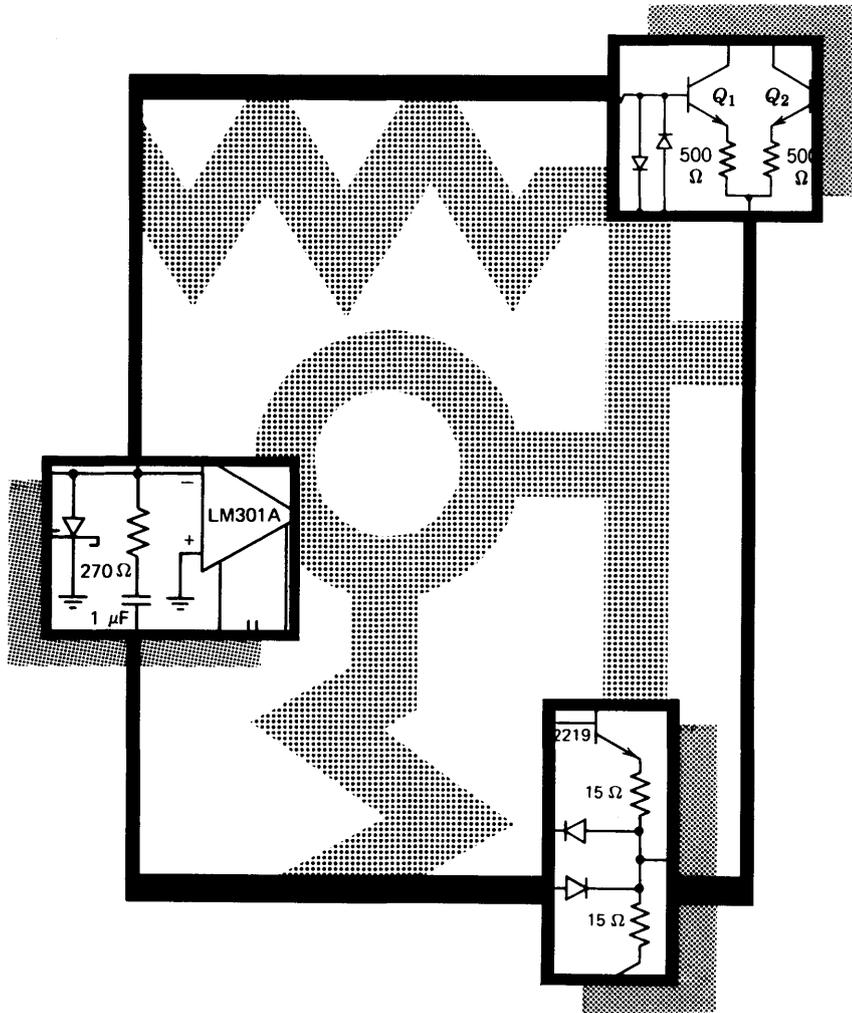


Stability

4

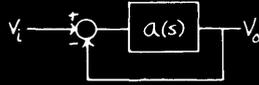


Blackboard 4.1

Stability (or lack thereof)
Bounded input \rightarrow Bounded Output

$$\int_{-\infty}^{\infty} |v_i(t)| < \infty$$

$$\int_{-\infty}^{\infty} |v_o(t)| < \infty$$



$$a(s) = \frac{a_0}{\tau_a s + 1}, a_0 \gg 1$$

$$A(s) \approx \frac{1}{\tau_a s + 1}$$

$$a(s) = \frac{a_0}{(\tau_a s + 1)(\tau_b s + 1)}, a_0 \gg 1$$

$$A(s) \approx \frac{1}{\tau_a \tau_b s^2 + (\tau_a + \tau_b) s + 1}$$

$$= \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1}$$

$$a(s) = \frac{a_0}{(\tau s + 1)^3}$$

$$A(s) = \frac{a_0}{\tau^3 s^3 + 3\tau^2 s^2 + 3\tau s + 1 + a_0}$$

poles @

$$s = -\frac{3}{\tau}, s = \pm j\frac{\sqrt{3}}{\tau}$$

With $a_0 = 8$,

$$A(s) @ s = \frac{j\sqrt{3}}{\tau} = \frac{8}{(j\sqrt{3} + 1)^3} = 1 \angle -180^\circ$$

4-1

Blackboard 4.2

Root Locus

$$A(s)f(s) = a_0 f_0 g(s)$$

$$A(s) = \frac{a(s)}{1 + a_0 f_0 g(s)}$$

poles @ $1 + a_0 f_0 g(s) = 0$

$$\text{Let } a(s)f(s) = \frac{a_0 f_0}{(\tau_a s + 1)(\tau_b s + 1)}$$

Characteristic equation:

$$1 + \frac{a_0 f_0}{(\tau_a s + 1)(\tau_b s + 1)} = 0$$

$$\tau_a \tau_b s^2 + (\tau_a + \tau_b) s + 1 + a_0 f_0 = 0$$

$$s_1, s_2 =$$

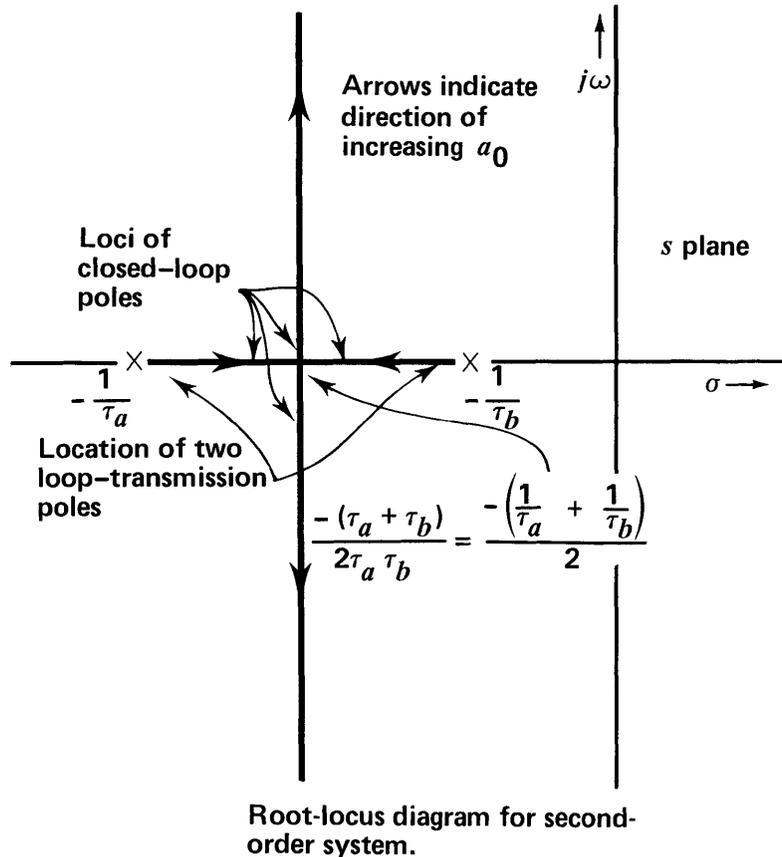
$$\frac{-(\tau_a + \tau_b) \pm \sqrt{(\tau_a + \tau_b)^2 - 4\tau_a \tau_b (1 + a_0 f_0)}}{2\tau_a \tau_b}$$

$$\text{meet @ } -\frac{(\tau_a + \tau_b)}{2\tau_a \tau_b} =$$

$$-\frac{1}{2} \left(\frac{1}{\tau_a} + \frac{1}{\tau_b} \right)$$

4-2

Viewgraph 4.1



This lecture provides our introduction to the stability of feedback systems. We will see again and again that the effective design of feedback systems hinges on the successful resolution of the compromise between desensitvity and speed of response on one hand and stability on the other.

Examples of first-, second-, and third-order systems illustrate how the difficulty of achieving a given degree of stability increases dramatically as the order of the system increases.

Comments

Corrections

I left out dt in both of the integrals used in the definition of stability on Blackboard 4.1. These integrals should read

$$\int_{-\infty}^{\infty} |v_i(t)| dt < \infty \quad \text{and} \quad \int_{-\infty}^{\infty} |v_o(t)| dt < \infty$$

I also left out a t in the test generator signal on the same blackboard. The relationship should be

$$K \sin \frac{\sqrt{3}}{\tau} t = V_i$$

I may have left the wrong impression concerning evaluation of stability by means of loop-transmission frequency response. If the loop transmission is exactly $+1$ at some frequency, there is certainly a closed-loop pair of poles on the imaginary axis at that frequency. The point is that closed-loop pole locations (and particularly the important question concerning the number of closed-loop poles in the right-half of the s plane) can generally not be resolved with loop-transmission information at isolated frequencies. This quantity must be known at all frequencies to answer the stability question.

Reading

Textbook: Chapter 4 through page 120.

Problems

Problem 4.1 (P4.1)

Problem 4.2 (P4.2)

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