Markov processes – 1

- checkout counter example
- Markov process definition
- n-step transition probabilities
- classification of states

\[ \text{state}(t+1) = f(\text{state}(t), \text{noise}) \]
checkout counter example

- discrete time \( n = 0, 1, \ldots \)
- customer arrivals: Bernoulli(\( p \))
- customer service times: geometric(\( q \))
- “state” \( X_n \): number of customers at time \( n \)
discrete-time finite state Markov chains

- $X_n$: state after $n$ transitions
  - belongs to a finite set
  - initial state $X_0$ either given or random
  - transition probabilities:
    \[ p_{ij} = P(X_1 = j \mid X_0 = i) = P(X_{n+1} = j \mid X_n = i) \]
- Markov property/assumption:
  “given current state, the past doesn’t matter”
  \[ p_{ij} = P(X_{n+1} = j \mid X_n = i) = P(X_{n+1} = j \mid X_n = i, X_{n-1}, \ldots, X_0) \]
- model specification: identify states, transitions, and transition probabilities
n-step transition probabilities

• state probabilities, given initial state $i$:
  \[ r_{ij}(n) = \mathbb{P}(X_n = j \mid X_0 = i) = \mathbb{P}(X_{n+s} = j \mid X_s = i) \]

• key recursion: \[ r_{ij}(n) = \sum_{k=1}^{m} r_{ik}(n-1)p_{kj} \]

• random initial state:
  \[ \mathbb{P}(X_n = j) = \sum_{i=1}^{m} \mathbb{P}(X_0 = i)r_{ij}(n) \]
example

\[ r_{ij}(n) = P(X_n = j \mid X_0 = i) \]

<table>
<thead>
<tr>
<th></th>
<th>( n = 0 )</th>
<th>( n = 1 )</th>
<th>( n = 2 )</th>
<th>( n = 100 )</th>
<th>( n = 101 )</th>
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</thead>
<tbody>
<tr>
<td>( r_{11}(n) )</td>
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<td>( r_{12}(n) )</td>
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<td>( r_{22}(n) )</td>
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generic convergence questions

- does $r_{ij}(n)$ converge to something?

\[
\begin{align*}
\text{n odd: } & r_{22}(n) = \\
\text{n even: } & r_{22}(n) =
\end{align*}
\]

- does the limit depend on initial state?

\[
\begin{align*}
& r_{11}(n) = \\
& r_{31}(n) = \\
& r_{21}(n) =
\end{align*}
\]
recurrent and transient states

- state i is recurrent if “starting from i, and from wherever you can go, there is a way of returning to i”
- if not recurrent, called transient

recurrent class: a collection of recurrent states communicating only between each other
Resource: Introduction to Probability
John Tsitsiklis and Patrick Jaillet

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