Theories, imagination, and the generation of new ideas

Ullman vs. Schulz
August 27th, 2015
Child as intuitive scientist

Stochastic search algorithms?

Large theory spaces

P(T|D) ∝ P(D|T) P(T)
Outline of Debate

Background (Tomer)

What good are theories?
Representing a good theory
Finding a good theory – stochastic search

Imagination and issues with stochastic search (Laura)

Response (Tomer)

Response and summary (Laura)
What Good is a Theory?

Structured knowledge, “theories”

(Magnets, metals and non-magnetic)
What Good is a Theory?

Begin collecting observations
Sometimes nothing happens
What Good is a Theory?

Sometimes nothing happens
Sometimes objects **stick**
What Good is a Theory?

Explanation: Bag of data?

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What Good is a Theory?

Explanation: Theory

Concepts: “schmagnet”, “schmetal”

Rules:

Rule 1: \( \text{interacts}(X,Y) \leftarrow \text{schmagnet}(X) \land \text{schmagnet}(Y) \)
Rule 2: \( \text{interacts}(X,Y) \leftarrow \text{schmagnet}(X) \land \text{schmetal}(Y) \)
Rule 3: \( \text{interacts}(X,Y) \leftarrow \text{interacts}(Y,X) \)

Assign “schmagnets” & “schmetals”

Predict observed data
Rational inference problem

Out of all possible theories, find the one that ‘best’ explains the observed data

\[ P(T|D) \propto P(D|T) \cdot P(T) \]

(Tenenbaum, Griffiths, & Kemp, 2006)
Learning a Good Theory - Grammar

**Top level theory**

(S1) \( S \Rightarrow (\text{Law}) \land S \)

(S2) \( S \Rightarrow (\text{Tem}) \land S \)

(S3) \( S \Rightarrow \text{Stop} \)

**Random law generation**

(Law) \( \text{Law} \Rightarrow (P_{\text{left}} \leftarrow P_{\text{right}} \land \text{Add}) \)

(Add1) \( A \Rightarrow \text{P} \)

(Add2) \( A \Rightarrow \text{Stop} \)

**Predicate generation**

\( P_{\text{left}} \)

\( P_{\text{left}} \Rightarrow \text{surface1()} \)

\( \vdots \)

\( P_{\text{left}}(\alpha) \)

\( P_{\text{left}} \Rightarrow \text{surface\alpha()} \)

\( P_{\text{right}} \)

\( P_{\text{right}} \Rightarrow \text{surface1()} \)

\( \vdots \)

\( P_{\text{right}}(\alpha) \)

\( P_{\text{right}} \Rightarrow \text{surface\alpha()} \)

\( P_{\text{right}}(\alpha + 1) \)

\( P_{\text{right}} \Rightarrow \text{core1()} \)

\( \vdots \)

\( P_{\text{right}}(\alpha + \beta) \)

\( P_{\text{right}} \Rightarrow \text{core\beta()} \)

**Law templates**

(Tem1) \( \text{Tem} \Rightarrow \text{template1()} \)

\( \vdots \)

(Tem\gamma) \( \text{Tem} \Rightarrow \text{template\gamma()} \)
Examples of Theories

Universal Theory

Theory

Model

Data

Probabilistic Horn Clause Grammar

Magnetism

Core Predicates: p(X), q(X)
Surface Predicates: interacts(X,Y)

Laws:
interacts(X,Y) ← p(X) ∧ p(Y)
interacts(X,Y) ← p(X) ∧ q(Y)
interacts(X,Y) ← interacts(Y,X)

p(X): “magnets”
“non-magnetic objects”

q(X): “magnetic objects”

Taxonomy

Core Predicates: f(X,Y), g(X,Y)
Surface Predicates: has_a(X,Y), is_a(X,Y)

Laws:
is_a(X,Y) ← g(X,Y)
has_a(X,Y) ← f(X,Y)
has_a(X,Y) ← is_a(X,Z) ∧ has_a(Z,Y)
is_a(X,Y) ← is_a(Z,Y) ∧ is_a(Z,Y)

Kinship

Core Predicates: t(X), u(X,Y), v(X,Y)
Surface Predicates: female(X), parent(X,Y)
spouse(X,Y), child(X,Y),
father(X,Y), uncle(X,Y), ...

Laws:
female(X) ← t(X)
spouse(X,Y) ← u(X,Y)
child(X,Y) ← v(X,Y)
child(X,Y) ← child(X,Z) ∧ spouse(Z,Y)
father(X,Y) ← ¬female(X) ∧ child(X,Y)

Psychology

Core Predicates: desires(X,Y)
Surface Predicates: reaches_for(X,Y), location(X,Y)

Laws:
reaches_for(X,Y) ← desires(X,Z) ∧ location(Z,Y)

“a shark is a fish”
“a bird can fly”
“a canary can fly”
“a salmon can breathe”

“John is William’s father”
“John is Judith’s grandfather”
“Judith is Hamnet’s sister”
“Margaret is Judith’s aunt”

Agent 1

Agent 2

Finding a Good Theory – Ideal Level

Prior (grammar)

Data comes in

Theory Space
The Problem of Search

Stochastic Search

1. Theory A
- interacts(X,Y) ← p(X) ∧ p(Y)
- interacts(X,Y) ← p(X) ∧ q(Y)
- interacts(X,Y) ← interacts(Y,X)

2. Theory B
- interacts(X,Y) ← p(X) ∧ q(Y)
- interacts(X,Y) ← interacts(Y,X)

3. Compare current and proposed theories

4. Probabilistically accept proposal

1. Current weights

2. Find gradient

3. Move along gradient

4. New weights

Proposing Alternative Theories

Observed predicate: interacts(X,Y)

Rule 1: interacts(X, Y) ← p(X) ∧ p(Y)

Metropolis-Hastings algorithm proposes alternative theories by changing current theory (new rules, predicates, etc)

Accept or reject new theory with probability depending on score
Proposing Alternative Theories

Observed predicate: interacts(X,Y)

Rule 1: interacts(X,Y) \land p(X) \land p(Y)
Rule 2: interacts(X,Y) \leftarrow q(X) \land q(Y)

Metropolis-Hastings algorithm proposes alternative theories by changing current theory (new rules, predicates, etc)

Accept or reject new theory with probability depending on score
Observed predicate: interacts(X,Y)

Rule 1: interacts(X, Y) ← p(X) ∧ p(Y)
Rule 2: interacts(X, Y) ← p(X) ∧ q(Y)

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Proposing Alternative Theories

Observed predicate: interacts(X,Y)

Rule 1: interacts(X, Y) ← p(X) ∧ p(Y)
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Rule 3: interacts(X, Y) ← interacts(Y, X)

Metropolis-Hastings algorithm proposes alternative theories by changing current theory (new rules, predicates, etc)

Accept or reject new theory with probability depending on score
Proposing Alternative Theories

Observed predicate: interacts(X,Y)

Rule 2: interacts(X, Y) ← p(X) ∧ q(Y)
Rule 3: interacts(X, Y) ← interacts(Y, X)

Metropolis-Hastings algorithm proposes alternative theories by changing current theory (new rules, predicates, etc)

Accept or reject new theory with probability depending on score
Proposing Alternative Theories

Observed predicate: interacts(X,Y)

Rule 1: interacts(X, Y) ← p(X)\land p(Y)
Rule 2: interacts(X, Y) ← p(X)\land q(Y)
Rule 3: interacts(X, Y) ← interacts(Y, X)

Metropolis-Hastings algorithm proposes alternative theories by changing current theory (new rules, predicates, etc)

Accept or reject new theory with probability depending on score
Stochastic Search and Children

Rule 1: \[\text{interacts}(X,Y) \leftarrow p(X) \land p(Y)\]
Rule 2: \[\text{interacts}(X,Y) \leftarrow p(X) \land q(Y)\]
Rule 3: \[\text{interacts}(X,Y) \leftarrow \text{interacts}(Y,X)\]

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Ullman, Goodman & Tenenbaum, 2012
Denison, Bonawitz, Gopnik and Griffiths 2013
Theories are useful

Rich, structured theories define a rich landscape

Algorithmic solution: stochastic search in rich landscape

Application to children?
Handoff to Laura

*In Which*, following an elegant exposition of a formal model, attendant experiments and quantitative data, Laura proceeds to wave her hands around ...
What’s wrong with stochastic search?

the end of the Expo—what we’re talking about—then let me be the end. But if, every time I want to sit down for a little rest, I have to brush away half a dozen of Rabbit’s smaller friends-and-relations first, then this isn’t an Expo—whatever it is—at all,

it’s simply a Confused Noise. That’s what I say.”

“I see what Eeyore means,” said Owl. “If you ask me——”

“I’m not asking anybody,” said Eeyore. “I’m just telling everybody. We can look for the North Pole, or we can play ‘Here we go gathering Nuts and May’ with the end part of an ant’s nest. It’s all the

“All right,” said Eeyore. “We’re going. Onl Don’t Blame Me.”

So off they all went to discover the Pole. And a they walked, they chattered to each other of this an that, all except Pooh, who was making up a song.

“This is not the Pole,” said Eeyore. “It’s just some place where we can sit down and have a few tummies.”

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Issues with Stochastic Search
Problem 1: Even with prior knowledge, templates, and a bias towards simplicity, the search space is infinite.

“I know why you have to turn off your cell phone when you get on the airplane”

“Oh yeah? Why?”
Issues with Stochastic Search

Problem 1: Even with prior knowledge, templates, and a bias towards simplicity, the search space is infinite.

“Because when the plane takes off it’s too noisy to hear.”

“Oh yeah? Why?”

Because airplanes are made of metal and so are phones.
Issues with Stochastic Search

Problem 1: Even with prior knowledge, templates, and a bias towards simplicity, the search space is infinite.

There are innumerable logical, constitutive, causal, and relational hypotheses consistent with the grammar of our intuitive theories. How do we rapidly converge on ones that actually might explain the data?

“Oh yeah? Why?”

Because airplanes and phones are both made in Ohio
Not just toy problem. Modeling even relatively simple, well-understood problems takes long time.

Iterations spent searching in hopeless places

Issues with Stochastic Search

Problem 1: Even with prior knowledge, templates, and a bias towards simplicity, the search space is infinite.

Approximate Bayesian Image Interpretation using Generative Probabilistic Graphics Programs

Vikash K. Mansinghka* 1,2, Tejas D. Kulkarni* 1,2, Yura N. Perov1,2,3, and Joshua B. Tenenbaum1,2
Not just a toy problem. Modeling even relatively simple, well-understood problems takes long time.

Iterations spent searching in hopeless places
Issues with Stochastic Search

Problem 2: Stochastic search does not make use of knowledge and abilities we seem to have.

- We know a lot about our problems, well before we can solve them.

- Abstract representation of what the solution might look like could help guide searching the space.
Issues with Stochastic Search

Problem 2: Stochastic search does not make use of knowledge and abilities we seem to have.

“Sure to be a pole,” said Rabbit, “because of calling it a pole, and if it’s a pole, well, I should think it would be sticking in the ground, shouldn’t you, because there’d be nowhere else to stick it.”
Issues with Stochastic Search

Problem 2: Stochastic search does not make use of knowledge and abilities we seem to have.

**Form** of the problem as **input** to algorithm should increase the probability that it proposes useful ideas.

Consider the information contained in question words:

- **Who?**
- **What?**
- **Where?**
- **When?**
- **Why?**
- **How?**
- **Which?**
Issues with Stochastic Search
Problem 2: Stochastic search does not make use of knowledge and abilities we seem to have.

Models use abstract form to evaluate hypotheses (Kemp & Tenenbaum 2008)

BUT representation of the problem could also constrain space

Learners have rich constraints far beyond question words.

Kinds of problems & criteria for solving them derive from multiple sources:

– The kinds of problems we want to solve (e.g., navigation, explanation, etc.)
– Broader epistemic ends (persuading, instructing, deceiving, etc.)
– Non-epistemic ends (impressing, soothing, entertaining, etc.)

Goals are innumerable, ways to achieve goals are limited
Proposal: Goal-oriented hypothesis generation
Proposal: Goal-oriented hypothesis generation

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When we do not have an abstract representation of what might count as a solution to a problem we resort to very inefficient and often ineffective searches.

– Indeed, what it might mean for us to think that a problem is “tractable” might be to recognize that we don’t know the answer

– but we at least have a precise enough representation of the problem to guide the search.
Goal-oriented hypothesis generation

Representing what “counts” as a solution to a problem might explain....

– Sense of “being on the right track”

– “Great idea!”, even when we know it is wrong

Can constrain proposals based on how well....

– They fit prior knowledge & data [“TRUTH”]

– They solve problems if they were true [“TRUTHINESS”]
What does it mean to think of a new idea?

- Generating new ideas is not about radical concept/theory change

- It is the problem of ordinary, everyday, productive thinking

- Can reliably make up new – relevant – answers to any *ad hoc* question. Answers may be trivial and may be false, but they are...
  
  - Genuinely new (didn’t have them until we thought of them)
  - Genuinely made up (didn’t learn them from new evidence/testimony)
  - Answers to the question (not non-sequiters)

- Only possible if we can use the form of the question to guide search
What does it mean to think of a new idea?

What’s a good name for a new theater company?

How do they get the stripes on peppermints?
Goal-oriented hypothesis generation

Is there any evidence that information contained only in the abstract form of the problem can help learners converge on solutions? ("Look Ma. No data.")

Rachel Magid

Mark Sheskin
Is there any evidence that information contained only in the abstract form of the problem can help learners converge on solutions? ("Look Ma. No data.")

**Goal-oriented hypothesis generation**

Two visual effects
- **Continuous**: ball flowing up and down.
- **Discrete**: ball appearing at the bottom, disappearing, and then appearing at top

Two auditory effects
- **Continuous**: low tone (225 Hz) gradually rising in pitch to high tone (900 Hz) and back
- **Discrete**: low tone (225 Hz) alternating with high tone (900 Hz)
Experiment 1

- Do you see the ball? It’s going low, high, low. I’m using one of these parts to make the ball go low, high, low.
- Do you see the ball? It’s going higher and lower. I’m using one of these parts to make the ball go higher and lower.

“Which part made the ball go______?”

Half the children asked about continuous visual and discrete auditory
Half asked about discrete visual and continuous auditory
Experiment 2

• Do you see the ball? It’s gazzing. I’m using one of these parts to make the machine gazz.

• Do you see the ball? It’s blicking. I’m using one of these parts to make the machine blick.

“Which part made the machine _______?”
Half the children asked about continuous visual and discrete auditory
Half asked about discrete visual and continuous auditory

![Diagram of machine parts](image-url)
Goal-oriented hypothesis generation

- No fact of the matter. No covariation evidence.

Two visual stimuli

- **Continuous**: ball flowing up and down.
- **Discrete**: ball appearing at the bottom, disappearing, and then appearing at top

Two auditory stimuli

- **Continuous**: low tone (225 Hz) gradually rising in pitch to high tone (900 Hz) and back
- **Discrete**: low tone (225 Hz) alternating with high tone (900 Hz)
Experiments 1 and 2

Results

% of trials on which children made each response type

- Corresponding affordance
- Non-corresponding affordance

Experiment 1
Experiment 2

Four-six year-olds.
Mean: 62 months.
N = 16/Experiment

Magid, Sheskin, & Schulz, in press, *Cognitive Development*
So what? We said there was no fact of the matter and no covariation evidence.

If children don’t know the answer and there’s no way to find out, maybe they just use cross-modal mapping to map from the affordance to the stimuli.

We wanted to know if they were actually using the form of the problem to constrain the solution.

If so, they should give different answers given different problems.
Goal-oriented hypothesis generation

- Do you see (hear) the machine? It’s gazzing (flurping). I’m using one of these parts to make the machine gazz (flurp).
- Do you see (hear) the machine. It’s blicking (daxing). I’m using one of these parts to make the machine blick (dax).

Showed the children the continuous visual stimuli and asked them how to generate the auditory one, and vice versa.
Goal-oriented hypothesis generation

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Results

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<th>% of trials on which children made each response type</th>
<th>Experiment 1</th>
<th>Experiment 2</th>
<th>Experiment 3</th>
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<td>Corresponding affordance</td>
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<td>Non-corresponding affordance</td>
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Four-six year-olds. Mean: 62 months. N = 16/Experiment

Magid, Sheskin, & Schulz, in press, *Cognitive Development*
Children prefer causal processes that preserve the dynamic form regardless of the lower level features and the absence of covariation evidence (Tsividis, Tenenbaum, & Schulz in prep).

$p < .001$ by two-tailed binomial test.
Is this just analogical reasoning?

- Funny kind of analogy. Not a mapping between a known problem and a solution to a new problem and new solution.
- Instead a mapping between the form of the problem to the form of the solution.
- Also the argument is that this applies to any possible goal we might have, including cases where it is not obvious that analogical reasoning applies.
- “What’s a good name for a new theater company?”
Rather, children seem to have **data-independent criteria** for the evaluation of hypotheses -- criteria that extend beyond simplicity or compatibility with prior knowledge.

Children can consider the extent to which a hypothesis fulfills the abstract goals of a solution to a problem, not just the degree to which a hypothesis fits the data.
A mystery of human cognition:

So much time pretending and making up stories

Good stories do not have to be true, BUT

Pose problems, solve problems
“And that is really the end of the story, and as I am very tired after that last sentence, I think I shall stop there.”

Goal-oriented hypothesis generation and imagination

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"Run away, run away" clip from Monty Python and the Holy Grail removed due to copyright restrictions.
"Tis but a scratch" clip from *Monty Python and the Holy Grail* removed due to copyright restrictions.
Critique: “Stochastic search does not make use of, or account for, some abilities we know people have”

Rebuttal: You’re wrong(?)

Many hypotheses - only aware of (relatively) good ones

Requires ability to suggest many hypotheses

Steve Piantadosi
Stochastic search algorithms can be parallelized (in some cases)

Run many “chains” in parallel, not one chain for a long time

Take advantage of GPU architecture, not CPU

~30 times faster than CPU

GPU’s are cheap and plentiful, search scales in number of GPU’s.

Awesome
Response 2: Relevant Proposals

Owen Lewis (MIT)
Response 1: Relevant Proposals

Current Hypothesis

- Or
  - And
    - Square
  - Red
    - Circle

New example

- Triangle
  - Size 2

Stochastic description

And

Owen Lewis (MIT)
Response 2: Relevant Proposals

Current Hypothesis

Or

And

Red

Circle

Square

Or

And

Triangle

Size 2

New example

Bottom line: propose relevant hypotheses

Owen Lewis (MIT)
Response 2: Relevant Proposals

Owen Lewis (MIT)
Reminder: Templates as smart proposals

Template 1: \( P(X,Y) \leftarrow P(X,Z) \land P(Z,Y) \)
Template 2: \( P(X,Y) \leftarrow P(Z,X) \land P(Z,Y) \)
Template 3: \( P(X,Y) \leftarrow P(X,Z) \land P(Y,Z) \)
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Template 8: \( P(X,Y) \leftarrow P(Y,X) \land P(Y) \)
Template 9: \( P(X,Y) \leftarrow P(X) \land P(Y) \)
Template 10: \( P(X,Y) \leftarrow P(Y,X) \)
Template 11: \( P(X,Y) \leftarrow P(X,Y) \)
Template 12: \( P(X) \leftarrow P(X) \)
Template 13: \( P(X) \leftarrow P(X,Y) \land P(X) \)
Template 14: \( P(X) \leftarrow P(Y,X) \land P(X) \)
Template 15: \( P(X) \leftarrow P(X,Y) \land P(Y) \)
Template 16: \( P(X) \leftarrow P(Y,X) \land P(Y) \)

Useful when there are multiple problems:

\text{e.g. Transitivity}

BUT: Discovery of templates?
Eyal Dechter
(MIT)

Exploration Compression Algorithm

(Dechter, Malmaud, Adams & Tenenbaum 2013)

Takes stochastic grammar over programs & primitives:

- Generates function library
- Library ‘encapsulates’ useful concepts

Example: Boolean circuits

- Primitives: \{I, S, C, B, \}

  Learned concepts:

  - [NOT]
  - [AND]
  - [E2]

Bottom line: learn and re-use good ‘chunks’
Response 2 – Good Primitives

Program length
short

long

Effective search area
Response 4 – Relevant Spaces

Construct relevant spaces on the fly

“Good name for new romantic drama”

“Give me a paper title for SRCD”

Max Siegel (MIT)
Response 4 – Relevant Spaces

“Good name for new romantic drama”

Mini-Grammar

The

N

Crown

DP

of

Lies

MT

Christine of the Big Tops

Cupid's Fireman

The Crown of Lies

The Coming of Amos

Clothes Make the Woman

The Climbers
Response 4 – Relevant Spaces

“Good name for new romantic drama”
- Essence of Time
- Love Lightly
- Endless Love
- Girls in Ships
- Those we Meet Again
- Value of Love
- Land of Roses
- Legend of Paris

“Good name for new action movie”
- Hu: the Annihilation
- Jack Death
- The Chase
- The Edge
- The Oversight
- Swordsman in China III
- The Hit
- Among Heroes
- Hunchback of Monte Cristo
- Get it Did
- Belle of a Lesser God
- Eagle Shooting Heroes
- Tomb Raider: the Raging God of Violence
- Legend of Legend
Summary

Still a long way to go to model children, meet Laura’s critique

Hard to say what is hard (early days)

People in development should (continue) to care about search algorithms, to everyone’s benefit
Goal-oriented hypothesis generation and imagination

Very cool. Error-driven proposals. But still driven by the data. We seem to treat the problem itself as part of the “data”.

Also very cool. Explains how you develop new representational resources. But not all learning problems can be solved just by changing the representational format.

Might be true. “That’s what an expedition means. A long line of everybody.” But ... not as good a story.
Questions?