February 27, 2004

UNIFIED QUIZ 1MS
(Beam Handout, (equations of elasticity handout available on request) no books, no notes; calculators allowed)

- Put your name on the top page, and on each subsequent page of the exam if you disassemble the pages.
- Read all questions carefully.
- Do all work on that question on that page. Use back of that page if necessary.
- Show all your work, especially intermediate results. Partial credit cannot be given without intermediate results.
- Show the logical path of your work. Explain clearly your reasoning and what you are doing. In some cases, the reasoning is worth as much (or more) than the actual answers.
- Be sure to show the units as well (if necessary). Final answers are not correct without the units.
- State any assumptions you are making
- Report significant digits only.
- Box your final answers.

EXAM SCORING

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FINAL SCORE
PROBLEM #1 (24%)

The questions that follow should require only a few sentences as answers, or very simple calculations. Provide sufficient detail to support your answer, but be brief.

a.) Structural steel girders and aluminum wing spars are often made with an "I" cross section rather than a solid rectangular cross-section. Why is this?

b.) Aircraft structures are designed according to requirements of high strength and stiffness and low mass. Steel, aluminum and titanium alloys all have virtually identical stiffness to density ratios (E/ρ) and strength to density ratios (σ/ρ). And yet aluminum is the material predominantly used for commercial transport airframes. Why?
c) Estimate the maximum tensile stress in the structure below, loaded by a point load of 100 kN.
Simple beam theory is derived based on the assumption "plane sections remain planar and perpendicular to the midplane of the beam". This only strictly applies to beams under pure bending (i.e. with no shear forces). In general, beams will carry shear forces. Justify the applicability of simple beam theory to cases where shear forces are present.
PROBLEM #2 (25%) 

A uniform bar of length 2L and cross sectional area A (with a solid circular cross-section), made of a material of density \( \rho \), Young's modulus \( E \) and yield stress, \( \sigma_y \), rotates at constant angular velocity \( \omega \) about an axis through its center, perpendicular to its length (see figure below). You should assume that no transverse forces or moments or torques act on the bar.

![Diagram of a rotating bar](image)

a) Determine and sketch \( F(x) \), the internal axial force distribution along the bar. Hint. The centrifugal force, \( F \), required to keep a point mass \( m \) rotating in a circle radius \( r \) at a speed \( \omega \) is \( F = mr\omega^2 \)
b) Calculate the axial extension of the bar.
PROBLEM #3 (26%)

A simply supported beam is loaded by a point load $P$, $1/3$ of the way along its length as shown below:

\[ P \]

\[ \begin{array}{c}
A \\
C \\
B \\
\end{array} \]

\[ \begin{array}{c}
L/3 \\
2L/3 \\
\end{array} \]

a) Calculate and sketch the bending moment and shear force distributions as a function of position along the beam. Note the maximum values of each and their locations.
b) The beam has a rectangular cross-section, height h, breadth b. It is made of a material with a Young’s modulus E and a yield stress, τ_y. Outline the process you would use to calculate the deflection of point C, the loading point. Write down all relevant equations and quantities where known, but do not solve the resulting equations.
PROBLEM #4 (25%)

A beam of length $L$ and flexural rigidity $EI$ is clamped at each end. The beam has a point load of magnitude $P$ applied at the mid point of the beam.

Outline a solution approach to calculate the deflection of point $C$ of the midpoint of the beam. Write down the relevant (final) equations you will need to use, describe how you would use them, but do not solve them.
PROBLEM #4 (25%)