Three-Dimensional Wall Effects

In a freestream, recall that a lifting body can be modeled by a horseshoe vortex:

Consider a rectangular cross-section tunnel:

Flow is into page

The image system for this looks like:
The effect of these images is:

For fixed lift, such that $\Gamma^*$ is constant,

* an upwash exists due to images $\Rightarrow \alpha$ is effectively larger

\[
\alpha_{\text{eff}} \approx \alpha_{\text{tunnel}} + \Delta \alpha_i,
\]

where $\alpha_{\text{eff}}$ is the effective AOA, $\alpha_{\text{tunnel}}$ is the AOA of the model in the tunnel, and $\Delta \alpha_i$ is the correction due to the upwash induced by the images.

* Similarly, this creates decrease in induced drag relative to freestream flight:

Recall,

\[
C_{D_i} \propto C_L \alpha_i
\]

$\Rightarrow \Delta C_{D_i} = C_L \Delta \alpha_i$

$\Rightarrow C_{D_{\text{tot}}} = C_{D_{\text{tunnel}}} + \Delta C_{D_i}$

Or, since we are interested in the total drag:

\[
C_{D_{\text{tot}}} = C_{D_{\text{tunnel}}} + \Delta C_{D_i}
\]

Specific formulas derived in detailed analysis give that:

\[
\Delta \alpha_i = \delta \left( \frac{S}{C} \right) C_L
\]

where $S =$ reference area

$C =$ tunnel cross - sectional area

$\delta =$ factor which depends on tunnel & model geometry

Wright Brothers is an elliptic cross-section with dimensions 10 ft wide by 7 ft high.
Define:

\[ \lambda \equiv \frac{h}{B} \]

\[ k \equiv \frac{b}{B} \]

\[ b_e \equiv \text{effective span} \approx 0.9b \]

Values of \( \delta \) for a wing with uniform loading in a closed elliptical jet:

![Graph showing values of \( \delta \) for different values of \( K \).]