**Critical Mach Number**

We can estimate the freestream Mach number at which the flow first accelerates above $M > 1$ (locally) using the Prandtl-Glauert scaling and isentropic relationships.

Recall from P-G:

\[ C_p(M_\infty) = \frac{C_p(M_\infty = 0)}{\sqrt{1 - M^2_\infty}} \]

If we have $C_p(M_\infty = 0)$ say from an incompressible panel solution, we could then find $C_p$ anywhere on the airfoil for higher $M_\infty$ under the assumptions of P-G (linearized flow, $M_\infty < 1$).

We can also use isentropic relationships:

\[ C_p = -\frac{2}{\gamma M^2_\infty} \left( \frac{p}{p_\infty} - 1 \right) \]

\[ \Rightarrow C_p = \frac{2}{\gamma M^2_\infty} \left[ \frac{1 + \frac{1}{2}(\gamma - 1)M^2_\infty \gamma^{\frac{\gamma}{\gamma - 1}}} {1 + \frac{1}{2}(\gamma - 1)M^2_\infty} \right] - 1 \]

The $C_p$ for $M = 1$ at a given $M_\infty$ is:

\[ C_{p,cr} = C_p(M = 1, M_\infty) = \frac{2}{\gamma M^2_\infty} \left[ \frac{1 + \frac{1}{2}(\gamma - 1)M^2_\infty \gamma^{\frac{\gamma}{\gamma - 1}}} {1 + \frac{1}{2}(\gamma - 1)} \right] - 1 \]

This critical freestream $M_\infty$ occurs when $C_{p,P-G}(M_{cr}) = C_{p,cr}(M_{cr})$.

This critical $M_\infty$ can be found graphically or can be solved for with a root-finding method. Let’s look at what happens graphically:
1. Find minimum $C_p$ at $M_\infty = 0$
2. Plot $C_{p,\min}$ vs. $M_\infty$
3. Plot $C_{p,\text{cr}}$ from isentropic relationships