Problem Set #4

Problem 1 (50 points)
Consider the following quote, taken from (Barwise and Etchemendy, 1993):

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

You will represent this quote in propositional logic using the following propositional symbols:

- MY = “Mythical Unicorn”
- IU = “Immortal Unicorn”
- MM = “Mortal Mammal”
- HU = “Horned Unicorn”
- MG = “Magical Unicorn”

Part A Restate the above English sentence as a sentence in propositional logic.

Part B Reduce your propositional sentence to conjunctive normal form (i.e., a list of clauses).

- Walk through your reduction step by step, starting from sentence S and ending with your set of clauses. Your reduction should be similar in style to the reduction given in the appendix of the class notes.

- List your resulting clauses, labeling them C1 … Cn.
Part C: Given your propositional logic sentence, does the following logically follow?

the *unicorn is mythical*

Include a proof of your claim.

Part D: Given your propositional logic sentence, does the following logically follow?

the *unicorn is magical*

Include a proof of your claim.

Part E: Given your propositional logic sentence, does the following logically follow?

the *unicorn is horned*

Include a proof of your claim.

Problem 2 – Propositional Logic and Inference (50 points)

**Part A – Interpretations**

Let $S$ be the propositional sentence:

$$(A \text{ implies } B) \text{ implies } (C \text{ implies } D)$$

and let $I$ be the interpretation:

$A = \text{ True}, \ B = \text{ False}, \ C = \text{ True}, \ D = \text{ False}$

Is $I$ a model for $S$, that is, does $I$ satisfy $S$ (Yes or No)? Demonstrate the correctness of your answer.

**Part B – Reduction to Clauses (Conjunctive Normal Form)**
Reduce the following three propositional sentences to conjunctive normal form (CNF) (i.e., a set of clauses). Derive each result step by step:

**Part B.1** Convert \([\neg ((\neg A \text{ and } B) \text{ or } (C \text{ and } D))]\) to CNF.

**Part B.2** Convert \((A \text{ iff } A)\) to CNF

**Part B.3** Convert \([(A \text{ iff } B) \text{ or } C]\) to CNF

**NOTE:** Please indicate the time that you have spent for each problem.