Lecture #AC–3

Aircraft Lateral Dynamics

Spiral, Roll, and Dutch Roll Modes
Aircraft Lateral Dynamics

- Using a procedure similar to the longitudinal case, we can develop the equations of motion for the lateral dynamics

\[ \dot{x} = Ax + Bu, \quad x = \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix}, \quad u = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \]

and \( \dot{\psi} = r \sec \theta_0 \)

\[ A = \begin{bmatrix} \frac{Y_v}{m} & \frac{Y_p}{m} & \frac{Y_r}{m} - U_0 & g \cos \theta_0 \\ \frac{(L_v + I'_{zx} N_v)}{T_{xx}} & \frac{(L_p + I'_{zx} N_p)}{T_{xx}} & \frac{(L_r + I'_{zx} N_r)}{T_{xx}} & 0 \\ \frac{I'_{zx} L_v}{I_{zz}} & \frac{I'_{zx} L_p}{I_{zz}} & \frac{I'_{zx} L_r + N_r}{I_{zz}} & 0 \\ 0 & 1 & \tan \theta_0 & 0 \end{bmatrix} \]

where

\[ I'_{xx} = (I_{xx} I_{zz} - I_{zx}^2)/I_{zz} \]
\[ I'_{zz} = (I_{xx} I_{zz} - I_{zx}^2)/I_{xx} \]
\[ I'_{zx} = I_{zx}/(I_{xx} I_{zz} - I_{zx}^2) \]

and

\[ B = \begin{bmatrix} (m)^{-1} & 0 & 0 & 0 \\ 0 & (I'_{xx})^{-1} & I'_{zx} & 0 \\ 0 & I'_{zx} & (I'_{zz})^{-1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \end{bmatrix} \]
• The code gives the numerical values for all of the stability derivatives. Can solve for the eigenvalues of the matrix $A$ to find the modes of the system.

$$-0.0331 \pm 0.9470i$$
$$-0.5633$$
$$-0.0073$$

– Stable, but there is one very slow pole.

• There are 3 modes, but they are a lot more complicated than the longitudinal case.

| Slow mode | -0.0073 | ⇒ Spiral Mode |
| Fast real | -0.5633 | ⇒ Roll Damping |
| Oscillatory | $-0.0331 \pm 0.9470i$ | ⇒ Dutch Roll |

Can look at normalized eigenvectors:

<table>
<thead>
<tr>
<th></th>
<th>Spiral</th>
<th>Roll</th>
<th>Dutch Roll</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.0067</td>
<td>-0.0197</td>
<td>0.3269</td>
</tr>
<tr>
<td>$\dot{\beta}$</td>
<td>-0.0009</td>
<td>-0.0712</td>
<td>0.1198</td>
</tr>
<tr>
<td>$\hat{r}$</td>
<td>0.0052</td>
<td>0.0040</td>
<td>0.0368</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Not as enlightening as the longitudinal case.
Lateral Modes

Roll Damping - well damped.

- As the plane rolls, the wing going down has an increased $\alpha$
  (wind is effectively “coming up” more at the wing)

- Opposite effect for other wing.

- There is a difference in the lift generated by both wings
  $\rightarrow$ more on side going down

- The differential lift creates a moment that tends to restore the equilibrium

- After a disturbance, the roll rate builds up exponentially until the restoring moment balances the disturbing moment, and a steady roll is established.
Spiral Mode - slow, often unstable.

- From level flight, consider a disturbance that creates a small roll angle $\phi > 0$
- This results in a small side-slip $v$ (vehicle *slides downhill*)
- Now the tail fin hits on the oncoming air at an incidence angle $\beta$
  $\rightarrow$ extra tail lift $\rightarrow$ yawing moment
- The positive yawing moment tends to increase the side-slip
  $\rightarrow$ makes things worse.
- If unstable and left unchecked, the aircraft would fly a slowly diverging path in roll, yaw, and altitude $\Rightarrow$ it would tend to *spiral* into the ground!!

- Can get a restoring torque from the wing dihedral
- Want a small tail to reduce the impact of the spiral mode.
**Dutch Roll** - damped oscillation in yaw, that couples into roll.

- Frequency similar to longitudinal short period mode, not as well damped (fin less effect than the horizontal tail).
- Do you know the origins on the name of the mode?
- Consider a disturbance from straight-level flight
  - Oscillation in yaw $\psi$ (fin provides the *aerodynamic stiffness*)
  - Wings moving back and forth due to yaw motion result in oscillatory differential Lift/Drag (wing moving forward generates more lift)
  - Oscillation in roll $\phi$ that lags $\psi$ by approximately $90^\circ$
  - *Forward going wing is low*

Oscillating roll $\rightarrow$ sideslip in direction of low wing.

- Damp the Dutch roll mode with a large tail fin.
**Aircraft Actuator Influence**

- Transfer functions dominated by lightly damped Dutch-roll mode.
  - Note the rudder is physically quite high, so it also influences the A/C roll.
  - Ailerons influence the Yaw because of the differential drag

- Impulse response for the two inputs:
  - **Rudder input**
    - \( \beta \) shows a very lightly damped decay.
    - \( p, r \) clearly excited as well.
    - \( \phi \) oscillates around 2.5\(^\circ\)
    - \( \Rightarrow \) Dutch-roll oscillations are clear.
    - \( \Rightarrow \) Spiral mode ultimately dominates \( \phi \rightarrow 0 \) after 250 sec.

  - **Aileron input**
    - Large impact on \( p \)
    - Causes large change to \( \phi \)
    - Very small change to remaining variables.
    - Influence smaller than Rudder.

- Lateral approximate models are much harder to make (see discussion in Etkin and Reid). Not worth discussing at length.
Figure 1: Rudder impulse to flight variables. The rudder excites all modes. Dutch roll oscillations dominate initially. The spiral mode dominates longer term.
Figure 2: Aileron impulse to flight variables. Response primarily in $\phi$. 