Lecture #8

Examples Using Lagrange's Equations
**Example**

Given: Catapult rotating at a constant rate (frictionless, in the horizontal plane)

Find the EOM of the particle as it leaves the tube.
Derivatives:

\[ \frac{\partial T}{\partial \dot{r}} = m \dot{r}, \quad \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{r}} \right) = m \ddot{r}, \quad \frac{\partial T}{\partial r} = m r \omega^2 \]

External forces: None

Lagrange’s equation gives the equation of motion as \( \ddot{r} - r \omega^2 = 0 \)

What do we get if we solve this via Newton’s method?
Example

Mass particle in a frictionless spinning ring.

Ring spins at constant rate $\omega$

Spherical coordinate set (2-11)

Two holonomic constraints

- $r = \text{constant}$
- $\phi = \omega t + \phi_0$ which gives the spin rate of the tube

So only 1 DOF $\Rightarrow$ use $\theta$ as the generalized coordinate
Example

System of 3 “particles” suspended by pulleys.

(Neglect mass of pulleys.)
Example

2 particles in a frictionless tube held by springs. Assume that $s = 0$ and $a = 0$