Math Review

Differentiation

\[ df = \left( \frac{\partial f}{\partial x} \right)_y \, dx + \left( \frac{\partial f}{\partial y} \right)_x \, dy \quad \text{and} \quad tf = z(x, y) \]

\[ \left( \frac{\partial f}{\partial x} \right)_y = \left( \frac{\partial f}{\partial x} \right)_y + \left( \frac{\partial f}{\partial y} \right)_x \left( \frac{\partial y}{\partial x} \right)_z \]

\[ \left( \frac{\partial x}{\partial y} \right)_z = \frac{1}{\left( \frac{\partial y}{\partial x} \right)_z} \]

\[ \left( \frac{\partial x}{\partial y} \right)_z = -\left( \frac{\partial x}{\partial y} \right)_y \left( \frac{\partial y}{\partial x} \right)_z \quad \text{or} \quad \left( \frac{\partial y}{\partial x} \right)_z \left( \frac{\partial y}{\partial x} \right)_z \left( \frac{\partial z}{\partial x} \right)_y = -1 \]

If \( df = gdx + hd\) is exact then

\[ \left( \frac{\partial g}{\partial y} \right)_x = \left( \frac{\partial f}{\partial x} \right)_y \]

Examples:

\[ \frac{d}{dx} x^r = rx^{r-1} \]

\[ \frac{d}{dx} g^x = g^x \ln g \]

\[ \frac{d}{dx} e^{ux} = e^{ux} u \]

\[ \frac{d}{dx} \ln(x) = \frac{1}{x} \]

\[ \frac{d}{dx} \log_a(x) = \frac{1}{x \ln(a)} \]

**Sum Rule:**

\[ \frac{d}{dx} (5f + 3g) = 5 \frac{df}{dx} + 3 \frac{dg}{dx} \quad \text{f= f(x); g=g(x)} \]

**Product Rule:**

\[ \frac{d}{dx} f g = f \frac{dg}{dx} + g \frac{df}{dx} \]

**Quotient Rule:**
\[ \frac{d}{dx} \left( \frac{f}{g} \right) = \frac{g \frac{df}{dx} - f \frac{dg}{dx}}{g^2} \]

Integration

Most common integrals:

\[ \int_a^b \frac{1}{x} \, dx = \ln \left( \frac{b}{a} \right) \]

\[ \int_a^b \frac{1}{x^2} \, dx = -\frac{1}{x} \bigg|_a^b = -\frac{1}{a} - \frac{1}{b} \]

\[ \int_a^b x \, dx = \frac{1}{2} x^2 \bigg|_a^b = \frac{1}{2} (b^2 - a^2) \]

\[ \int_a^b e^{\alpha x} \, dx = \frac{1}{\alpha} e^{\alpha x} \bigg|_a^b = \left[ \frac{1}{\alpha} (e^{\alpha b} - e^{\alpha a}) \right] \]

Quadratic Equation: solving for \( x \)

\[ \alpha x^2 + bx + c = 0 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

→ Simplifications (kinetics, equilibrium problems)

The quadratic equation shows up in kinetics questions and equilibrium questions. Typically, it is not required to solve the quadratic equation IF \( X \) IS SMALL.

Example:

\[ \frac{x^2}{(x - a)(b - x)} = d \]

If \( x \) is assumed to be small, then,

\[ \frac{x^2}{(a)(b)} = d \]

If the answer is small for \( x \), then it is ok. If \( x \) turns out to be pretty big (0.4) then use the quadratic formulae!
Taylor Expansions: when $x$ is small (especially in kinetics questions!)

\[
\phi^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \quad -\infty < x < \infty
\]

\[
\ln(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots \quad \text{For } x < 1
\]

Partial Fractions

Example:

\[
\frac{x - 1}{(3x - 5)(x - 3)} = \frac{A}{3x - 5} + \frac{B}{x - 3}
\]

Multiply by denominator everywhere:

\[
\frac{(x - 1)(3x - 5)(x - 3)}{(3x - 5)(x - 3)} = \frac{A(3x - 5)(x - 3) + B(3x - 5)(x - 3)}{3x - 5} + \frac{B(3x - 5)(x - 3)}{x - 3}
\]

Simplify:

\[x - 1 = A(x - 3) + B(3x - 5)\]

Solve by setting values for $x$:

Let $x=3$; \ 2 = A(0) + B(4) \ \Rightarrow \ B = 0.5$

Let $x=0$; \ -1 = A(-3) + 0.5(-5) \ \Rightarrow \ A = -0.5$

Final Answer:

\[
\frac{x - 1}{(3x - 5)(x - 3)} = -0.5 \cdot \frac{3x - 5}{3x - 5} + 0.5 \cdot \frac{5}{x - 3} = \frac{-1}{2(3x - 5)} + \frac{1}{2(x - 3)}
\]
Math Tricks:

\[ \ln a - \ln b = \ln \left( \frac{a}{b} \right) \]

*when you have to equations in terms of \( \ln \), subtract the equations from each other and use the trick

Example:

\[
\begin{align*}
\ln(p_1) &= -\frac{3000}{T} + 13.1 \\
\ln(p_2) &= -\frac{4000}{T} + 16.4 \\
\end{align*}
\]

given \( \frac{p_1}{p_1} = 1 \) **(what point is this?)

To solve for \( T \) and you are given the ratio of the pressures, subtract the equations:

\[
\ln \left( \frac{p_1}{p_2} \right) = -\frac{3000}{T} + 13.1 - \left( -\frac{4000}{T} + 16.4 \right)
\]

\[
\ln(xy) = \ln x + \ln y
\]

\[
\delta \left( \frac{1}{p} \right) = -\frac{1}{p^2} \delta p
\]

\[
\frac{1}{p} \delta p = \delta \ln p
\]

\[
e^a e^b = e^{a+b}
\]