**1.020 Ecology II: Engineering for Sustainability**

**Lectures 08_15 Outline: Discounting, Building Energy Revisited**

**Motivation/Objective**
Determine the economic benefits of installing more energy efficient windows in the small house analyzed in Problem Set 3.

**Approach**
1. Use the model of Problem Set 3 to determine the heating fuel consumed with and without improved (higher R value) windows.
2. Add present value and amortization to house energy MATLAB code
3. Compare present value of energy costs saved with improved windows vs. cost of purchasing and installing the windows.
4. Alternatively, compare amortized monthly payment on loan to purchase windows with monthly savings in heating bill

**Concepts and Definitions Needed:**
Temporal allocation of benefits and costs
- Net present value: convert all benefits and costs to a single value at beginning of time period.
- Amortization: Convert present benefits/costs to equal periodic (e.g. annual) payments

**Equations:**
- Interest rate for each time period = $r$, number of time periods = $N$
- Present value $B_{PV}$ of benefit $B_t$ at end of period $t$: $B_{PV} = (1 + r)^{-t} B_t$
- Present value $C_{PV}$ of benefit $C_t$ at end of period $t$: $C_{PV} = (1 + r)^{-t} C_t$

Net present value of a stream of benefits (or costs) accrued over $N$ periods:

$$B_{NPV} = B_0 + \sum_{t=1}^{N} (1 + r)^{-t} B_t$$
$$C_{NPV} = C_0 + \sum_{t=1}^{N} (1 + r)^{-t} C_t$$

For equal benefits (or costs) incurred every period after initial time:

$$B_{NPV} = (CRF)B_1$$
$$C_{NPV} = (CRF)C_1$$
$$CRF = \frac{1 - (1 + r)^{-N}}{r}$$

Amortized amount $C_t$ (payment over each period) of amount $C_0$ borrowed or received at initial time: $C_t = \frac{C_0}{CRF}$

Total paid over $N$ periods: $C_{\text{tot}} = \frac{NC_0}{CRF}$

**Implications**
Classical exponential discounting minimizes advantages of future benefits (e.g. abundant clean water for the next generation) and disadvantages of future costs (e.g. increased sea levels due to climate change). For this reason classical discounting has been criticized by some environmental economists. Classical exponential discounting should be used with caution.

**Modeling Example -- Improved Windows**
Note how much energy must be saved for investment in new windows to be justified.