Question 1

Pricing a Trucking Service

A small trucking company serves two OD pairs as represented in Figure 1. Assume that daily costs are represented by the following function:

\[ C = 100 + 5 Y_1 + 3 Y_2 - 0.001 Y_1 Y_2 \]

Furthermore, demand at each OD pair is given by:

\[ Y_1 = 50 - P_1 \]
\[ Y_2 = 25 - 2P_2 \]

where \( Y_i \) is in tons/day, \( P_i \) is in $/ton, and \( C \) is in $/day.

Answer the following questions:

a. Find the fares that:
   i. maximize social welfare
   ii. maximize profit
   iii. maximize social welfare subject to no losses (note: this will require the use of the Excel or other numerical solver)

Compare and discuss your results.

b. Assume that cost complementarity between both flows is zero (i.e. the cost function is \( C = 100 + 5 Y_1 + 3 Y_2 \)). Calculate the new optimal fares for each case in (a) and discuss the differences in your results (between (a) and (b)).
Question 2
The Use of Pricing to Control Flow on Urban Expressways

Travel on urban expressways can be described by the following simple linear
relationship between the actual speed at which traffic flows (S, measured in miles per
hour) and the flow of traffic (Q, measured in vehicles per hour in each traffic lane):

$$S = S_0 - aQ$$

where $S_0$ represents the speed at which vehicles would travel under uncrowded
conditions if there were no speed limit, and $a$ is a parameter of the relationship.

The Highway Capacity Manual suggests values of 70 miles per hour for $S_0$ and 0.015 for
$a$. These values are quite close to those estimated from actual traffic speed and flow
data for urban expressways in California, which are $S_0=74.7$ miles per hour and
$a=0.0169$.

Answer the following questions:

a. Using the estimated California values for $S_0$ and $a$, and assuming that the fixed
operation expenses for a typical automobile total 20 cents per mile, derive an
expression for the average private cost per mile of operating an automobile on an
urban expressway when the auto is occupied by $n$ persons, each valuing travel time
at $V$ dollars per hour. Compute and plot this cost for a range of traffic flows from 0
to 2000 cars per lane per hour, using $n = 1.4$ and $V = $10.00 per hour.

b. Using the same assumptions, write an equation for the marginal social cost imposed
by each vehicle using the facility, defined as the change in the total private costs
incurred by all travelers on the road when an additional vehicle enters the flow.
(Total private costs are the product of average private cost per vehicle and the
number of vehicles in the flow, $Q$.) Compute and plot this cost over the same range
of traffic flows as in part (a) above, again using the values $n = 1.4$ and $V = $10.00 per
hour.

c. Suppose an urban expressway characterized by these cost functions initially
experiences flows of 2000 cars per lane per hour during the peak travel hours. How
large is the “external” cost per mile that is imposed on other travelers by each
automobile using the facility? How large is it as a proportion of the costs that are
privately borne by each vehicle? If automobiles average 20 miles per gallon of fuel,
how does it compare to 36 cents per gallon in federal and state gasoline taxes?

d. Answer the following:

i. Based on your calculations in (c), what is the toll that should be charged by the
government for use of the road?

ii. If the elasticity of demand for travel on this facility during the peak hours with
respect to the cost of doing so is $-0.2$, what flow would results from a policy of
charging motorists this toll?

iii. How fast would vehicles travel at this new flow? What is the toll that should be
charged at the new flow? Are motorists over-charged?

iv. Compute the equilibrium optimal toll, taking into consideration the demand
change in response to the toll.
Question 3
Airline Revenue Management

Consider a flight from Boston, MA to Minneapolis, MN. Table 5 describes the different booking classes on the flight and the predicted demand on each class.

<table>
<thead>
<tr>
<th>Booking class</th>
<th>Mean Predicted demand</th>
<th>Fare</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15</td>
<td>1000</td>
</tr>
<tr>
<td>B</td>
<td>30</td>
<td>650</td>
</tr>
<tr>
<td>C</td>
<td>50</td>
<td>350</td>
</tr>
</tbody>
</table>

The demand for booking classes A and B are characterized by the discrete distributions shown in Table 6. The total number of available seats on the aircraft is 100.

<table>
<thead>
<tr>
<th>Booking Class A</th>
<th>Demand</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>19</td>
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<tr>
<td></td>
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<td>0.11</td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>0.11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Booking Class B</th>
<th>Demand</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
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<td>0.2</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Note: These are discrete distributions, so demand on booking classes A and B can only take on the values shown in the table. For example, the probability that demand for booking class B is 23 or 44 is zero.

Answer the following question:
Apply the EMSRb model to calculate the number of seats that should be protected in each booking class. (10 points)

A Note on Numerical Solvers

A few parts of this problem set will require the basic use of numerical solvers. Excel has sufficient functionality and is likely the easiest method for those unfamiliar with such tools. Basic use of the Excel solver may be covered by the TA in a recitation, but otherwise you should make an effort to ask the TA or one of your classmates how to use it in advance of doing this problem set.