III. INTERPARTICLE FORCES: Components and Interaction 

1. COMPONENTS OF EFFECTIVE STRESS (II 2-25) 

1.1 Physico-Chemical Effective Stress Eqn (Ladd 1961) 

1) Objective - General idea of how \( \sigma' \) is transmitted between particles in cohesive soil. 

2) Assume "contact" forces act when 
   \( 2d \leq 20A \) (rather arbitrary upper area ratio \( A_c \)) 
   Long range double layer type forces act at \( 2d > 20A \) 

3) Eqn: 
   \[ \sigma' = \text{net contact stress} + \text{net long range stress} \]
   \[ = \bar{\sigma} \cdot a_c + R - A \]
   \[ = (\bar{\sigma}_r - \bar{\sigma}_a) a_c + R - A \]

4) Long range stresses (\( 2d > 20A \)) 
   - Double layer (osmotic) repulsion \( R = f(\frac{1}{d}) \) - Shear A, B 
   - Long range van der Waals attraction \( A = f(\frac{1}{d}) \)
   \[ \rho_a (\text{parallel particle}) = \frac{A''}{48 \pi \pi} \left( \frac{1}{d^3} + \frac{1}{(d+5)^3} - \frac{2}{(d+5)^3} \right) \]
   \( A'' = \text{Hamaker constant} = 2 \times 10^{-20} J \) (JKM '91, p. 124) 
   \( d = \text{particle thickness} \)
   \( \rho_a = \text{half spacing between particles} \)
   \[ A \propto \rho_a^3 \] increases with \( \rho_a \) thought to be indep of pore fluid
5) Contactennes (2d < 20 A)

Repulsion $F_r = \text{displacement of "adsorbed" water (pore fluid when not H}_2O$)

+ Born repulsion of mineral contact
  (+ edge-to-face repulsion for reg. edge charge)

Attractive $\varphi_a = \text{short range van der Waals}$

+ edge-to-face electrostatic attraction
  (+ edge in reg. face)

+ primary volume bonding of mineral contact
  (ionic & covalent)

+ cementation (like Carbonate Iron Oxides...)

1.2 Discussion

1) Components for granular scale

- Is there a DL?
- $\sigma = \varphi_a \cdot A_c$

2) Effects of pore fluid on $R_A$ for cohesive soils with "high" SSA

Decreasing $R = f(P_r)$ for

(i) EC. value $\nu$

(ii) Bulk $C_0$

(iii) Dielectric cont. $D$

For clay in sea water ($35g/l=1M+\text{C}_0=0.6M$): $R \gtrsim A$?

For clay in alcohol ($D=20 > 80$ in $H_2O$): $R \gtrsim A$?
3) Effects of pore fluid on \( \bar{\sigma}_c \) for cohesive soils

What trends for:

1. \( \text{mo, pH} \)
2. \( \text{mo, CO} \)
3. \( \text{V, high strain velocity} \)
4. \( \text{Deo, O} \)

2. PARTICLE INTERACTION

2.1 Energy Diagrams

\( V \) = energy/unit area to change minimum distance (2d) between two particles

- 2a) Always increasing net repulsion (repulsive magnets)
- 2b) Always increasing net attraction (attractive magnets)

\[ DL \rightarrow \Delta V / \Delta 2d \]

Net force = \( \Delta V / \Delta 2d \)

\( V \) = energy/unit area to change minimum distance (2d) between two particles

**Need to apply energy to**: decrease spacing

**Need to apply energy to**: increase spacing
2.2 Energy Diagram for Hypothetical Contact (Sheet C)

1) Overview for illitic clay at low salt concentration.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Dominant Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$R$ = osmotic repulsion</td>
</tr>
<tr>
<td>B</td>
<td>$\sigma_a$ due to electrostatic attraction</td>
</tr>
<tr>
<td>C</td>
<td>$\sigma_t$ due to adsorbed HSO 4 (Acid)</td>
</tr>
<tr>
<td>D</td>
<td>$\sigma_a$ due to V.D. forces, then primary valence</td>
</tr>
<tr>
<td>E</td>
<td>$\sigma_r$ due to Born repulsion</td>
</tr>
</tbody>
</table>

2) Significance of numbered pts

2) 3 5 Metastable

$\sigma' = \ldots$ but slight $\Delta V$ to

1) Energy Sink

$\sigma' = \ldots$ but need significant $\Delta V$ to

2.3 Source of True Cohesion

Must put energy into system to increase spacing

1) $\Delta V = b - a$

2) $\Delta V = c$
Supplement on Double Layer Repulsion

Single Double Layer: Ion conc. vs. distance Na⁺ Ca Montmorillonite

From Mitchell (1993) Figs. 7.8/7.11

\[ t_D = \text{Debye thickness} \]
\[ = 333 \, \text{Å Na} \]
\[ = 167 \, \text{Å Ca} \]

\[ n_C = n_a \]
\[ C_0 = 0.83 \times 10^{-4} \, \text{M} \]
\[ = 6 \times 10^{-3} \times \frac{1}{1000} \]
\[ = 5 \times 10^{-6} \]

\[ M = \text{mole/l} \]
\[ 6 \times 10^{-3} \text{ Ammonium} \]
\[ \text{Calcium} \]

Interacting Double Layer: (valence \( \nu^+ = \nu^- = 2 \); \( C_0 = \text{bulk concentration}, M = \text{mole/l} \))

\[ P_r = \frac{R_T}{C_0} \left( \frac{C_C + C_C - 2 C_0}{2} \right) \]
\[ T = 273 + ^\circ C \]
\[ R_T = 8.314 \, \text{J} \, \text{K}^{-1} \, \text{mol}^{-1} \]
\[ C + M, \text{mole/l} \]

\[ P_r (\text{bar}) = 24.37 (C_C + C_C - 2 C_0) \text{ at } 20^\circ C, D = 80 \text{ ln H}_2O \]

<table>
<thead>
<tr>
<th>( C_0 (\text{M}) )</th>
<th>( \nu )</th>
<th>( 2d (\text{Å}) )</th>
<th>( P_r (\text{bar}=6\text{cm}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{-3} )</td>
<td>1</td>
<td>200</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td>0.8</td>
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<td></td>
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<td></td>
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<td>12</td>
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<tr>
<td>( 10^{-5} )</td>
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<td>25</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

Fig 2-1

1.361 Part II 2

1.361 Part III 2
Double Layer Repulsion ($P_r$)

$T = 20^\circ C \quad \nu = cation \ valence, \ M = \text{mol/liter}$

From Bolt (1956) $x_0 = 41 \nu$ for Mont.

$c = D - x_0 = 2I \nu$ for Kool.

$c = I \nu$ for Illite

(Gouy-Chapman with Stern layer)

van der Waals Attraction ($P_a$)

Calculated for particle thickness, $S = 100 \AA$ and $A'' = 2.5 \times 10^{-20} J$

Fig. II2-1 Double Layer (Osmotic) Repulsion and van der Waals Attraction vs. Half Spacing for Parallel Infinite Plates (after Ladd, 1961 Sc.D Thesis)
Hypothetical Relationship of Energy vs. Interparticle Spacing
(Illitic clay at low salt concentration)

Must put energy into system to form contact
Must put energy into system to break contact

(Donnan)
Electrostatic Attraction
van der Waals Attraction
Absorbed Water Repulsion
Osmotic repulsion
Hydration
Orientation
Primary Valence

Figure by MIT OCW.