IV-2A TWO-DIMENSIONAL FLOW

(Objective: to know how to interpret flow nets)

1. Basic Equation
   1.1 Assumptions
   1.2 Physical Model for 3-D Flow with Changing Volume
   1.3 Solution for 2-D Flow and No Volume Change

2. Drawing Flow Nets
   2.1 Problem
   2.2 Stepwise Drawing Flow Net
      (For information only)
   2.3 Eq. for Flow per Unit Length

3. Miscellaneous
   3.1 Singularities
   3.2 Non-Homogeneous Sink
   3.3 Anisotropic Soil
1. BASIC EQUATION

1.1 Assumptions

Darcy's law \( q = k \alpha \) + constant \( k \) + incompressible fluid \( S = 100\% \)

1.2 Physical Model for 3-D Flow with Changing Volume

- Unit volume, with axes coinciding
- Partial differentials since \( h = f(x, y, z) \) with \( k \) axes

\[
\begin{align*}
\Delta q_x &= k_x \left( \frac{\partial h}{\partial x} \right)_x - \left( \frac{\partial h}{\partial x} \right)_{x_2} \\
&= k_x \left( \frac{\partial^2 h}{\partial x^2} \right) dx dy dz \text{ per unit volume}
\end{align*}
\]

(2) Summation in 3 directions for unit volume:

\[
\begin{align*}
&k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = \frac{1}{1 + \alpha} \frac{\partial e}{\partial t} \\
\{ \text{3-D consolidation} \}
\end{align*}
\]

\[
\begin{align*}
\text{Net flow into element/unit time} &\quad \text{Volumetric change/unit time}
\end{align*}
\]

(2) Solution for 2-D Flow \& No Volume Change \( (\partial e/\partial t = 0) \)

(1) \( k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0 \rightarrow \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \) for isotropic soil \( (k_x = k_y) \)

Change in \( h \) gradient per unit distance in \( x \) direction

\[
\begin{align*}
\therefore &\quad \text{Ratio } b/l \text{ is constant} \\
\text{For convenience select } b/l = 1 \rightarrow \text{"squares"}
\end{align*}
\]

Wherein solution \( \rightarrow \) series of lines intersecting at 90°:

- Equipotential lines (constant \( h \))
- Ratio \( b/l \) is constant
- Flow lines

\[
\begin{align*}
h_1 > h_2 > h_3
\end{align*}
\]
2. DRAWING FLOW NETS

2.1 Problem \((k_h = k_v)\)

2.2 Steps in Drawing Flow Net (If computer program not available)

1. Draw problem in ink.
2. Draw in known equipotential & flow boundary lines.
3. Sketch in 2 or 3 flow lines (experience helps!)
4. Draw corresponding equipotential lines.
   - Check for "squares" \& L intersections.
5. Keep adjusting (and adjusting ...)

2.3 Eq. For Flow Per Unit Length

\[ q = \frac{K^3}{h} \]

where \( n_f = \) no. of flow paths; \( n_l = \) no. of equipotential drops.

- Even sloppy flow nets (such as above) \(\Rightarrow\) reasonable \(\$\) and flow estimates given uncertainty in \(k\) values.
- BUT: sloppy nets can \(\Rightarrow\) large errors in \(i = \frac{\Delta h}{\Delta x}\) at critical locations.
3. MISCUELLANEous
3.1 Singularities

(i) When angle between flow and equipotential lines (on side containing the flow):
   - $a < 90^\circ \rightarrow \ell = 0$ (5-sided)
   - $b > 90^\circ \rightarrow \ell = \infty$ (3-sided)

(ii) When $\ell = \infty$, real surface, wrong about piping if Quick condition.

3.2 Non-Homogeneous Soils

- See Sheet A for example that illustrates constant $f$ for varying $n/k$. ! 62

3.3 Anisotropic Soil ($k_x \neq k_y$)

(i) Transformed section:
   - Decrease $\ell$ in direction of $k_{\max} \sqrt{k_{\min}/k_{\max}}$ or
   - Increase $\ell$ in direction of $k_{\min} \sqrt{k_{\max}/k_{\min}}$

(ii) Draw flow net $\rightarrow f$

   \[ Q = k \ell \frac{h (D L)(h)}{W_d} \]

   where $k = \sqrt{k_{\min} k_{\max}}$

(iii) To obtain $\ell$, from transformed section ($k_y = \ell k$), need $\ell$ natural

\[ i = \frac{\Delta h}{\ell_N} = \frac{\text{shortest distance}}{\text{between equipotential lines}} \]
Three forms of one flow net.

### Zone 1
- \( q_1 = k_1 \frac{h}{a}(35)(i) = 0.44 k_i h \)
- \( \Delta h = h/8; n_d = 8 \)
- \( n_f - 1 = 3.5 \)
- \( k_2 = 5k_1 \)

### Zone 2
- \( q_2 = k_2 \frac{h}{b}(35)(i) = 0.088 k_i h = 0.44 k_i h \)
- \( n_f - 2 = b/a = 0.7 \)
- \( \Delta h = h/8; n_d = 8 \)
- \( n_f - 2 = 0.7 \)

### Example
- Applying \( q = kh \left( \frac{n_f}{n_d} \right) \) to seepage in non-homogeneous soils.

Note: \( a = 1 \) in notes (i.e. \( b/a = \frac{b}{a} \))