The problem is to determine the optimum (highest net benefit) amount of groundwater to pump from four wells screened in a confined groundwater aquifer. The decision variables are the pumping rates and pumping lifts (depths to the water table) at the four wells:

\[ x = [x_1, x_2, x_3, x_4] \quad \text{Pumping rates (m}^3/\text{sec) at wells 1, 2, 3, and 4} \]

\[ y = [y_1, y_2, y_3, y_4] \quad \text{Pumping lifts (m.) at wells 1, 2, 3, and 4} \]

Net benefit is measured in terms of the difference between the annual revenues obtained from irrigated crops and the annual costs associated with pumping and delivery:

Objective: \[ F = R - C \]

Revenue: \[ R = \sum_{i=1}^{4} a x_i (b - x_i) \]

Cost: \[ C = \sum_{i=1}^{4} e x_i y_i \]

Note that the marginal revenue \( dR/dx_i \) for each well decreases to zero as the quantity pumped increases to \( b/2 \). This reflects the decreasing demand for increasing amounts of pumped water. The water cost is proportional to the electrical energy required to pump at a rate \( x_i \) over a depth \( y_i \) for a growing season of 4 months.

Pumping at each well affects the drawdown at the other wells. This effect may be quantified with a groundwater model or through pumping tests. In either case, there is a linear relationship between the vectors \( x \) and \( y \) if the aquifer is confined (as we assume here). If we also assume the groundwater system is in steady-state the pumping-lift relationship may be summarized with a symmetric response matrix \( A \). Each element \( A_{ij} \) of this matrix relates the steady state lift \( y_i \) at well \( i \) (in m.) to the steady-state pumping rate \( x_j \) at well \( j \) (m\(^3\)/sec):

\[ y_i = A_{ij} x_j \quad \text{where } A = \begin{bmatrix} 2000 & 600 & 300 & 200 \\ 600 & 3000 & 500 & 400 \\ 300 & 500 & 1500 & 500 \\ 200 & 400 & 500 & 2000 \end{bmatrix} \text{sec/m}^2 \]
When the response matrix relationship is substituted in the objective function expressions above the result is a quadratic objective. For a GAMS solution to this problem the response matrix equation and the definitions of \( R \) and \( C \) may be included as a set of equality constraints. Then the objective function can be written in terms of the intermediate decision variables \( R \) and \( C \).

Additional problem constraints are:

\[
    x_i \leq \frac{b}{2} \quad y_i \leq 30 \quad x_i \geq 0 \quad y_i \geq 0 \quad \text{for all } i
\]

Assume that:

\[
a = 4.0 \times 10^8 \frac{\text{$/m}^6/\text{sec}^2}{b = .01 \text{ m}^3/\text{sec}}
\]

In order to solve this problem, carry out the following tasks:

1). Derive the cost coefficient \( e \), given that the price of electricity is $0.20 / kwhr.

2). Check to see if a local maximum for this problem is also global (i.e. is the objective function concave?). To check this you will need to construct a Hessian matrix and use MATLAB to test its eigenvalues for your particular set of problem inputs.

3). Solve the problem using GAMS. If the objective function is not concave try a few different initial feasible solutions to provide confidence that the GAMS solution is a global maximum.

4). Evaluate the shadow prices associated with any pumping lift \( (y_i) \) constraints that are active at the GAMS solution.

Please hand in only enough information to adequately document your solution.