The Tropical Atmosphere: Hurricane Incubator
A One-Dimensional Description of the Tropical Atmosphere
Elements of Thermal Balance: Solar Radiation

• Luminosity: \(3.9 \times 10^{26} \text{ J s}^{-1} = 6.4 \times 10^7 \text{ Wm}^{-2}\) at top of photosphere

• Mean distance from earth: \(1.5 \times 10^{11} \text{ m}\)

• Flux density at mean radius of earth

\[
S_0 \equiv \frac{L_0}{4\pi d^2} = 1370 \text{ Wm}^{-2}
\]
Stefan-Boltzmann Equation: \( F = \sigma T^4 \)
\[ \sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{K}^{-4} \]

Sun: \( \sigma T^4 = 6.4 \times 10^7 \text{ Wm}^{-2} \)
\[ \rightarrow T \approx 6,000 \text{ K} \]
Disposition of Solar Radiation:

Total absorbed solar radiation = $S_0 \left( 1 - \alpha_p \right) \pi r_p^2$

$\alpha_p \equiv \text{planetary albedo} \ (\approx 30\%)$

Total surface area = $4\pi r_p^2$

Absorption per unit area = $\frac{S_0}{4} \left( 1 - \alpha_p \right)$

Absorption by clouds, atmosphere, and surface
Terrestrial Radiation:

Effective emission temperature:

\[ \sigma T_e^4 = \frac{S_0}{4} \left( 1 - \frac{a}{p} \right) \]

Earth: \[ T_e = 255K = -18^\circ C \]

Observed average surface temperature = 288K = 15°C
Highly Reduced Model

- Transparent to solar radiation
- Opaque to infrared radiation
- Blackbody emission from surface and each layer
Radiative Equilibrium:

Top of Atmosphere:

\[ \frac{\sigma T_A^4}{4} \left( 1 - \alpha_p \right) = \sigma T_e^4 \]

\[ \rightarrow T_A = T_e \]

Surface:

\[ \sigma T_s^4 = \sigma T_A^4 \left( 1 - \alpha_p \right) + \frac{S_0}{4} = 2 \sigma T_e^4 \]

\[ \rightarrow T_s = 2^{\frac{1}{4}} T_e = 303 \text{ K} \]
Surface temperature too large because:

- Real atmosphere is not opaque
- Heat transported by convection as well as by radiation
**Extended Layer Models**

**TOA:** \( \sigma T_2^4 = \sigma T_e^4 \rightarrow T_2 = T_e \)

**Middle Layer:** \( 2\sigma T_1^4 = \sigma T_2^4 + \sigma T_s^4 = \sigma T_e^4 + \sigma T_s^4 \)

**Surface:** \( \sigma T_s^4 = \sigma T_e^4 + \sigma T_1^4 \)

\( \rightarrow \) \( T_s = 3^{\frac{1}{4}} T_e \) \hspace{1cm} \( T_1 = 2^{\frac{1}{4}} T_e \)
Effects of emissivity $< 1$

Surface: \[ 2\varepsilon_A \sigma T_A^4 = \varepsilon_A \sigma T_1^4 + \varepsilon_A \sigma T_s^4 \]
\[ \rightarrow T_A = \left(\frac{5}{2}\right)^{\frac{1}{4}} T_e \approx 321K < T_s \]

Stratosphere: \[ 2\varepsilon_t \sigma T_t^4 = \varepsilon_A \sigma T_2^4 \]
\[ \rightarrow T_t = \left(\frac{1}{2}\right)^{\frac{1}{4}} T_e \approx 214K < T_e \]
Full calculation of radiative equilibrium:
Problems with radiative equilibrium solution:

- Too hot at and near surface
- Too cold at a near tropopause
- Lapse rate of temperature too large in the troposphere
- (But stratosphere temperature close to observed)
Missing ingredient: Convection

• As important as radiation in transporting enthalpy in the vertical
• Also controls distribution of water vapor and clouds, the two most important constituents in radiative transfer
When is a fluid unstable to convection?

• Pressure and hydrostatic equilibrium
• Buoyancy
• Stability
Hydrostatic equilibrium:

\[ \text{Weight: } -g \rho \delta x \delta y \delta z \]

\[ \text{Pressure: } p \delta x \delta y - (p + \delta p) \delta x \delta y \]

\[ F = MA: \quad \rho \delta x \delta y \delta z \frac{dw}{dt} = -g \rho \delta x \delta y \delta z - \delta p \delta x \delta y \]

\[ \frac{dw}{dt} = -g - \alpha \frac{\partial p}{\partial z}, \quad \alpha = \frac{1}{\rho} = \text{specific volume} \]
Pressure distribution in atmosphere at rest:

**Ideal gas:** \( \alpha = \frac{RT}{p} \), \( R \equiv \frac{R^*}{\bar{m}} \)

**Hydrostatic:**
\[
\frac{1}{p} \frac{\partial p}{\partial z} = \frac{\partial \ln(p)}{\partial z} = -\frac{g}{RT}
\]

**Isothermal case:** \( p = p_0 e^{-z/H} \), \( H \equiv \frac{RT}{g} = "scale\ height" \)

Earth: \( H \sim 8 \text{ Km} \)
Buoyancy:

\[ F = MA: \quad \rho_b \delta x \delta y \delta z \frac{dw}{dt} = -g \rho_b \delta x \delta y \delta z - \delta p \delta x \delta y \]

\[ \frac{dw}{dt} = -g - \alpha_b \frac{\partial p}{\partial z} \quad \text{but} \quad \frac{\partial p}{\partial z} = -\frac{g}{\alpha_e} \]

\[ \rightarrow \quad \frac{dw}{dt} = g \frac{\alpha_b - \alpha_e}{\alpha_e} \equiv B \]
Buoyancy and Entropy

Specific Volume: \[ \alpha = \frac{1}{\rho} \]

Specific Entropy: \[ s \]

\[ \alpha = \alpha(p,s) \]

\[
\left( \frac{\delta \alpha}{\delta s} \right)_p = \left( \frac{\partial \alpha}{\partial s} \right)_p \delta s = \left( \frac{\partial T}{\partial p} \right)_s \delta s
\]

\[
B = g \frac{\left( \frac{\delta \alpha}{\delta s} \right)_p}{\alpha} = \frac{g}{\alpha} \left( \frac{\partial T}{\partial p} \right)_s \delta s = - \left( \frac{\partial T}{\partial z} \right)_s \delta s \equiv \Gamma \delta s
\]

Maxwell: \[ \left( \frac{\partial \alpha}{\partial s} \right)_p = \left( \frac{\partial T}{\partial p} \right)_s \]
The adiabatic lapse rate:

First Law of Thermodynamics:

\[ \dot{Q} = T \frac{ds_{rev}}{dt} = c_v \frac{dT}{dt} + p \frac{d\alpha}{dt} \]

\[ = c_v \frac{dT}{dt} + \frac{d(\alpha p)}{dt} - \alpha \frac{dp}{dt} \]

\[ = (c_v + R) \frac{dT}{dt} - \alpha \frac{dp}{dt} \]

\[ = c_p \frac{dT}{dt} - \alpha \frac{dp}{dt} \]

Adiabatic: \[ c_p dT - \alpha dp = 0 \]

Hydrostatic: \[ c_p dT + gdz = 0 \]

\[ \rightarrow \left( \frac{dT}{dz} \right)_s = - \frac{g}{c_p} \equiv -\Gamma^d \]
Earth’s atmosphere:

\[ \Gamma = \frac{g}{c_p} \]

\[ \Gamma = \frac{1 \text{K}}{100 \text{m}} \]
Model Aircraft Measurements
(Renno and Williams, 1995)
Radiative equilibrium is unstable in the troposphere.
Re-calculate equilibrium assuming that tropospheric stability is rendered neutral by convection:

Radiative-Convective Equilibrium
Better, but still too hot at surface, too cold at tropopause
Above a thin boundary layer, most atmospheric convection involved phase change of water:

**Moist Convection**
Moist Convection

- Significant heating owing to phase changes of water
- Redistribution of water vapor – most important greenhouse gas
- Significant contributor to stratiform cloudiness – albedo and longwave trapping
Stability Assessment using Tephigrams:

![Graph showing stability assessment using Tephigrams. The graph plots pressure (mb) against temperature (°C). The axes are labeled as follows: Pressure (mb) on the y-axis and Temperature (°C) on the x-axis. Various lines represent different conditions or stability levels.](image-url)
Convective Available Potential Energy (CAPE):

\[ CAPE_i \equiv \int_{p_n}^{p_i} (\alpha_p - \alpha_e) dp \]

\[ = \int_p^{p_i} R d \left( T_{\rho_p} - T_{\rho_e} \right) d \ln(p) \]
Tropical Soundings: Virtually no CAPE
Annual Mean Kapingamoronga
Tropical Cyclones: Steady State Physics. Where does the Energy Come From?
Angular momentum per unit mass

\[ M = rV + \Omega \sin(\theta) r^2 \]
Energy Production

Equivalent potential temperature (K), from 334.4955 to 373.3983
Carnot Theorem: Maximum efficiency results from a particular energy cycle:

- Isothermal expansion
- Adiabatic expansion
- Isothermal compression
- Adiabatic compression

Note: Last leg is not adiabatic in hurricane: Air cools radiatively. But since environmental temperature profile is moist adiabatic, the amount of radiative cooling is the same as if air were saturated and descending moist adiabatically.

Maximum rate of working:

\[ W = \frac{T_s - T_o}{T_s} \dot{Q} \]
Total rate of heat input to hurricane:

\[
\dot{Q} = 2\pi \int_0^{r_0} \rho \left[ C_k |V| (k_0^* - k) + C_D |V|^3 \right] r dr
\]

In steady state, Work is used to balance frictional dissipation:

\[
W = 2\pi \int_0^{r_0} \rho \left[ C_D |V|^3 \right] r dr
\]
Plug into Carnot equation:

$$\int_0^{r_0} \rho \left[ C_D \left| V \right|^3 \right] r dr = \frac{T_s - T_o}{T_o} \int_0^{r_0} \rho \left[ C_k \left| V \right| \left( k_0^* - k \right) \right] r dr$$

If integrals dominated by values of integrands near radius of maximum winds,

$$\rightarrow \quad \left| V_{\text{max}} \right|^2 \simeq \frac{C_k}{C_D} \frac{T_s - T_o}{T_o} \left( k_0^* - k \right)$$
Graph of Solution for particular values of coefficients:

Maximum Wind Speed (m/s)

$H = 0.75 \quad C_k/C_D = 1.2$
Numerical simulations
Relationship between potential intensity (PI) and intensity of real tropical cyclones
Wind speed (m/s)

Potential wind speed (m/s)

V (m/s)

Time after maximum intensity (hours)
Evolution with respect to time of maximum intensity

\[(a)\]

- **Atlantic**
- **Pacific**

\[V \text{ (m/s)}\]

\[\text{Time after maximum intensity (hours)}\]

Graph showing the evolution of wind speed over time for Atlantic and Pacific regions post-maximum intensity.
Evolution with respect to time of maximum intensity, normalized by peak wind.
Evolution curve of Atlantic storms whose lifetime maximum intensity is limited by declining potential intensity, but not by landfall.
Evolution curve of WPAC storms whose lifetime maximum intensity is limited by declining potential intensity, but not by landfall.
CDF of normalized lifetime maximum wind speeds of North Atlantic tropical cyclones of tropical storm strength (18 m s\(^{-1}\)) or greater, for those storms whose lifetime maximum intensity was limited by landfall.
CDF of normalized lifetime maximum wind speeds of Northwest Pacific tropical cyclones of tropical storm strength (18 m s$^{-1}$) or greater, for those storms whose lifetime maximum intensity was limited by landfall.
Evolution of Atlantic storms whose lifetime maximum intensity was limited by landfall
Evolution of Pacific storms whose lifetime maximum intensity was limited by landfall
Composite evolution of landfalling storms