Propagation Medium

• Propagation:
  – Signal propagation from satellite to receiver
  – Light-time iteration
  – Basic atmospheric and ionospheric delays
  – Propagation near receiving antenna
Propagation

• Basics:
  – Signal, tagged with time from satellite clock, transmitted.
  – About 60 msec (20,000 km) later the signal arrives at GPS receiver. Satellite has moved about 66 m during the time it takes signal to propagate to receiver.
  – Time the signal is received is given by clock in receiver. Difference between transmit time and receive time is pseudorange.
  – During the propagation, signal passes through the ionosphere (10-100 m of delay, phase advance), and neutral atmosphere (2.3-30 m depending on elevation angle).
Propagaiton

• To determine an accurate position from range data, we need to account for all these propagation effects and time offsets.

• In later lectures, examine ionospheric and atmospheric delays, and effects near antenna.

• Basic clock treatment in GPS
  – True time of reception of signal needed
  – True time of transmission needed (af0, af1 from broadcast ephemeris initially OK)
  – Position of satellite when signal transmitted
Times

• RINEX data files, tag measurements by reception time as given by the receiver clock. The error in the receiver time must be determined iteratively.

• For linearized least squares or Kalman filter need to establish non-linear model and then estimator determines adjustments to parameters of model (e.g. receiver site coordinates) and initial clock error estimates that “best” match the data.
Non-linear model

• Basics of non-linear model:
  – Rinex data file time tags give approximate time measurement was made.
  – Using this time initially, position of satellite can be computed
  – Range computed from receiver and satellite position
  – Difference between observed pseudorange and computed ranges, gives effects of satellite and receiver clock errors. In point positioning, satellite clock error is assumed known and when removed from difference, error in receiver clock determined.
  – With new estimate of receiver clock, process can be iterated.
  – If receiver position poorly known, then whole system can be iterated with updated receiver coordinates.
Sensitivities

• Satellites move at about 1km/sec, therefore an error of 1 msec in time results in 1 m satellite position (and therefore in range estimate and receiver position).

• For pseudo-range positioning, 1 msec errors OK. For phase positioning (1 mm), times needed to 1 μsec.

• (1 μsec is about 300 m of range. Pseudorange accuracy of a few meters in fine).
“Light-time-iteration”

• To compute theoretical range; two basic methods used
  – (a) “Doppler shift corrections” ie. Account for rate of change of range during propagation time
  – (b) “Light-time-iteration” Method most commonly used.

• Light time iteration: Basic process is to compute range using simple Cartesian geometry but with position of receiver at receive time and position of transmitter at transmit time.
Light-time-iteration

• Light time iteration must be computed in a non-rotating frame

• Reason: Consider earth-fixed frame: one would simply compute Earth fixed coordinates at earlier time. In non-rotating frame, rotation to inertial coordinates would be done at two different time (receiver when signal received; transmitted when signal transmitted).

• Difference is rotation of Earth on \(~60\) msec. Rotation rate \(~460\) m/sec; therefore difference is about 30 meters.
Clock errors

PRN 03 (June 14)

Clock errors

Clock SA (ns) 1999
Clock NoSA (ns) 2000

Clock error (ns)

Time (hrs)

04/05/2010 12.540 Lec 14
Relativistic effects

• General relativity affects GPS in three ways
  – Equations of motions of satellite
  – Rates at which clock run
  – Signal propagation

• In our GPS analysis we account for the second two items

• Orbits only integrated for 1-3 days and equation of motion term is considered small
Clock effects

- GPS is controlled by 10.23 MHz oscillators
- On the Earth’s surface these oscillators are set to 10.23x(1-4.4647x10^{-10}) MHz (39,000 ns/day rate difference)
- This offset accounts for the change in potential and average velocity once the satellite is launched.
- The first GPS satellites had a switch to turn this effect on. They were launched with “Newtonian” clocks
Propagation and clock effects

• Our theoretical delay calculations are made in an Earth centered, non-rotating frame using a “light-time” iteration i.e., the satellite position at transmit time is differenced from ground station position at receive time.

• Two corrections are then applied to this calculation
Corrections terms

- Propagation path curvature due to Earth’s potential (a few centimeters)

\[ \Delta \tau = \frac{2GM}{c^3} \ln \left( \frac{R_r + R_s + \rho}{R_r + R_s - \rho} \right) \]

- Clock effects due to changing potential

\[ \Delta \tau = \frac{-\sqrt{GM}}{c^2} e \sqrt{a \sin E} \]

- For \( e = 0.02 \) effect is 47 ns (14 m)
Effects of General Relativity

![Graph showing clock error and GR effect over time.](image)

PRN 03 Detrended; e=0.02
Tests of General Relativity

• After some back of the envelope derivations, in the parameterized post-Newtonian formulation, the time delay expression* becomes:

\[ \Delta \tau = \frac{-2\sqrt{GM}}{c^2} \frac{(2 + \gamma)}{3} e\sqrt{a \sin E} \]

• In PPN, \( \gamma \) is the gravitational curvature term. In general relativity \( \gamma = 1 \). It usually appears as \((1+\gamma)/2\) but this has been modified as explained next.

• The clock estimates from each GPS satellite allow daily estimates of \( \gamma \)
PPN formulation

• The clock rate correction is made of two parts:
  – Special relativity term that depends on $\frac{v^2}{2c^2}$ where $v$ is the satellite velocity and $c$ is the speed of light.
  – General relativity contribution that depends on $\frac{2U}{c^2}$ where $U$ is potential

• The periodic variations arise from the eccentricity of the orbit: At perigee
  – $v$ is fastest which slows clock
  – $U$ is largest which also slows clock (decreasing potential increases the clock rate).

• The contribution to $\Delta \tau$ from the special relativity term is half that of the general relativity term and thus the $\gamma$ formulation.
Using GPS to determine $\gamma$

- Each day we can fit a linear trend and once-per-revolution sin and cos terms to the clock estimates of each of the 28-32 GPS satellites.
- Comparison between the amplitude and phase (relative to sin(E)) allows an estimate of gamma to be obtained.
- Quadrature estimates allows error bound to be assessed (cos(E) term).
- Problems:
  - Once-per-orbit perturbations are common. However should not be proportional to eccentricity.
  - Also possible thermal effects on the clocks in the satellites. (This is probably largest effect as we will see).
  - General quality of clocks in the satellites.
- There are multiple groups that process global GPS data using different processing software and analysis methods (double differencing versus one-way clock estimation).
Example satellite clock (PRN 07)

Results from different analysis groups
Eccentricity is 0.011; $\tau_g \sim 25$ ns.
Differences between groups

- RMS differences are about 0.2% of relativistic effect (measure of "processing" noise).
- Clock variation is ~4%.
- Eccentricity ~0.011: $\Delta \tau$ 25 ns
PRN 28 MIT Analysis

Error bars from RMS fit to clock estimates (linear, sinE and cosE terms only). Red line is fit to annual signal.
PRN 28 GFZ analysis

Results are similar but “annual” is slightly different
Examples of receiver clock behavior

• Examples of satellite and station clock behaviors can be found at:
• [http://geoweb.mit.edu/~tah/MITClk](http://geoweb.mit.edu/~tah/MITClk)
• Directories are by GPS week number and directories ending in W are total clock estimates; folders ending in D are differences between IGS analysis centers
• Now examine some examples
Receiver clocks: ASC1

![Graph showing the trend of ASC1_Clk(m) over days from 14.0 to 15.5 with some data points scattered along the trend.](image-url)
Receiver Clock: HOB2 Hydrogen Maser

![Graph showing the relationship between Day and HOB2_clk_(m) with a downward trend.]
ASC/HOB2 Linear trends removed

Detrended (m)

ASC1 detrended (m)
HOB2 detrended (m)

Day

04/05/2010 12.540 Lec 14
Summary of clocks

- In some cases, clocks are well enough behaved that linear polynomials can be used.
- Most commonly: receiver clocks are estimated at every measurement epoch (white noise clocks) or GPS data is differenced to remove clock (as in question 2 of HW 2).
- Errors in receiver clocks are often thousands of km of equivalent time.
- More detailed analysis of gamma estimates in following slides.
Analysis of result

• Not all satellites look like PRN 28. Only a few satellites are so well behaved.

• The following figures show some examples that show:
  – Change in character when satellite changed (PRN02)
  – Effects of change from Cs to Rb clocks (PRN10)
  – Effects of changing eccentricity of orbit (PRN18) - change from 0.001 to 0.010
  – Period of oscillation: Draconic period: Period of orbital plane relative to sun (precession of node makes this different from 365.25 days). Since all satellites in ~55 deg inclination and precess at the same rate, period is 351.4 days.
Change of satellite

PRN 02 ND 2625 Gamma  0.0001 +/− 0.0010 RMS  0.0531

PRN 02 ND 2625 Quad   0.0001 +/− 0.0013 RMS  0.0649
Change of clock type from Cs to Rb

Change from Cs to Rb
Effects of increasing eccentricity

PRN 18 ND 2906 Gamma  0.0088 +/- 0.0009 RMS  0.0490

PRN 18 ND 2906 Quad  -0.0028 +/- 0.0009 RMS  0.0470
Annual versus Draconic Period

PRN 01 ND 2641 Gamma -0.0056 +/- 0.0082 RMS 0.4223

PRN 01 ND 2641 Gamma -0.0123 +/- 0.0063 RMS 0.3249
12.540 Principles of the Global Positioning System
Spring 2012