12.540 Principles of the Global Positioning System
Lecture 04

Prof. Thomas Herring

http://geoweb.mit.edu/~tah/12.540
Review

• So far we have looked at measuring coordinates with conventional methods and using gravity field

• Today lecture:
  – Examine definitions of coordinates
  – Relationships between geometric coordinates
  – Time systems
  – Start looking at satellite orbits
Coordinate types

• Potential field based coordinates:
  – Astronomical latitude and longitude
  – Orthometric heights (heights measured about an equipotential surface, nominally mean-sea-level (MSL))

• Geometric coordinate systems
  – Cartesian XYZ
  – Geodetic latitude, longitude and height
Astronomical coordinates

• Astronomical coordinates give the direction of the normal to the equipotential surface

• Measurements:
  – Latitude: Elevation angle to North Pole (center of star rotation field)
  – Longitude: Time difference between event at Greenwich and locally
Astronomical Latitude

• Normal to equipotential defined by local gravity vector
• Direction to North pole defined by position of rotation axis. However rotation axis moves with respect to crust of Earth!
• Motion monitored by International Earth Rotation Service IERS [http://www.iers.org/](http://www.iers.org/)
Astronomical Latitude

\[ \phi_a = Z_d - \delta \]

- Rotation Axis
- Zenith distance = 90-elevation
- Geiod
- \( \delta \) = declination
- To Celestial body
Astronomical Latitude

• By measuring the zenith distance when star is at minimum, yields latitude

• Problems:
  – Rotation axis moves in space, precession nutation. Given by International Astronomical Union (IAU) precession nutation theory
  – Rotation moves relative to crust
Rotation axis movement

- Precession Nutation computed from Fourier Series of motions
- Largest term 9” with 18.6 year period
- Over 900 terms in series currently (see http://geoweb.mit.edu/~tah/mhb2000/JB000165_online.pdf)
- Declinations of stars given in catalogs
- Some almanacs give positions of “date” meaning precession accounted for
Rotation axis movement

- Movement with respect to crust called “polar motion”. Largest terms are Chandler wobble (natural resonance period of ellipsoidal body) and annual term due to weather.

- Non-predictable: Must be measured and monitored.
Evolution (IERS C01)

Pole Position (arcsec)

1" = 31 m

Year

1860 1880 1900 1920 1940 1960 1980 2000

PMX
PMY - 0.5"

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Evolution of uncertainty

![Graph showing the evolution of uncertainty with time, with a scale of 1" = 31 m. The graph plots Sigma Pole Position in arc sec against Year, from 1860 to 2000. The data is represented with thin black lines, and the trend shows fluctuations over time. The graph includes a legend indicating Sig X and Sig Y.]
Recent Uncertainties (IERS C01)

Sigma Pole Position (arc sec)

1"=31 m

Sig X

Sig Y

Year

Astronomical Longitude

• Based on time difference between event in Greenwich and local occurrence
• Greenwich sidereal time (GST) gives time relative to fixed stars

\[ GST = 1.0027379093\,UT1 + \vartheta_0 + \Delta \psi \cos \varepsilon \]

\[ \vartheta_0 = 24110.54841 + 8640184.812866 \frac{T}{\text{Julian Centuries}} + 0.093104T^2 - 6.2 \times 10^{-6}T^3 \]
Universal Time

• UT1: Time given by rotation of Earth. Noon is “mean” sun crossing meridian at Greenwich
• UTC: UT Coordinated. Atomic time but with leap seconds to keep aligned with UT1
• UT1-UTC must be measured
Length of day (LOD)

LOD = Difference of day from 86400. seconds
Recent LOD

LOD (ms)

Year

LOD (ms)


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LOD compared to Atmospheric Angular Momentum
LOD to UT1

- Integral of LOD is UT1 (or visa-versa)
- If average LOD is 2 ms, then 1 second difference between UT1 and atomic time develops in 500 days
- Leap second added to UTC at those times.
UT1-UTC

- Jumps are leap seconds, longest gap 1999-2006. Historically had occurred at 12-18 month intervals.
- Prior to 1970, UTC rate was changed to match UT1.
Transformation from Inertial Space to Terrestrial Frame

• To account for the variations in Earth rotation parameters, as standard matrix rotation is made

\[ x_i = P N S W x_t \]

Inertial: Precession, Nutation, Spin, Polar Motion, Terrestrial
Geodetic coordinates

• Easiest global system is Cartesian XYZ but not common outside scientific use
• Conversion to geodetic Lat, Long and Height

\[ X = (N + h) \cos \phi \cos \lambda \]
\[ Y = (N + h) \cos \phi \sin \lambda \]
\[ Z = \left( \frac{b^2}{a^2} N + h \right) \sin \phi \]
\[ N = \frac{a^2}{\sqrt{a^2 \cos^2 \phi + b^2 \sin^2 \phi}} \]
Geodetic coordinates

- WGS84 Ellipsoid:
  - $a=6378137$ m, $b=6356752.314$ m
  - $f=1/298.2572221$ ($=[a-b]/a$)

- The inverse problem is usually solved iteratively, checking the convergence of the height with each iteration.

- (See Chapters 3 & 10, Hofmann-Wellenhof)
Heights

- Conventionally heights are measured above an equipotential surface corresponding approximately to mean sea level (MSL) called the geoid.
- Ellipsoidal heights (from GPS XYZ) are measured above the ellipsoid.
- The difference is called the geoid height.
Geiod Heights

- National geodetic survey maintains a web site that allows geiod heights to be computed (based on US grid)
  - [http://www.ngs.noaa.gov/cgi-bin/GEOID_STUFF/geoid99_prompt1.prl](http://www.ngs.noaa.gov/cgi-bin/GEOID_STUFF/geoid99_prompt1.prl)
- New Boston geiod height is -27.688 m
NGS GEIOD09


N = 8574241
Mean = -29.91
SD = 9.93

Min = -50.68
Max = 3.44
Spherical Trigonometry

- Computations on a sphere are done with spherical trigonometry. Only two rules are really needed: Sine and cosine rules.
- Lots of web pages on this topic (plus software)
- [http://mathworld.wolfram.com/SphericalTrigonometry.html](http://mathworld.wolfram.com/SphericalTrigonometry.html) is a good explanatory site
Basic Formulas

A B C are angles
a b c are sides
(all quantities are angles)

Sine Rule
\[ \frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \]

Cosine Rule sides
\[
\begin{align*}
\cos a &= \cos b \cos c + \sin b \sin c \cos A \\
\cos b &= \cos c \cos a + \sin c \sin a \cos B \\
\cos c &= \cos a \cos b + \sin a \sin b \cos C
\end{align*}
\]

Cosine Rule angles
\[
\begin{align*}
\cos A &= -\cos B \cos C + \sin B \sin C \cos a \\
\cos B &= -\cos A \cos C + \sin A \sin C \cos b \\
\cos C &= -\cos A \cos B + \sin A \sin B \cos c
\end{align*}
\]
Basic applications

• If $b$ and $c$ are co-latitudes, $A$ is longitude difference, $a$ is arc length between points (multiply angle in radians by radius to get distance), $B$ and $C$ are azimuths (bearings)

• If $b$ is co-latitude and $c$ is co-latitude of vector to satellite, then $a$ is zenith distance (90-elevation of satellite) and $B$ is azimuth to satellite

• (Colatitudes and longitudes computed from $\Delta XYZ$ by simple trigonometry)
Summary of Coordinates

• While strictly these days we could realize coordinates by center of mass and moments of inertia, systems are realized by alignment with previous systems
• Both center of mass (1-2cm) and moments of inertia (10 m) change relative to figure
• Center of mass is used based on satellite systems
• When comparing to previous systems be cautious of potential field, frame origin and orientation, and ellipsoid being used.
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