Lectures 19-20: Savings and Technology

- Review

- Cont.: Change in Saving rate

- Technological progress
Solow’s Growth Model

\[ A = 1, \ N = 1 \]

\[ Y = y = f(k) \]

\[ S = sY \]

\[ I = S \]

\[ K(t+1) = (1-d) \ K(t) + I(t) \]

\[ \Rightarrow \]

\[ k(t+1) - k(t) = s \ f(k(t)) - d \ k(t) \]
Steady State and the Saving Rate

In steady state: \( k(t+1) = k(t) = k^* \)

\[
k(t+1) - k(t) = s \, f(k(t)) - d \, k(t)
\]

\[
=>
\]

\[
sf(k^*) = d \, k^*
\]

\[
g_y^* = 0 \quad (\text{if } n > 0, \ g_y^* = 0 \Rightarrow g_Y = g_K = n > 0)
\]

In steady state, the saving rate does NOT matter for per-capita growth.

It does matter, however, for the level of per-capita output and transitional dynamics

Figures 11-3, 11-4
Some numbers

• $Y = (KN)^{0.5} \implies y = (K/N)^{0.5} = k^{0.5}$

• $k(t+1) - k(t) = s \cdot k(t)^{0.5} - dk(t)$

• St.St: $k^* = (s/d)^2 ; \quad y^* = (s/d)$

• $s_0 = d = 0.1; \quad s_1 = 0.2 \implies$

• $k^*$ goes from 1 to 4 and $y^*$ from 1 to 2.

• Higher saving $\implies$ need to maintain more capital

• $c^* = y^* - dk^*$

• The Golden Rule: Table 11-1
Dynamics

• Dynamics: \( k(1) = 1 + 0.2 - 0.1 = 1.1 > 1 \)
• … and so on
• Figure 11-7
Technological Progress

• Table 12-2
• \( Y = F(K,N,A) \) ..... \( Y = F(K,NA) \)
• \( y = \frac{Y}{NA} = F(K/NA,1) = f(K/NA) = f(k) \)
• \( \frac{I}{AN} = s \frac{Y}{AN} \)
• In order to maintain a given \( k \), we need to invest at least:
  \[
  (d + g_A + g_N) K
  \]
Technological Progress

\[ \frac{I}{AN} > (d + g_A + g_N) \left( \frac{K}{AN} \right) \]

\[ \Rightarrow k \text{ grows} \]

Figure 12-2
Table 12-1
Figure 12-3 / 12-4
A Decline in $g_A$

- Table 12-2
- Table 12-1
- (use) Figure 12-2
- Why? (we don’t know…)
  - Measurement error?
  - The rise of the Service Sector?
    - Figure 12-5
  - Decreased R&D Expenditure?
    - Table 12-3
### The New Economy and Productivity Growth

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<td>Private Non-Farm Business</td>
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<td>Labor productivity</td>
<td>2.9</td>
<td>1.2</td>
<td>1.4</td>
<td>1.6</td>
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<tr>
<td>Multifactor productivity</td>
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<td>0.4</td>
<td>0.3</td>
<td>0.6</td>
<td>1.1</td>
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<tr>
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<td>-0.6</td>
<td>1.1</td>
<td>1.3</td>
<td>2.1</td>
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<tr>
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<td>0.2</td>
<td>3.2</td>
<td>3.1</td>
<td>5.8</td>
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<td>Electronic Mach.</td>
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<td>1.0</td>
<td>3.0</td>
<td>6.0</td>
<td>7.4</td>
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</tbody>
</table>

Source: BLS.
Investment Has Increased

Figure by MIT OCW. After source: BEA; Datastream; St. Louis Federal Reserve.
The Price of New Capital

Real Interest Rates
(Constant maturity nominal rates minus Un. of Michigan inflation expectations)

Change in Relative Price of Equipment
(1996 Relative price normalized to one; log changes)

Figure by MIT OCW. After source: BEA; Datastream; St. Louis Federal Reserve.