Bank Run Model (40 points)

This exercise leads you to go through the derivation of the bank run model.

There are 3 periods. At $t = 0$, all agents start with 1 unit of endowment and can invest in 2 different technologies:

1. A short term technology that delivers 1 unit of output at $t = 1$ for each unit of output invested at $t = 0$.

2. A long term technology that delivers $R > 1$ units of output at $t = 2$ for each unit of output invested at $t = 0$. However, if this technology is liquidated at $t = 1$ it delivers $L < 1$ units of output.

There are two types of agents (patient and impatient) differing in their consumption preference. Patient agents are indifferent between consuming at $t = 1$ or at $t = 2$. Impatient agents only consume at $t = 1$.

To simplify the setting, we further assume – differently from the DD model – that it is impossible to store goods between $t = 1$ and $t = 2$. Hence, if a patient agent receives some goods in $t = 1$, she must consume them readily in $t = 1$.

a). Consider a simple case: if agents know their type before investing, what is the optimal investment strategy for patient and impatient types? (5 points)

b). Let $\alpha$ be the amount of endowment invested in the long-term technology. What is the consumption at $t = 1$ and $t = 2$ for patient and impatient types? (5 points)

c). If the goal is to maximize expected consumption, what is the optimal value for $\alpha$? (5 points)

d). Suppose agents can trade bond contract at $t = 1$ (see lecture notes). The price of the bond is $p$. If trade occurs, it must be $\frac{L}{R} \leq p \leq 1$, why? (7 points)

e). Calculate the equilibrium $p$ and $\alpha$ that maximize expected consumption. Show that expected consumption in the trading equilibrium is higher than the autarky case. (10 points)
f). If agents cannot trade, but all agents delegate bank to make investment. Show that the bank can achieve the trading equilibrium. (8 points)

g). (Bonus) If the bank’s goal is to maximize agent’s expected consumption, what is the optimal $\alpha$? Why this is not achievable in a pure trading economy. (10 points)

1 IS-LM Model (30 points)

a). Suppose that the economy has no money market and that $I, G, T$ are given. Let $C = c_0 + c_1(Y - T)$. Derive equilibrium output and the government spending multiplier. (8 points)

b). Assume now that investment is $I = b_0 + b_1Y - b_2i$. Suppose that the interest rate is exogenously fixed. Derive equilibrium output and the government spending multiplier. How does this spending multiplier compare to the one you found in part a)? (8 points)

c). Introduce a money market in which $MP = d_1Y - d_2i$. Derive equilibrium output. Derive the government multiplier. Is it bigger or smaller that the one you computed in part b)? Provide some intuition for this result. (14 points)

2 The Amplification Mechanism (30 points)

To answer this question, you need to read page 74 – 83 of the lecture notes. Assume $f'(k) < \alpha$, namely, the marginal return facing landlords is lower than the marginal return facing entrepreneurs when all capital is invested by landlords.

a). Consider the benchmark case with no frictions, so entrepreneurs do not face collateral constraint (the equation on page 77). What is the equilibrium price? (10 points)

b). Consider the general case with collateral constraint. If entrepreneur’s personal wealth is $w_t$ at time $t$, what is the maximum amount of capital he can purchase? (10 points)

c). What happens to capital invested by the entrepreneur if $w$ is reduced by $\Delta w$? (10 points)