Product Differentiation

Linear city (Hotelling, 1929)

- two firms selling (physically) identical objects
- customers are distributed uniformly on a unit interval, with one firm located at 0 and the other at 1
  - e.g., imagine a beach with two ice cream vendors, with one located on each end
  - or pedestrians walking on Newbury St between two similar stores
- both firms have \( mc = c \)
- consumers incur a cost \( t \) to travel a unit distance, so a consumer at \( x \) incurs \( xt \) to go to firm 0 and \( (1-x)t \) to go to firm 1
- consumers have unit demands and enjoy utility \( V - xt - P_0 \) if they are located at \( x \) and purchase one unit from firm 0
- firms set prices
- assume firm prices are not different enough so that one firm sells nothing
- also assume that prices are not so high relative to \( V \) so that some consumers buy nothing
- what are \( P_0, P_1 \)?

- a consumer indifferent to firm 0 & firm 1 is located at \( x \) where \( x \) is given by
\[ P_0 + tx = P_1 + t(1 - x) \]
\[ tx = P_1 - P_0 + t - tx \]
\[ x = \frac{P_1 - P_0 - t}{2t} \Rightarrow \text{firm 0's demand} \]
likewise \( (1 - x) = \frac{P_0 - P_1 + t}{2t} \Rightarrow \text{firm 1's demand} \)
\[ \Pi^i(P_0, P_1) = \frac{1}{2t}(P_i - c)(P_j - P_i + t) \]
\[ \Pi = \frac{1}{2t}(P_iP_j - P_i^2 + P_i t - cP_j + cP_i - ct) \]
\[ \frac{\partial \Pi}{\partial P_i} = \frac{1}{2t}(P_j + c + t - 2P_i) \]
first-order condition: \( P_j + c + t - 2P_i = 0 \)
so \( P_0^* = \frac{P_1^* + c + t}{2} \) \( P_0^* = P_1^* = c + t \)
\( P_1^* = \frac{P_0^* + c + t}{2} \) \( \Rightarrow \) and \( \Pi_0^* = \Pi_1^* = \frac{t}{2} \)

Notes:
- physically identical goods, firms competing in price, but they price above \( mc \) and make positive \( \Pi \) because of difference in location
- allows firms to escape the Bertrand Paradox
- differences in physical location are a metaphor for any type of product differentiation
- markup is a function of "transportation cost", or how hard it is for a consumer to move away from preferred location in product space
- as \( t \) increases, competition becomes less intense, as consumers close to each firm become more captive, giving the firm increased market (or monopoly) power.
- as \( t \) decreases, competition intensifies and approaches Bertrand as \( t \) goes to 0

Discussion
- Are transportation costs always exogenous?
- Is there anything firms can do to increase them?
  - change physical characteristics of product to differentiate from rivals' products
Entry

Circular city (Salop, 1979)

- firms selling (physically) identical objects
- consumers distributed uniformly on a circle with circumference = 1 and they can only travel on the circle

- think about gas stations on the beltway around Indianapolis
- this is a stylized model with some odd assumptions, but it is designed to let us focus on the issue of entry and resulting competition

- firms have $mc = c$ and fixed cost of entry $f$
- consumers incur travel costs $t$ to travel unit distance (around circle)
- consumers have unit demands and enjoy utility $V - xt - P_0$ if they are located at $x$ and purchase from firm 0
- two-stage game:
  - potential entrants sequentially choose whether to enter – let $n$ denote the number who do enter
  - and they are exogenously spaced equidistantly
  - firm chooses prices

- solve backwards
  - $n$ firms enter, where $n$ is large enough so that the market is covered
    * we need this assumption to ensure we don’t have a bunch of monopolists
  - since they are equidistance, consider equilibria $\exists$ all firms charge equal prices
  - firm $i$ chooses $P_i$

consumer located at $x \in (0, \frac{1}{n})$ from firm $i$ is indifferent between firms $i$ and $(i-1)$ if:

$$P_i + tx = P + t(\frac{1}{n} - x)$$
then firm $i$ faces demand:

$$D_i(P, P) = \frac{2x}{t} = \frac{P + \frac{t}{n} - P_i}{t}$$

so firm $i$ maximizes:

$$\max P_i : (P_i - c)(P + \frac{t}{n} - P_i) - f$$

after differentiating wrt $P_i$, setting $P_i = P$, and setting $= 0$, we get:

$$P = c + \frac{t}{n} \text{ (where } \frac{t}{n} \text{ is the margin above marginal cost)}$$

* this is similar to linear city result, but here $P$ is also a function of $n$

- what is $n$?

firms correctly anticipate what competition in the second stage is going to look like, so firms enter until they foresee that the margin they can charge above cost will not cover their fixed costs.

because the circumference is 1 and each firm gets $\frac{1}{n}$ of that:

$$(P - c)\frac{1}{n} - f = 0$$

plugging in for $P$, we get:

$$\frac{t}{n^2} - f = 0$$

so $n^* = \sqrt{\frac{t}{f}}$

and $P^* = c + \sqrt{tf}$

Notes:

- Hotelling gave us intuition that markups depend on differentiation, and Salop tells us that supernormal profits get dissipated through entry. In the real world, both effects are present and important.
- entry is countervailing force to product differentiation leading to higher markups.
- firms do price above marginal cost but only enough to cover their fixed costs (up to integer constraint).
- increase in $f \Rightarrow$ decrease in $n$, increase in margins.
- increase in $t \Rightarrow$ increase in margins, but also increase in $n$ so firms lose some of their market.

Discussion

- what if firms have different $f$?
- can firms do anything to affect $f$ or $t$ for other firms?
- what if firms can be located at more than one place?
14.27 Economics and E-Commerce
Fall 2014

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