14.382 MIDTERM 2006

Answer as if your try to explain the material to your fellow student.

Consider the model, where \( Y = X\beta + \epsilon \), where for each \( t \), \( \epsilon_t \sim \sigma(e_t - 1) \), where \( \epsilon_t \) is standard exponential variable such that \( E[\epsilon_t] = 1 \) and \( Var[\epsilon_t] = 1 \). Assume that \( X \) are independent of \( \epsilon \). Suppose that \( (x_t, \epsilon_t) \) are i.i.d. across \( t \).

1. (10) Do Gauss-Markov assumptions hold for this model?
2. (10) Consider the least squares estimator \( \hat{\beta} \). Compute \( E[\hat{\beta}|X] \) and \( Var[\hat{\beta}|X] \). Is \( \hat{\beta} \) normally distributed in finite samples, conditional on \( X \)?
3. (10) Carefully, but briefly, explain the label "BLUE". Is OLS BLUE in this set-up?
4. (10) Consider estimating the following effect
   \[ E[y_t|x_t = x''] - E[y_t|x_t = x'] = (x'' - x')'\beta \]
   Give an economic example where such an effect might be of interest. Is \( (x'' - x')'\hat{\beta} \) BLUE for this effect? Why or why not?
5. (10) Is OLS the BUE (best unbiased estimator) in this model? A brief answer suffices.
6. (15) What is the large sample distribution of \( \hat{\beta} \)? Make any additional primitive assumptions you might need. [Note: high level assumptions will receive partial credit.]
7. (10) Construct a consistent estimator for the large sample variance of \( \hat{\beta} \). Prove its consistency by making any additional assumptions you need.
8. (10) Suppose we want to test the null hypothesis \( H_0 : \beta_j = 0 \) vs \( H_A : \beta_j < 0 \). Construct a t-statistic for testing this hypothesis. Derive its limit distribution and describe how to select critical value for this test to maintain the level of significance equal to 5%.
9. (15) Suppose the sample size \( n = 6 \). Do you expect the large sample distribution to be a good approximation to the exact distribution of the t-statistic in question 8. Discuss how to get the exact distribution of the t-statistic. How would you generate p-values (or critical values) for checking the hypothesis of question 8 that would be valid even for \( n = 6 \)?
10. (Extra Points) Can you come up with better estimators than OLS for this model? Hint: think about the trivial case first, where \( X_t = 1 \).