Demand Shocks with Dispersed Information

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Nominal rigidities: imperfect information

- How to model demand shocks in a baseline environment with imperfect info?
- Need consumer’s decisions to be richer:
  - Forward looking
  - No fully revealing prices

1. Embed in something closer to neo-keynesian benchmark
2. Add shocks to expected productivity

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Ingredients

Model of “fundamental” and “sentiment” shocks

- Fundamental information is dispersed across the economy
- Agents know “potential output” in their own sector, but not the aggregate
- Demand shocks: shifts in average beliefs about aggregate potential output
Households: consumer/producer on $[0, 1]$.

Preferences:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left( \log C_{it} - \frac{1}{1+\eta} N_{it}^{1+\eta} \right)$$

$$C_{it} = \left( \int_{J_{it}} C_{ijt}^{\frac{\sigma-1}{\sigma}} \, dj \right)^{\frac{\sigma}{\sigma-1}}$$

random consumption basket: $J_{it} \subset [0, 1]$

Technology:

$$Y_{it} = A_{it} N_{it}$$
Individual productivity (private signal) is

\[ a_{it} = \log A_{it} = a_{t-1} + \theta_{it} \]

aggregate component and idiosyncratic component

\[ \theta_{it} = \theta_t + \varepsilon_{it} \]

Aggregate productivity is

\[ a_t = a_{t-1} + \theta_t \]
Public signal about aggregate innovation

\[ s_t = \theta_t + e_t \]

- news
- aggregate statistics
- stock market
- ...

\[ \theta_t = \text{fundamental shock} \]
\[ e_t = \text{sentiment shock} \]
Agents have nominal balances $B_{it-1}$ with CB (\textit{cashless economy})

- Before observing current shocks: state contingent contracts
- CB sets nominal interest rate on balances $R_t$
- Producer set price $P_{it}$
- Consumer observes prices in consumption basket $P_{jt}$ for $j \in J_{it}$
- Consumer buys goods
- All shocks publicly revealed, state contingent contracts settled
Budget constraint

\[ B_{it} = R_t \left( B_{it-1} + (1 + \tau) P_{it} Y_{it} - \bar{P}_{it} C_{it} + Z_{it} (h_t) - T_t \right) \]
\[ - \int q_t (\tilde{h}_t) Z_{it} (\tilde{h}_t) d\tilde{h}_t. \]

- \( \bar{P}_{it} \) price index for goods in \( J_{it} \)
- \( Z \) state contingent contracts
- subsidy \( \tau \) to correct for monopolistic distortion
- \( T_t \) lump sum tax to finance subsidy

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Random consumption baskets

Figure 1:

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Random consumption baskets

Figure 3:

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Random consumption baskets

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Random consumption baskets (continued)

\[
\bar{\theta}_{it} = \{ \theta_{jt} : j \in J_{it} \}
\]

additional idiosyncratic shock: sampling shock \( v_{it} \)

\[
\bar{\theta}_{it} = \theta_t + v_{it}
\]
Monetary policy rule

Interest rate rule

\[ r_t = r + \xi (p_{t-1} - p_{t-1}^*) \]

Price target

\[ p_t^* = \phi_\theta \theta_t + \phi_s s_t \]

- no superior information
- only trying to keep nominal prices stable
- \( \xi > 1 \) 'active' rule
- all lowercase = logs
Linear equilibrium

Individual prices and consumption

\[ p_{it} = \phi_0 + \phi_\theta \theta_{it} + \phi_s s_t \]
\[ c_{it} = \psi_0 + a_{t-1} + \psi_\varepsilon \theta_{it} + \psi_v \bar{\theta}_{it} + \psi_s s_t \]

- in equilibrium \( p_t = p^*_t \)
- interest rate constant

Proposition

*Linear equilibrium exists under given policy rule, determinate if \( \xi > 1 \)*
Linear equilibrium (continued)

Potential output

\[ c_t^* = \psi_0^* + a_{t-1} + \theta_t \]

- aggregate output under first best allocation
- = aggregate output under full information (with right \( \tau \))
- = linear equilibrium iff

\[ \psi_\theta = 1 \quad \psi_s = 0 \]
Linear equilibrium (continued)

Mechanics and remark 1

- full insurance + normal sampling shocks + iso-elastic preferences
  ⇒ closed form linear equilibrium

- e.g.: the price index for consumer \( i \) is

\[
\bar{P}_{it} = V_p \exp \{\rho t + \phi \theta v_i\}
\]

where

\[
V_p = \exp\left\{\frac{1 - \sigma}{2} \phi \theta^2 \hat{\sigma}^2\right\}
\]
Linear equilibrium (continued)

Mechanics and remark 2

- consumers observe whole distribution $P_{jt}$ for $j \in J_{it}$

- a sufficient statistic is $\bar{\theta}_{it}$

- this is like having two noisy signals of $\theta_t$:

$$\theta_{it} = \theta_t + \varepsilon_{it}$$
$$\bar{\theta}_{it} = \theta_t + \nu_{it}$$
Linear equilibrium (continued)

Mechanics and remark 2

• consumers observe whole distribution $P_{jt}$ for $j \in J_{it}$

• a sufficient statistic is $\overline{\theta}_{it}$

• this is like having two noisy signals of $\theta_t$:

$$\theta_{it} = \theta_t + \varepsilon_{it}$$

$$\overline{\theta}_{it} = \theta_t + \nu_{it}$$

• information structure is independent of monetary policy
**Pricing**

Optimality condition

\[ p_{it} = \eta \left( \mathbb{E}_{it} \left[ c_t + \sigma (p_t - p_{it}) \right] - a_{it} \right) + \]
\[ + \left( \mathbb{E}_{it} \left[ \bar{p}_{it} + c_{it} \right] - a_{it} \right) + \eta \left( \psi_v + \sigma \phi_\theta \right) \mathbb{E}_{it} \left[ \nu_{jt} \right] \]

- \( \mathbb{E}_{it} \) expectation at pricing stage
- high demand relative to production → high price
- high consumption relative to production → high price

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Consumption

Euler equation

\[ c_{it} = E^\prime_{it} \left[ a_{t+1} - (r - p_{t+1} + p_i) \right] \]

- \( E^\prime_{it} \): expectation at consumption stage
Demand shocks

Properties of monetary regime

- $E_t[p_{it+1}] = 0$
- stable price level in expectation
- equilibrium $r_t$ constant

Simple case

$$\frac{\sigma_e}{\sigma_\theta} \to \infty$$

agents disregard their private info

$$E_t^P[.] = E[.|a_{t-1}, s_t]$$

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Effects of $e_t$ and $\theta_t$

$$p_t = \frac{1 + \eta}{1 + \sigma \eta} (E_t^P [a_t] - a_t)$$

$$y_t = \lambda E_t^P [a_t] + (1 - \lambda) a_t$$

Effects of $e_t > 0$

- only temporary effects
- raise $c_t$, $p_t$ and $n_t$

Effects of $\theta_t > 0$

- permanent effects
- raise $c_t$
- lower $p_t$ and $n_t$

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Dynamic responses to $u_t$ (solid) and $e_t$ (dash).

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What restrictions does the theory impose?

- evidence on 'signals' gives testable implications
- evidence on aggregate beliefs
- basic restrictions on joint behavior of error and actual series

\[ y_t = \lambda E_t^P[a_t] + (1 - \lambda) a_t \]

fraction of variance of \( y_t \) due to demand shocks over total variance is **bounded**
\[ \frac{\sigma_e^2}{\sigma_u^2} = 0.2 \]

\[ \frac{\sigma_e^2}{\sigma_u^2} = 1.0 \]

\[ \frac{\sigma_e^2}{\sigma_u^2} = 5 \]

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Changing $\sigma_e$

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A richer policy rule

Interest rate rule

\[ r_t = r + \xi (p_{t-1} - p_{t-1}^*) \]

Price target

\[ p_t^* = \mu a_t + \phi \theta_t + \phi s s_t \]

- use past information
- \( p_t \) aggregate price index
- note the term \( \mu a_{t-1} \) inertial rule
Monetary Policy (continued)

Consumption under $\mu \neq 0$

Euler equation

$$c_{it} = \mathbb{E}_{it}^I \left[ a_{t+1} - \left( r_t - p_{t+1} + \bar{p}_{it} \right) \right]$$

where $c_{it}$ is the consumption, $r_t$ is the interest rate, $p_{t+1}$ is the expected future price, and $\bar{p}_{it}$ is the inflation rate.
A richer policy rule

Interest rate rule

\[ r_t = r + \xi (p_{t-1} - p_{t-1}^*) \]

Price target

\[ p_t^* = \mu a_{t-1} + \phi \theta_t + \phi_s s_t \]

- use past information
- \( p_t \) aggregate price index
- note the term \( \mu a_{t-1} \) inertial rule
Power of policy rule

Agents have different expectations about future output

...but also different expectations about real interest rate

$$E^H_{it}[r - \mu_\theta \theta_t + \bar{p}_{it}]$$

2 crucial ingredients:

- agents forward looking
  - in the future more information than now

→ policy rule allows to ’manage expectations’
Power of policy rule (continued)

The choice of $\mu_\theta$ feeds back into optimal prices $\bar{p}_{it}$

It also affects response to $s_t$ and response of relative prices

An increase in $\mu_\theta$

- increases $\psi_\theta$
- reduces $\phi_\theta$
- increases $\phi_s$
- decreases $\psi_s$
Achievable linear equilibria

vector $\psi_\theta, \phi_\theta, \phi_s, \psi_s$

s.t.

$$
\psi_v = \psi_\varepsilon \delta_v / \delta_\varepsilon - \phi_\theta \\
(1 + \sigma \eta) \phi_\theta = \eta ((\psi_\theta + \sigma \phi_\theta) \beta_\theta - 1) + ((\psi_\theta + \phi_\theta) \beta_\theta / \delta_\theta - 1) + \\
+ \eta (\psi_v + \sigma \phi_\theta) \gamma (1 - \beta_\theta), \\
0 = \eta (\psi_\theta + \sigma \phi_\theta) \beta_s + (1 + \eta) \psi_s + \\
+ (\psi_\theta + \phi_\theta) (\beta_s - \delta_s) / \delta_\theta - \eta (\psi_v + \sigma \phi_\theta) \gamma \beta_s,
$$
Another divine coincidence?

Proposition

There is a $\mu^f_s$ that achieves full stabilization:

$$\psi_\theta = 1 \quad \psi_s = 0$$

- here output is always equal to potential
- induce agents to respond more to private productivity
4.4 Full stabilization

As noticed above, monetary policy can affect equilibrium behavior, but its scope is limited by the fact that prices and consumption decisions are individually optimal. The following proposition shows that, in spite of that, monetary policy can achieve full aggregate stabilization, that is, a path for aggregate output, $y_t$, which is identical to potential output, $y^*_t$.

**Proposition 5**

There exist a subsidy, $\tau_{fs}$, and a monetary policy rule, $\mu_{fs}$, that, jointly, achieve full aggregate stabilization, $y_t = y^*_t$. This corresponds to a linear equilibrium with parameters: $\psi_s = 0$, $\psi_\theta = 1$.

To illustrate this result, Figure 1 shows the equilibrium relation between $\mu_\theta$ and the parameters $(\psi_\theta, \psi_s)$. As discussed above, a positive value for $\mu_\theta$ increases the response of the economy to the fundamental shock. However, due to the equilibrium response of optimal prices, an increase in $\mu_\theta$ also reduces the response of the economy to the public signal. The surprising result is that when $\mu_\theta = \mu_{fs}$ we reach an equilibrium where, at the same time, $\psi_\theta$ reaches 1 and $\psi_s$ reaches 0.
More on the relation between $\psi_\theta$ and $\phi_\theta$

- increase response of output to fundamental
- increase response of demand to local productivity
- reduce price adjustment ($\phi_\theta < 0$)
Welfare

4 components:

\[-(1 + \eta) \mathbb{E} \left[ (c_t - c^*_t)^2 \mid a_{t-1} \right] - (1 + \eta) \operatorname{Var} (n_{it}) +
- \operatorname{Var} (c_{jt} + \sigma \bar{p}_{jt} \mid j \in \tilde{J}_{it}) + \sigma (\sigma - 1) \operatorname{Var} (p_{jt} \mid j \in J_{it})\]

1. aggregate output gap (-)
2. labor supply cross sectional dispersion (-)
3. demand cross sectional dispersion (-)
4. relative price dispersion (+)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>7</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma^2_\theta$</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table: Parameters for the example
of prices observed by a given consumer, in particular the variance of the (log) prices in the basket of consumer $i$ is $(1 - \gamma)$ times the aggregate cross-sectional price variance. A value of 0.5 is chosen as a conventional middle point.

Table 1: Parameters for the benchmark example

To illustrate the properties of optimal monetary policy, Figure 2 plots the four elements in (12) for this example. The first panel depicts $w_1$, capturing the gains associated with the stabilization of aggregate output. The maximum for this curve is reached at $\mu_{fs}$, that is, at the point where aggregate output is equal to potential. In accordance with Proposition 5, $w_1$ is zero at $\mu_{fs}$.

The effects on the remaining three components can be interpreted considering the effects of monetary policy on equilibrium price dispersion. An increase in $\mu_{\theta}$, starting from the simple price-targeting rule, $\mu_{\theta} = 0$, has the effect of reducing the dispersion of relative prices by reducing the value of $|\phi_{\theta}|$, which determines the response of individual prices to individual productivity shock. This reduction in price dispersion can be interpreted as follows. A household facing a positive productivity shock, $\theta_i > 0$,...

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Figure 3: Optimal monetary policy increases its consumption more if $\mu_\theta > 0$. As a consequence the incentive to increase labor effort is dulled, and, at the price-setting stage, household $i$ has a smaller incentive to reduce prices. Therefore, relative prices respond less. This reduction in price dispersion has several effects. Given the assumptions made about parameters it tends to reduce the dispersion in demand, although there is a countervailing effect due to the fact that consumption is more responsive to the idiosyncratic shock. Moreover, it tends to reduce the dispersion in labor supply. Both effects have positive welfare effects. This can be seen in the second panel of Figure 2, where both curves $w_2$ and $w_3$ are increasing at $\mu_\theta = 0$. Finally, from the point of view of the consumer, the reduction in price dispersion decreases welfare, as I argued above. This effect is illustrated in the last panel curve of Figure 2.

In the example considered, the first component and the last component are the relevant ones from a quantitative point of view. The optimal monetary policy $\mu_\theta^*$ balances the welfare costs of reducing price dispersion, against the welfare gains of increasing aggregate volatility. As a consequence, $\mu_\theta^*$ is smaller than $\mu_{fs}$. In this case, it is optimal to reduce the aggregate volatility due to demand disturbances, but it is not optimal to completely eliminate this source of volatility. This is illustrated in Figure 3 that plots total welfare and shows that $\mu_\theta^* < \mu_{fs}$. Note that at the optimal monetary policy the corresponding decreasing if the following inequality holds:

$$\beta_\theta \delta_\theta + \eta \beta_\theta > \eta \gamma (1 - \beta_\theta) \mu_1 - \delta_\theta \delta_\theta.$$

Both conditions are satisfied in this example, in a neighborhood of $\mu_\theta = 0$.16

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Optimal monetary policy

Proposition

*Full stabilization is typically not optimal*

*Some accommodation of demand shocks is optimal*

- It is optimal $\mu^* < \mu^{fs}$
- It is optimal to partially accommodate $\psi_s > 0$
- Price dispersion is larger at optimal monetary policy than under full stabilization
Special case

$$\eta = 0$$

- now it is optimal $$\psi_\theta = 1$$

- $$\phi_\theta = -1$$

- decreasing prices proportionally to productivity gives:
  1. right relative prices
  2. right response of consumption
Special case (continued)

\[ p_{it} = \left( E_i \left[ \bar{p}_{it} + c_{it} \right] - a_{it} \right) \]
\[ c_{it} = E_i^{II} \left[ a_{t+1} + p_{t+1} \right] - \bar{p}_{it} \]

- unit intertemporal elasticity of substitution
- proportional response is optimal
Transparency

Is better public information good? (Morris and Shin (2002))

- Effect on output gap may be bad
- Total effect always good
Effect on welfare

![Graph showing the effect on welfare](image-url)

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Effect on output gap volatility

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Compare with Hellwig (2005)

Lucas style model with unobserved money supply shocks

- more precision about monetary shocks is good:
  - reduce output gap
  - reduce price variance (spurious)

Here uncertainty about real shocks

- more precision is good:
  - ambiguous on output gap
  - increase price variance (good)
  - second effect dominates

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Expectations’ shocks and business cycles


- **Problem 1**: in neoclassical setting ‘demand disturbances’ have hard time generating right response of hours/consumption/investment

- Euler equation

\[
    c_t = E_t \left[ a_{t+1} - (r_t - p_{t+1} + p_t) \right]
\]

- with flexible prices the real rate increases automatically

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Expectations’ shocks and business cycles (continued)

- Nominal rigidity can help (Christiano, Motto and Rostagno (2006))
- **Problem 2**: monetary policy accommodation of demand shocks is typically suboptimal
- Euler equation

\[
c_t = \mathbb{E}_t \left[ a_{t+1} - (r_t - p_{t+1} + p_t) \right]
\]

- with full information optimal to increase \( r \)

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Expectations’ shocks and business cycles (continued)

- Imperfect information + nominal rigidity can help

- **Problem 3**: policy rules still able to wipe out demand shocks

- ...but this is not optimal

- a theory of demand shocks that survive optimal policy
Concluding

- Future superior information + forward looking consumers → policy can induce efficient use of dispersed information


- Efficient use of dispersed information ≠ full stabilization output gap
  - Still some offsetting of demand shocks is feasible and desirable
  - Clearly this requires commitment, which may be tough (bubble example)