1.3 **Intro to *q* theory**

- Hayashi (1982)

- Firm with initial stock of capital $k_0$

- Maximize expected present value of dividends

$$E \sum_{t=0}^{\infty} \beta^t d_t$$

- Flow of funds constraint

$$d_t + G(k_{t+1}, k_t) = A_t F(k_t, l_t) - w_t l_t$$

Cite as: Guido Lorenzoni, course materials for 14.462 Advanced Macroeconomics II, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].
• $G$ is investment costs + adjustment costs, e.g.

$$G(k_{t+1}, k_t) = k_{t+1} - (1 - \delta) k_t + \frac{\xi (k_{t+1} - k_t)^2}{2 k_t}$$

• Assumption:

$G$ and $F$ are constant returns to scale

• Stochastic process for $A_t$ and $w_t$

$$A_t = A(s)$$
$$w_t = w(s)$$

$$s' = \Gamma(s, \epsilon')$$.
• Firm’s problem

\[ V(k, s) = \max_{d', l, k'} d + \beta E \left[ V(k', s') \right] \]
\[ \text{s.t.} \quad d + G(k', k) \leq A(s) F(k, l) - w(s) l \]

• From sequence problem and CRS: \( V(k, s) \) is linear

\[ V(k, s) = R(s) k \]
• f.o.c.

\[ 1 = \lambda \]

\[ \lambda G_1 (k', k) = \beta E [V_1 (k', s')] \]

• envelope

\[ V_1 (k, s) = \lambda \left( A(s) F_1 (k, l) - G_2 (k', k) \right) \]
1.3.1 Marginal $q$

\[ q^m (s) = G_1 (k', k) = \beta E [V_1 (k', s')] \]

\[ q^m (s) = G_1 \left( \frac{k'}{k}, 1 \right) \]

- one-to-one correspondence between $q^m$ and investment
• it is also true that investment equalizes marginal return on capital to cost of funds

\[
\frac{\beta E[V_1(k', s')]}{q^m(s)} = 1
\]

\[
\frac{E[(A(s') F_1(k', l') - G_2(k''', k')]}{q^m(s)} = 1/\beta
\]

where \(1/\beta\) interest rate
1.3.2 Average $q$

Define the value of the firm: total present value of future claims

$$p_t = E \sum_{j=1}^{\infty} \beta^j d_{t+j}$$

$$p (k, s) = V (k, s) - d$$

From envelope+linearity of $V$ function

$$V (k, s) = \left[ A (s) F_1 (k, l) - G_2 (k', k) \right] k$$

From budget constraint

$$d = A (s) F (k, l) - w (s) l - G (k', k)$$

$$= A (s) F_1 (k, l) k - G_1 (k', k) k' - G_2 (k', k) k$$
\[ p(k, s) = G_1(k', k) k' \]

Ratio of firms’ value to the capital invested

\[ q(s) = \frac{p(k, s)}{k'} = G_1(k', k) = q^m(s) \]

- average \( q \) = marginal \( q \)

- average \( q \) sufficient statistic for investment
1.3.3 Used capital market

Market for used capital $q^o$

\[ d + G(k', k^o) + q^o k^o \leq A(s) F(k, l) - w(s) l + q^o k \]

- Same as equilibrium above with

\[ q^o = -G_2(k', k) \]

- Total liquidation value of the firm

\[ A(s) F(k, l) - w(s) l + q^o k = R(s) k \]
• Total investment cost

\[ G\left(k', k^o\right) + q^o k^o = q^m k' \]

• Then compact program

\[
V(k, s) = \max \quad d + \beta \mathbb{E} \left[ V\left(k', s'\right) \right] \\
\text{s.t.} \quad d + q^m(s) k' \leq R(s) k
\]

• \(R(s)\) gross return on investment per unit of capital

• \(q^m(s)\) shadow cost of new capital
• Immediate

\[ V(k, s) = R(s)k \]

\[ p(k, s) = R(s)k - d = q^m(s)k' \]

and

\[ \frac{\mathbb{E}[R(s')]}{q^m} = \frac{1}{\beta} \]
1.4 Empirical performance of $q$ theory

- Run regressions

$$(I/K)_{j,t} = ... + a_1 q_{j,t} + a_2 (CF/K)_{j,t} + e_{j,t}$$


Discussion: maybe $q$ is mis-measured and $CF$ better predictor of future returns

- Gilchrist and Himmelberg (1995)
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• Construct artificial value of the firm

\[ \hat{p}_t = E \sum_{j=1}^{\infty} d_{t+j} \]

and use it to get

\[ \tilde{q}_t = \frac{\hat{p}_t}{k_{t+1}} \]

• results do not change
Approach II: identify exogenous shocks to internal funds


- components of cash flow that are exogenous to investment opportunities
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