14.662 Recitation 10

Goldberger (1984): Reverse Regression and Salary Discrimination

Peter Hull

Spring 2015
Regressing Discrimination

- Often (both in academia and the “real world”) discrimination is diagnosed by regressions of the form

\[ y = x' a + bz + e \]  \hspace{1cm} (1)

where \( z \) indicates a sex/race and \( x \) are other relevant “qualifications”

- Another approach is the “reverse” regression of, for \( q \equiv x'a \):

\[ q = cy + dz + u \]  \hspace{1cm} (2)

- A naïf might expect \( d < 0 \) if \( a > 0 \) (“if men earn more than equally-qualified women, they’re less qualified than equally-paid women”)
  - But that’s only true for deterministic relationships

- We might think (2) \( \succ \) (1) if qualifications are measured with error (suppose, for some reason, we’re not worried about OVB)
  - Goldberger (1984) shows this preference may be ill-founded
Forward and Reverse Regressions

A. Median Income by Schooling

<table>
<thead>
<tr>
<th>Schooling (years)</th>
<th>None</th>
<th>1–4</th>
<th>5–7</th>
<th>8</th>
<th>High School</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>1569</td>
<td>1962</td>
<td>3240</td>
<td>3981</td>
<td>5013</td>
<td>6104</td>
</tr>
<tr>
<td>Nonwhite</td>
<td>1042</td>
<td>1565</td>
<td>2353</td>
<td>2900</td>
<td>3253</td>
<td>4029</td>
</tr>
</tbody>
</table>

Dollars

White          | 5529 | 6104 |
Nonwhite       | 4840 | 7779 |

B. Median Schooling by Income

<table>
<thead>
<tr>
<th>Income ($1000s)</th>
<th>None</th>
<th>0–1</th>
<th>1–2</th>
<th>2–3</th>
<th>3–4</th>
<th>4–5</th>
<th>5–6</th>
<th>6–7</th>
<th>7–9</th>
<th>10+</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>8.4</td>
<td>8.0</td>
<td>8.4</td>
<td>8.7</td>
<td>9.5</td>
<td>10.5</td>
<td>11.4</td>
<td>12.1</td>
<td>12.4</td>
<td>14.0</td>
</tr>
<tr>
<td>Nonwhite</td>
<td>6.9</td>
<td>5.1</td>
<td>6.5</td>
<td>7.8</td>
<td>8.7</td>
<td>9.3</td>
<td>10.4</td>
<td>11.2</td>
<td>12.1</td>
<td>12.8</td>
</tr>
</tbody>
</table>

Source: Hashimoto and Kochin [20].

- Both $a > 0$ and $d > 0$ (Hashimoto and Kochin call this a “riddle”)

© John Wiley & Sons, Inc. All rights reserved. This content is excluded from our Creative Commons license.
For more information, see http://ocw.mit.edu-help/faq-fair-us/
Reverse ≥ Forward?

- Reverse regression often suggests less discrimination (in favor of men, whites, etc.), and sometimes even reverse discrimination
  - Conway and Roberts (1983): \( a = 0.15, d = -0.01 \) in a sex regression for 274 bank employees, education/experience/age controls
  - Abowd, Abowd, and Killingsworth (1983): \( a, d > 0 \) in a race regression from the 1976 Survey of Income and Education

- Conway and Roberts (1983): “The problem of omitted job qualifications points to the weakness of a direct-regression-adjusted income differential [relative to reverse regression]”

- Goldberger shows this is true only in very special case where salary is a deterministic function of productivity and gender
  - In a more general EIV model, forward reg. will be upward-biased and reverse reg. will be downward-biased
  - ...but in another “proxy variable” model forward can be unbiased while reverse is still downward-biased
Multivariate EIV

Data-generating process:

\[ y = \alpha z + p + \nu \]
\[ p = \beta x^*, \quad x^* = \mu z + u, \quad x = x^* + \epsilon \]

where \( \nu, u, \) and \( \epsilon \) are all white noise terms.
Forward Regression

- Estimate $E[y|x, z] = az + bx$ (we normalize everything to be mean-zero for women)

$$b = \frac{\text{Cov}(y, \tilde{x})}{\text{Var}(\tilde{x})} = \frac{\text{Cov}(\alpha z + \beta x^* + v, x^* + \epsilon - \mu z)}{\text{Var}(x^* + \epsilon - \mu z)}$$

$$= \frac{\text{Cov}((\alpha + \beta \mu)z + \beta u + v, u + \epsilon)}{\text{Var}(u + e)} = \beta \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}$$

- So

$$a = E[y|z = 1] - bE[x|z = 1]$$

$$= \alpha + \beta E[x^*|z = 1] - bE[x|z = 1]$$

$$= \alpha + (\beta - b)\mu$$

$$= \alpha + \beta \mu \frac{\sigma_e^2}{\sigma_u^2 + \sigma_e^2}$$

- Regression puts more weight on a positive correlate to a noisy signal.
Reverse Regression

- By substitution,
  \[ y = (\alpha + \beta \mu)z + \beta u + \nu \]

- Estimate \( E[x|y, z] = cy + dz \)
  \[ c = \frac{Cov(x, \tilde{y})}{Var(\tilde{y})} = \frac{Cov(\mu z + \varepsilon + u, \beta u + \nu)}{Var(\beta u + \nu)} = \frac{\beta \sigma_u^2}{\beta^2 \sigma_u^2 + \sigma_v^2} \]

- So
  \[ d = E[x|z = 1] - cE[y|z = 1] \]
  \[ = \mu - c(\alpha + \beta \mu) \]
  \[ = \frac{\sigma_v^2}{\beta^2 \sigma_u^2 + \sigma_v^2} \mu - c\alpha \]

- Implied discrimination coefficient: \(-d/c = \alpha - \mu \sigma_u^2 / (\beta \sigma_v^2) < \alpha\)
Comparing Forward and Reverse

- Forward regression gives an upper bound on $\alpha$, while reverse regression gives a lower bound
  - Bounds are tighter when $\mu$ is smaller (so $z$ and $x^*$ are less correlated)
  - $\alpha$ is closer to $a$ when $\beta$ is smaller or $\sigma_u^2$ is larger
  - $\alpha$ is closer to $-d/c$ when $\beta$ is larger or $\sigma_u^2$ is smaller

- If $\sigma_v^2 = 0$ (deterministic salary function), $d = -c\alpha$ and reverse regression is indeed unbiased (but not otherwise)
  - Dempster (1982): “[we are] somewhat skeptical about the existence of a chance mechanism whereby the employer creates a random disturbance an adds it”
  - Are mismeasured qualifications fallible measures of true productivity ($\sigma_v^2 = 0$) or of its determinants ($\sigma_v^2 > 0$)?
x as a “Proxy” for True Qualification

Data-generating process:

\[ y = \alpha z + p + \nu \]

\[ p = \beta x + \varepsilon, \quad x = \mu z + u \]

where \( \nu \) and \( \varepsilon \) are white noise terms.
Forward and Reverse Regression

- Note that by substitution

\[ y = \alpha z + \beta x + \varepsilon + \nu \]

since \( \varepsilon \) and \( \nu \) are white noise, forward regression will be unbiased

- Reverse regression: for \( E[x|y,z] = cy + dz \)

\[
    c = \frac{\text{Cov}(x, \tilde{y})}{\text{Var}(\tilde{y})} = \frac{\text{Cov}(\mu z + u, \beta u + \varepsilon + \nu)}{\text{Var}(\beta u + \varepsilon + \nu)} = \frac{\beta \sigma_u^2}{\beta^2 \sigma_u^2 + \sigma^2_\varepsilon + \sigma^2_\nu}
\]

and \( d = \frac{\sigma^2_\nu}{\beta^2 \sigma_u^2 + \sigma^2_\varepsilon + \sigma^2_\nu} \mu - c\alpha \) as before; \(-d/c < \alpha\)

- Now reverse regression bias persists even if the salary function is deterministic: we have \( \sigma^2_\varepsilon + \sigma^2_\nu > 0 \) even if \( \sigma^2_\nu = 0 \)

- Bias may be large enough that the reverse regression estimate may be of the wrong sign
Goldberger (1984) is a nice illustration of how discrimination regressions may be hard to interpret
- Whether forward or reverse is correct depends on assumed DGP
- This kind of regression gymnastics builds character!

Today we would likely care much more about OVB/misspecification
- If the true wage CEF is nonlinear, forward regression may be sensible and reverse may not (Racine and Rilstone, 1994)
- If men and women have unobservably different productivity, everything goes out the window

Is it clear we want to control for productivity?
- May capture a narrow definition of discrimination (Lundberg and Startz, 1983)
14.662 Labor Economics II
Spring 2015

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.