Helpful readings for this homework: Chapter 12.1-5, 12.7, Chapter 13.1-3

**Exercise 10.1:** Exercise 12.3 from Chapter 12 of A&L (page 695).

**Exercise 10.2:** Exercise 14.4 from Chapter 14 of A&L (page 824). Hint: Use the impedance method.

**Problem 10.1:** In the network shown below, the inductor and the capacitor have zero current and voltage, respectively, prior to $t = 0$. At $t = 0$, a step in voltage from 0 to $V_o$ is applied by the voltage source indicated.

![A step-driven series RLC circuit.](image)

(a). Find $v_C$, $v_L$, $v_R$, $i$, $di/dt$ just after the step at $t = 0$.

(b). Argue that $i = 0$ at $t = \infty$ so that $i(t)$ has no constant component.

(c). Find a second-order differential equation which describes the behavior of $i(t)$ for $t > 0$.

(d). Following parts (a) and (b), the current $i(t)$ takes the form $i(t) = I \sin(\omega t + \phi)e^{-\alpha t}$. Find $I$, $\omega$, $\phi$ and $\alpha$ in terms of $V_o$, $R$, $L$ and $C$.

(e). Suppose that the input is a voltage impulse with area $\Lambda_o$, where $\Lambda_o = \tau V_o$, $V_o$ is the amplitude of the voltage step shown in Figure 1 and $\tau$ a given time constant. Repeat parts (a), (b), (c)
and (d) for the network shown in Figure 1. Hint: Assume that the current \( i(t) \) takes the form
\[
i(t) = [A \cos(\omega t) + B \sin(\omega t)]e^{-\alpha t}.
\]

(f). Using the expression for \( i(t) \) found in part (d), verify your answer to part (e) by considering the relation between step and impulse responses.

Save copies of your work for the pre-lab of Lab 3.

**Problem 10.2:** The network shown in Figure 2 is driven in steady-state by the sinusoidal current \( i_{IN}(t) = I_{in} \cos(\omega t) \). The output of the network is the voltage \( v_{OUT}(t) \), which takes the form
\[
v_{OUT}(t) = V_{out} \cos(\omega t + \phi).
\]
Find \( V_{out} \) and \( \phi \) as functions of \( \omega \) as follows.

(a). Find a differential equation that can be solved for \( v_{OUT}(t) \) given \( i_{IN}(t) \). Hint: consider how \( v_{OUT}(t) \) is related to the inductor current.

(b). Let \( i_{IN}(t) = Re\{I_{in}e^{j\omega t}\} \). Also let \( v_{OUT}(t) = Re\{\hat{V}_{out}e^{j\omega t}\} \), where \( \hat{V}_{out} \) is a complex function of the circuit parameters, \( \omega \) and \( I_{in} \). With these definitions, find \( \hat{V}_{out} \).

(c). An alternative way to write \( v_{OUT}(t) \) is as \( v_{OUT}(t) = Re\{|\hat{V}_{out}|e^{j(\omega t + \angle \hat{V}_{out})}\} \). Determine \( |\hat{V}_{out}| \) and \( \angle \hat{V}_{out} \) as functions of the circuit parameters, \( \omega \) and \( I_{in} \). Then, find \( V_{out} \) and \( \phi \) for the original cosine input, again both as functions of the circuit parameters \( \omega \) and \( I_{in} \).

(d). Sketch and clearly label \( V_{out}/I_{in} \) and \( \phi \) as functions of \( \omega \). Identify the low-frequency and high-frequency asymptotes on the sketch.

**Problem 10.3:** Parts (a), (b) and (c) of Problem 14.16 from Chapter 14 of A&L (page 834).