Objectives and Outcomes for 6.011
Signals, Systems and Inference

6.011 will enable you to understand how signals, systems and inference combine in prototypical tasks of communication, control and signal processing. More specifically, you will:

**Objective 1** Learn the use of random-process (or stochastic-process) models to represent non-deterministic signals and noise, and extract from these models the time-domain and frequency-domain structure of the signals and noise;

**Objective 2** Analyze the response of linear, time-invariant dynamic systems to random input signals or noise, and understand how the resulting outputs reflect input and system characteristics;

**Objective 3** Use probabilistic characterizations of random signals and noise, and measurements derived from these signals, to make optimal inferences about related signals and systems;

**Objective 4** Understand the notion of state for a causal system, the relation of state to system input and output signals, and the use of state in inference and feedback control for the system;

**Objective 5** Analyze systems – commonly found in communication, control and signal processing – that combine discrete-time and continuous-time components, as well as deterministic and stochastic behavior.

Mastering the material in this subject will correspondingly enable you to:

**Outcome 1**
- compute and interpret means, correlations/covariances of random variables;
- compute and interpret auto- and cross-correlation/covariance functions of random processes;
- recognize whether a random process is wide-sense stationary (WSS), and whether two processes are jointly WSS;
- compute and interpret the power spectral density (PSD) of a WSS random process, recognize the properties a valid PSD must satisfy, understand what white noise is, compute the cross-PSD of two WSS processes and recognize the constraints it must satisfy;
- determine whether a WSS random process is ergodic in mean value;

**Outcome 2**
- compute the correlation/covariance function of the output process for a linear time-invariant (LTI) system driven by a WSS input process, compute the cross-
correlation/covariance of output with input, and compute all the corresponding PSDs;
b. generate a WSS process with a specified auto-correlation/covariance function or PSD by appropriate LTI filtering of a process of known PSD (e.g., white noise), and understand how all-pass filtering leaves PSD unchanged;
c. given (rational) PSDs of the input and output processes of an LTI system, determine the set of possible transfer functions for the system;

**Outcome 3** Formulate and solve minimum conditional-expected-cost inference problems, particularly for
a. square-error costs, leading to
   • minimum mean-square-error (MMSE) estimation for continuous random variables, estimates and estimators, linear MMSE estimation, the normal equations;
   • non-causal and causal Wiener filters for random processes;
b. all-or-none costs, leading to
   • minimum probability-of-error choices (decisions) among discrete random variables (hypotheses), the likelihood ratio test, probabilities of false alarm, miss and detection, Neyman-Pearson detection, receiver operating characteristics;
   • matched filtering for signal-to-noise ratio maximization and for optimal detection of known signals in Gaussian noise;

**Outcome 4**
a. identify appropriate state variables in particular problems, write corresponding state-space equations, compute equilibrium solutions, obtain linearized state-space models describing small deviations from equilibrium;
b. analyze the behavior of LTI state-space models, compute modal solutions, identify unreachable and unobservable modes, track down hidden modes;
c. design state feedback controls, observers for state estimation, and observer-based controls;

**Outcome 5** analyze systems for
a. discrete-time (DT) processing of continuous-time (CT) signals;
b. DT control of CT systems via sampled-data control;
c. CT communication of DT signals via pulse-amplitude modulation (PAM), pulse shaping and filtering to combat channel distortions (frequency-dependent group delay and amplitude response, noise).