1. Example 9.1, page 463 in textbook
Romeo and Juliet start dating, but Juliet will be late on any date by a random amount $X$, uniformly distributed over the interval $[0, \theta]$. The parameter $\theta$ is unknown. Assuming that Juliet was late by an amount $x$ on their first date, find the ML estimate of $\theta$ based on the observation $X = x$.

2. Example 9.4, page 464 in textbook
Estimate the mean $\mu$ and variance $\nu$ of a normal distribution using $n$ independent observations $X_1, \ldots, X_n$.

3. Example 9.8, page 474 of textbook
We would like to estimate the fraction of voters supporting a particular candidate for office. We collect $n$ independent sample voter responses $X_1, \ldots, X_n$, where $X_i$ is viewed as a Bernoulli random variable, with $X_i = 1$ if the $i$th voter supports the candidate. We conducted a poll of 1200 people in North Carolina, and found that 684 were supporting the candidate. We would like to construct a 95% confidence interval for $\theta$, the proportion of people who support the candidate.

As we saw in lecture, using the central limit theorem, an (approximate) 95% confidence interval can be defined as

$$\hat{\Theta} - 1.96 \sqrt{\frac{\nu}{n}}, \quad \hat{\Theta} + 1.96 \sqrt{\frac{\nu}{n}}$$

where $\nu = \text{Var}(X_i)$, and $\hat{\Theta}_n = (X_1 + \ldots + X_n)/n$. Unfortunately, we don’t know the value for $\nu$. Construct confidence intervals for $\theta$ using the following three ways of estimating or bounding the value for $\nu$ (in each case simply assume that $\nu$ is equal to the given estimate; note that this is a further approximation in cases (a) and (b)).

(a)

$$\hat{\nu}_n^2 = \frac{1}{n - 1} \sum_{i=1}^{n} (X_i - \hat{\Theta}_n)^2$$

(b)

$$\hat{\Theta}_n(1 - \hat{\Theta}_n)$$

(c) The most conservative upper bound for the variance.