Problems for Recitation 22

1 Properties of Variance

In this problem we will study some properties of the variance and the standard deviation of random variables.


b. Show that for any random variable $R$ and constants $a$ and $b$, $\text{Var}[aR + b] = a^2 \text{Var}[R]$.
   Conclude that the standard deviation of $aR + b$ is $a$ times the standard deviation of $R$.

c. Show that if $R_1$ and $R_2$ are independent random variables, then
   \[ \text{Var}[R_1 + R_2] = \text{Var}[R_1] + \text{Var}[R_2]. \]

d. Give an example of random variables $R_1$ and $R_2$ for which
   \[ \text{Var}[R_1 + R_2] \neq \text{Var}[R_1] + \text{Var}[R_2]. \]

e. Compute the variance and standard deviation of the Binomial distribution $H_{n,p}$ with parameters $n$ and $p$.

f. Let’s say we have a random variable $T$ such that $T = \sum_{j=1}^{n} T_j$, where all of the $T_j$’s are mutually independent and take values in the range $[0, 1]$. Prove that $\text{Var}(T) \leq \text{Ex}(T)$.
   We’ll use this result in lecture tomorrow. **Hint:** Upper bound $\text{Var}[T_j]$ with $\text{E}[T_j]$ using the definition of variance in part (a) and the rule for computing the expectation of a function of a random variable.