Problem 1: The inductance and phase velocity are:

\[ L = CZ_0^2 = 2 \times 10^{-10} \times 900 \approx -0.18 \mu \text{H/m} \]

\[ c = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{2 \times 10^{-10} \times 1.8 \times 10^{-7}}} \approx 1.6667 \times 10^8 \text{m/s} \]

Before the load is connected, voltage is uniform and equal to the source and current is zero. So:

\[ V_+ = \frac{V}{2} \]
\[ V_- = \frac{V}{2} \]
\[ I_+ = \frac{V}{2Z_0} \]
\[ I_- = -\frac{V}{2Z_0} \]

When the switch is thrown, at the load end, \( V_- \) becomes zero, since the load end is matched to the line. For the first interval, the situation is as shown in Figure 1. The voltage at the matched end is \( V_+ \). When the reverse going pulse gets to the sending end, \( V_+ \) becomes equal to the source, since \( V_- \) is equal to zero. That takes another period of time to propagate to the receiving end, at which point the voltabe is equal to sending end voltage. This is shown in Figure 2. Transit time is:

\[ T_t = \frac{L}{u} = \frac{100,000}{1.6667 \times 10^8} \approx 600 \mu \text{s} \]

And it takes \( 2T_t \) for the transient to finish.
Figure 1: Pulses along the line

Figure 2: Termination Voltage
Problem 2: Voltage at the receiving end and impedance at the sending end are:

\[
V_r = V_s \frac{Z_t}{Z_t \cos k\ell + jZ_0 \sin k\ell} \\
Z_s = Z_0 \frac{jZ_t \sin k\ell + Z_0 \cos k\ell}{jZ_t \sin k\ell + Z_0 \cos k\ell}
\]

If the line is open at the receiving end,

\[
V_r = \frac{V_s}{\cos k\ell} \\
Z_s = jZ_t \coth k\ell
\]

The rest of the calculations are carried out by Matlab (source appended) and the results are:

<table>
<thead>
<tr>
<th>Line Open at Receiving End</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Receiving end voltage</td>
<td>46176.3 V</td>
</tr>
<tr>
<td>Sending End Current</td>
<td>345.2 A</td>
</tr>
<tr>
<td>Line loaded with 60 Ohms</td>
<td></td>
</tr>
<tr>
<td>Receiving End Voltage</td>
<td>45873.6 V</td>
</tr>
<tr>
<td>Sending end Current</td>
<td>820.2 A</td>
</tr>
<tr>
<td>Receiving End Power</td>
<td>35073062.2 W</td>
</tr>
<tr>
<td>Sending End Real Power</td>
<td>35073062.2 W</td>
</tr>
<tr>
<td>Sending End Reactive Power</td>
<td>-11498241.4 VAR</td>
</tr>
</tbody>
</table>
Problem 3: (Chapter 6, Problem 6 from text)

Turns ratio is \( N = \frac{13.800}{\sqrt{3} \times 480} \approx \frac{796.4}{480} \approx 16.60 \).

If the primary voltages are:

\[
V_B = V e^{-j \frac{2\pi}{3}} \\
V_C = V e^{j \frac{2\pi}{3}}
\]

Then, on the secondary side, with respect to Figure 3:

\[
V_{bc} = \frac{V}{N} e^{-j \frac{2\pi}{3}} \\
V_{ca} = \frac{V}{N} e^{j \frac{2\pi}{3}} \\
V_{ab} = -V_{ca} - V_{bc} = -\frac{V}{N}
\]

![Figure 3: Equivalent Circuit with Currents](image)

The actual phase voltages are shown in Figure 4

![Figure 4: Secondary Side Voltages](image)

The phase voltages are:
\[
V_a = \frac{V}{\sqrt{3}N} e^{-j\frac{\pi}{6}} \\
V_b = \frac{V}{\sqrt{3}N} e^{j\frac{2\pi}{3}} \\
V_c = \frac{V}{\sqrt{3}N} e^{-j\frac{\pi}{2}}
\]

If the loads are unity power factor and of magnitude \(I_0\),

\[
I_a = I_0 e^{-j\frac{\pi}{6}} \\
I_b = I_0 e^{-j\frac{2\pi}{3}} \\
I_c = I + 0e^{j\frac{\pi}{2}}
\]

On the wye side of the transformer:

\[
I_A = 0 \\
I_B = \frac{I_0}{N} \frac{I_0 e^{-j\frac{2\pi}{3}}}{N} \\
I_C = -\frac{I_0}{N} \frac{I_0 e^{j\frac{5\pi}{6}}}{N}
\]

This is shown in Figure 5

![Figure 5: Primary Side Currents](image)

As a test, we can see if the real and reactive powers are the same on the primary and secondary sides. On the primary side:

\[
P_A = 0 \\
P_B = \frac{V I_0}{N} e^{-j\frac{\pi}{6}}
\]
\[ P_C = \frac{V I_0 e^{j\frac{\pi}{3}}}{N} \]

\[ P_A + P_B + P_C = \frac{\sqrt{3}V I_0}{N} \]

On the secondary side, power is:

\[ P = 3 \frac{V I_0}{\sqrt{3}N} = \frac{\sqrt{3}V I_0}{N} \]

A few numbers are:

Problem 6.6
Turns Ratio = 16.5988
Primary Power = 83138.4 + j 0
Secondary Power = 83138.4 + j 2.01948e-28

Finally, if the ground is not connected on the primary side, the thing becomes a single phase circuit, with \( V_B - B_C = -j\sqrt{3}V \) Then: \( V_{BC} = -V_{CA} = j\sqrt[3]{3V2N} \) And this drives a resistance of \( R_{eq} = \frac{3}{2R} \).
**Problem 4:** Essentially the whole story is contained in the attached script. The first part is straightforward. The second part is done by first doing the delta-wye equivalent, finding the currents in the delta, re-assembling them into the wye and then transforming them across the transformer. The third part is a little tricky, but note that the voltages can be found by:

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} =
\begin{bmatrix}
R_a + R_n & R_n & R_n \\
R_n & R_b + R_n & R_n \\
R_n & R_n & R_c + R_n
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]

This is easily inverted in Matlab to get the currents. A summary of what is calculated is:

Turns Ratio = 5.05181  
Line-Neutral Voltage = 2424.87  
Phase C current = 13.1966  
Phase C real power = 32000  
Phase A real power = 8000  
Low side power = 48000  
High side power = 48000  
High side reactive = 13856.4

```
>> open p4.m
>> p6_6
Problem 6.6
Turns Ratio = 16.5988  
Primary Power = 83138.4 + j 0  
Secondary Pwr = 83138.4 + j 2.01948e-28
```

```
>> p6_7
Part A: Secondary Side Grounded
Secondary Currents
Ia = 5.54256 + j 0 = 5.54256 angle 0
Ib = -4.6188 + j -8 = 9.2376 angle -2.0944
Ic = -4.6188 + j 8 = 9.2376 angle 2.0944
Primary Currents
IA = 0.676923 + j -0.532939 = 0.861538 angle -0.666946
IB = -0.676923 + j -0.532939 = 0.861538 angle -2.47465
IC = 2.9584e-16 + j 1.06588 = 1.06588 angle 1.5708
Check: Primary P = 6656 Q = 0 Secondary P = 6656 Q = 0
```

```
Part B: Secondary Side Ungrounded
Rab = 130  Rbc = 78  Rca = 130
Secondary Currents
Ia = 6.39526 + j -6.66134e-16 = 6.39526 angle -1.0416e-16
Ib = -3.19763 + j -8 = 8.61538 angle -1.95105
Ic = -3.19763 + j 8 = 8.61538 angle 1.95105
Primary Currents
IA = 0.639053 + j -0.532939 = 0.832113 angle -0.695102
IB = -0.639053 + j -0.532939 = 0.832113 angle -2.44649
```

7
\[
R_1 = 0.78 \times 0.03 \approx 0.0234 \\
R_2 = 1 \\
R_3 = \sqrt{2} \approx 1.4142 \\
R_4 = 1
\]

So
\[
\text{GMD} = \sqrt[4]{0.0234 \times \sqrt{2}} \approx 0.4265 \text{m}
\]
% 6.061 Problem Set 5 Problem 2 2/26/11

L = 1e5; % line length 100 km
Z0 = 30; % characteristic impedance
Cap = 2e-10; % capacitance
om = 120*pi; % frequency
Vs = 45000; % sending end voltage
Ind = Cap * Z0^2; % line inductance
u = 1/sqrt(Ind*Cap); % phase velocity
k = om/u; % wavenumber

% part 1: line open
Vr = Vs / cos(k*L); % receiving end voltage
Is = j*tan(k*L)*Vs/Z0; % sending end voltage

fprintf('Line Open at Receiving End\n')
fprintf('Receiving end voltage = %.1f V\n', Vr)
fprintf('Sending End Current = %.1f A\n', abs(Is))

% part 2: resistive loading at 60 ohms
Zr = 60;
Vr = Vs * Zr/(Zr * cos(k*L) + j*Z0 * sin(k*L));
Zs = Z0 * (Zr*cos(k*L) + j*Z0*sin(k*L))/(j*Zr*sin(k*L) + Z0 * cos(k*L));
Is = Vs/Zs;

Ss = Vs*conj(Is);
Ps = real(Ss);
Qs = imag(Ss);
Pr = abs(Vr)^2/Zr;

fprintf('Line loaded with 60 Ohms\n')
fprintf('Receiving End Voltage = %.1f V\n', abs(Vr))
fprintf('Sending end Current = %.1f A\n', abs(Is))
fprintf('Sending end Power = %.1f W\n', Pr)
fprintf('Sending End Real Power = %.1f W\n', Ps)
fprintf('Sending End Reactive Power = %.1f VAR\n', Qs)

% chapter 6, problem 6

V_0 = 13800/sqrt(3);
N = V_0/480;
V_A = V_0;
V_B = V_0 * exp(-j*2*pi/3);
V_C = V_0 * exp(j*2*pi/3);
\[ V_s = \frac{480}{\sqrt{3}}; \]
\[ V_a = V_s \exp(-j\pi/6); \]
\[ V_b = V_s \exp(-j5\pi/6); \]
\[ V_c = V_s \exp(j\pi/2); \]
\[ I_a = 100 \exp(-j\pi/6); \]
\[ I_b = 100 \exp(-j5\pi/6); \]
\[ I_c = 100 \exp(j\pi/2); \]
\[ S_s = V_a \text{conj}(I_a) + V_b \text{conj}(I_b) + V_c \text{conj}(I_c); \]
\[ I_P = 100/N; \]
\[ I_B = I_P \exp(-j5\pi/6); \]
\[ I_C = I_P \exp(j5\pi/6); \]
\[ S_P = V_B \text{conj}(I_B) + V_C \text{conj}(I_C); \]

\text{fprintf('
Problem 6.6
')}
\text{fprintf('Turns Ratio = \%g\n', N)}
\text{fprintf('Primary Power = \%g + j \%g\n', real(S_P), imag(S_P))}
\text{fprintf('Secondary Power = \%g + j \%g\n', real(S_s), imag(S_s))}

\% 6.061 Problem Set 5, Problem 4 (6.7 from Text)

\% This is mostly book-keeping

\[ V_p = 4160; \quad \% \text{primary side, line-line} \]
\[ V_s = 480; \quad \% \text{secondary side, line-line} \]
\[ V_{sn} = V_s/\sqrt{3}; \quad \% \text{secondary side, line-neutral} \]
\[ R_a = 50; \quad \% \text{three phase resistors} \]
\[ R_b = 30; \]
\[ R_c = 30; \]
\[ N = V_p/V_{sn}; \quad \% \text{turns ratio: delta-wye} \]
\[ a = \exp(j2\pi/3); \quad \% 120 degree rotation \]

\% Preliminary: Primary Side
\[ V_{pn} = V_p/\sqrt{3}; \]
\[ V_A = V_{pn} \exp(-j\pi/6); \quad \% \text{connection rotates +30deg} \]
\[ V_B = V_A^\ast2; \]
\[ V_C = V_A^\ast; \]

\% part a: secondary side grounded

\[ V_a = V_{sn}; \quad \% \text{set 'real' to the secondary side} \]
\[ V_b = V_{sn}a^\ast2; \]
\[ V_c = V_{sn}a; \]
Ia = Va/Ra;
Ib = Vb/Rb;
Ic = Vc/Rc;

SS = Va*conj(Ia)+Vb*conj(Ib)+Vc*conj(Ic);

% translate to primary side:

IA = (Ia-Ic)/N;
IB = (Ib-Ia)/N;
IC = (Ic-Ib)/N;

SA = VA*conj(IA);
SB = VB*conj(IB);
SC = VC*conj(IC);
SP = SA + SB + SC;

% output
fprintf('Part A: Secondary Side Grounded\n')
fprintf('Secondary Currents\n')
fprintf('Ia = %g + j %g = %g angle %g\n', real(Ia), imag(Ia), abs(Ia), angle(Ia))
fprintf('Ib = %g + j %g = %g angle %g\n', real(Ib), imag(Ib), abs(Ib), angle(Ib))
fprintf('Ic = %g + j %g = %g angle %g\n', real(Ic), imag(Ic), abs(Ic), angle(Ic))
fprintf('Primary Currents\n')
fprintf('IA = %g + j %g = %g angle %g\n', real(IA), imag(IA), abs(IA), angle(IA))
fprintf('IB = %g + j %g = %g angle %g\n', real(IB), imag(IB), abs(IB), angle(IB))
fprintf('IC = %g + j %g = %g angle %g\n', real(IC), imag(IC), abs(IC), angle(IC))
fprintf('Check: Primary P = %10.0f Q = %10.0f Secondary P = %10.0f Q = %10.0f\n', real(SP), imag(SP), real(P), imag(P))

% Part B: secondary side ungrounded

Rab = (Ra*Rb + Ra*Rc + Rb*Rc)/Rc;  % find equivalent delta
Rbc = (Ra*Rb + Ra*Rc + Rb*Rc)/Ra;
Rca = (Ra*Rb + Ra*Rc + Rb*Rc)/Rb;

fprintf('Part B: Secondary Side Ungrounded\n')
fprintf('Rab = %g Rbc = %g Rca = %g\n', Rab, Rbc, Rca)

Iab = (Va - Vb)/Rab;  % currents in the delta
Ibc = (Vb - Vc)/Rbc;
Ica = (Vc - Va)/Rca;

Ia = Iab - Ica;  % now get currents in the wye
Ib = Ibc - Iab;
Ic = Ica - Ibc;

% repeat the rest of part a: it is now the same

SS = Va*conj(Ia)+Vb*conj(Ib)+Vc*conj(Ic);

% translate to primary side:

IA = (Ia-Ic)/N;
IB = (Ib-Ia)/N;
IC = (Ic-Ib)/N;

SA = VA*conj(IA);
SB = VB*conj(IB);
SC = VC*conj(IC);
SP = SA + SB + SC;

fprintf('Secondary Currents\n')
fprintf('Ia = %g + j %g = %g angle %g\n', real(Ia), imag(Ia), abs(Ia), angle(Ia))
fprintf('Ib = %g + j %g = %g angle %g\n', real(Ib), imag(Ib), abs(Ib), angle(Ib))
fprintf('Ic = %g + j %g = %g angle %g\n', real(Ic), imag(Ic), abs(Ic), angle(Ic))
fprintf('IA = %g + j %g = %g angle %g\n', real(IA), imag(IA), abs(IA), angle(IA))
fprintf('IB = %g + j %g = %g angle %g\n', real(IB), imag(IB), abs(IB), angle(IB))
fprintf('IC = %g + j %g = %g angle %g\n', real(IC), imag(IC), abs(IC), angle(IC))
fprintf('Check: Primary P = %10.0f Q = %10.0f Secondary P = %10.0f Q = %10.0f\n', real(SP), imag(SP), real(SP), imag(SP))

% Part C: Secondary grounded through a resistor
Rn = 10;

RM = [Ra+Rn Rn Rn;Rn Rn+Rn Rn Rn Rn Rn Rc+Rn]; % matrix of resistances
Vv = [Va; Vb; Vc;]; % vector of voltages
Is = RM\Vv; % these should be the currents

Ia = Is(1);
Ib = Is(2);
Ic = Is(3);

% repeat the rest of part a: it is now the same

SS = Va*conj(Ia)+Vb*conj(Ib)+Vc*conj(Ic);

% translate to primary side:

IA = (Ia-Ic)/N;
IB = (Ib-Ia)/N;
IC = (Ic-Ib)/N;

SA = VA*conj(IA);
SB = VB*conj(IB);
SC = VC*conj(IC);
SP = SA + SB + SC;

fprintf('Part C: secondary grounded through a resistor\n')
fprintf('Secondary Currents\n')
fprintf('Ia = %g + j %g angle %g\n', real(Ia), imag(Ia), abs(Ia), angle(Ia))
fprintf('Ib = %g + j %g angle %g\n', real(Ib), imag(Ib), abs(Ib), angle(Ib))
fprintf('Ic = %g + j %g angle %g\n', real(Ic), imag(Ic), abs(Ic), angle(Ic))
fprintf('Primary Currents\n')
fprintf('IA = %g + j %g angle %g\n', real(IA), imag(IA), abs(IA), angle(IA))
fprintf('IB = %g + j %g angle %g\n', real(IB), imag(IB), abs(IB), angle(IB))
fprintf('IC = %g + j %g angle %g\n', real(IC), imag(IC), abs(IC), angle(IC))
fprintf('Check: Primary P = %10.0f Q = %10.0f Secondary P = %10.0f Q = %10.0f\n', real(SP), im