6.301 Solid-State Circuits

Recitation 2: Device Physics and Modeling
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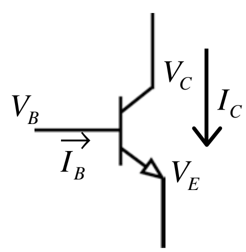
A few of you have expressed concern over the 6.012 material for this class. There is no need to worry. You will not need much of 6.012, and the parts you do need will become obvious because of how often they come up. The purpose of this recitation is to help you come to grips with the transistor physics that you will need.

We start modeling by facing the ugly reality of a highly nonlinear device whose inner workings make our heads spin:



$$\Rightarrow \quad I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)$$
$$\approx I_S e^{\frac{qV_{BE}}{kT}}$$

Right away, we have a problem. We can handle linear systems. We cannot handle nonlinear systems!*

So what do we do? We close our eyes, pretend that it is a linear device, and then arrange our design so that we're not punished for our crimes.

*The math in these situations is clumsy at best.

You know this as "linearizing," or "deriving a small signal model." We take an expression like

$$I_C = I_S e^{\frac{qV_{BE}}{kT}} = F(V_{BE})$$

And write instead

$$\Delta i_C = k \cdot \Delta v_{BE}$$

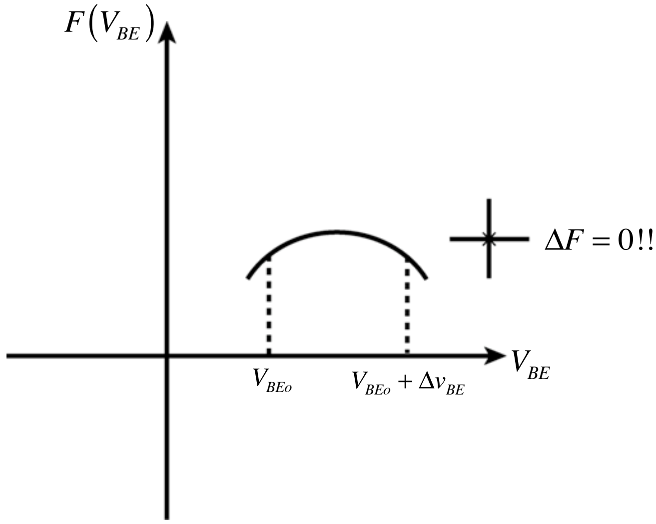
Hmmm. We need to find an appropriate *k* somehow, even though our approximate expression looks nothing like the original.

Based on what we've written, we say

$$k = \frac{\Delta i_C}{\Delta v_{BE}} = \frac{F(V_{BE0} + \Delta v_{BE}) - F(v_{BE})}{\Delta v_{BE}}$$

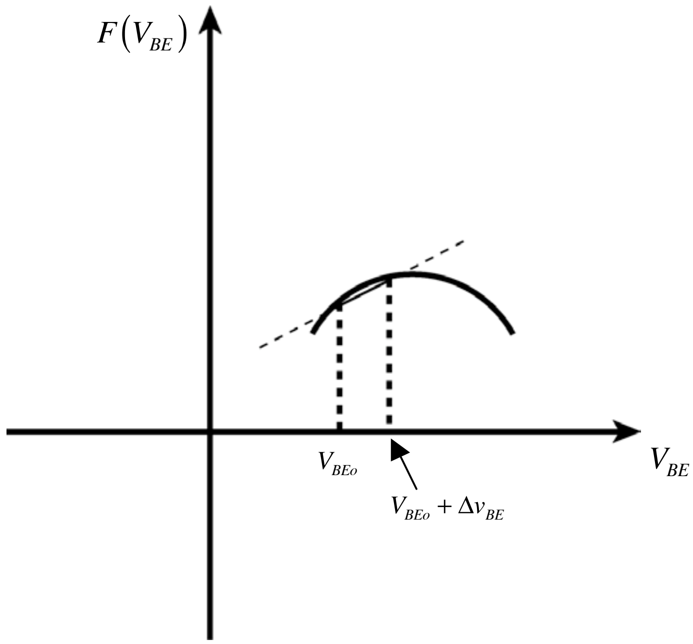
Clearly we're taking a risk here. We're throwing away all of the rich nonlinear behavior in favor of a simple proportionality factor k. What if we choose our interval and operating point badly?

Consider:



Our linear model would say that F does not depend on V_{BE} !

We can minimize our error by restricting the size of Δv_{BE} .



Over a small range, not such a bad approximation! And we do better by making Δv_{BE} even smaller:

$$k = \lim_{\Delta v_{BE} \to 0} \frac{F\left(V_{BE0} + \Delta v_{BE}\right) - F\left(v_{BE}\right)}{\Delta v_{BE}}$$

Recognize this? Think high school calculus:

$$k = \frac{dF}{dv_{BE}}\bigg|_{v_{BE} = V_{BE0}}$$

So we approximate the nonlinear behavior as a linear function, with the caveat that we stay "close" to V_{BE} . In the case of our original function:

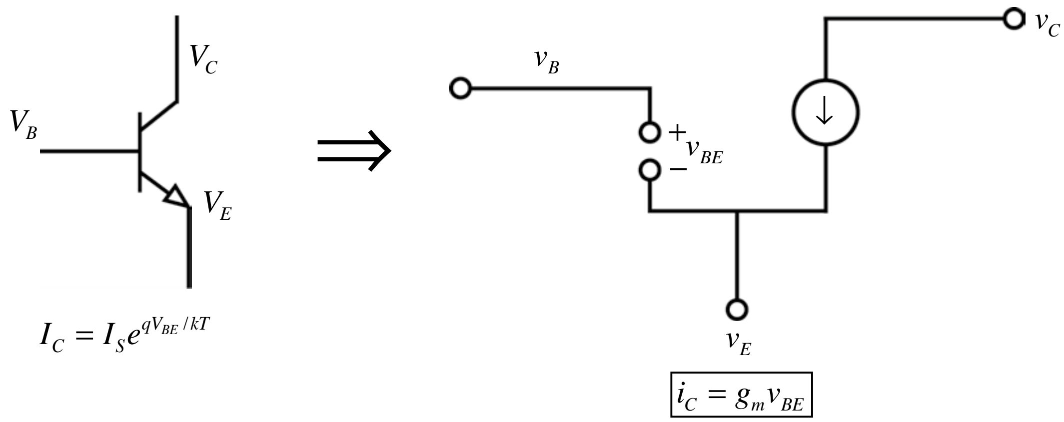
$$I_C = I_S e^{\frac{qV_{BE}}{kT}}$$

$$i_C = \left[\frac{d}{dv_{BE}} \left(I_S e^{\frac{qV_{BE}}{kT}} \right) \right] v_{BE}$$

$$= \left[\frac{q}{kT} I_S e^{\frac{qv_{BE}}{kT}} \right] v_{BE}$$

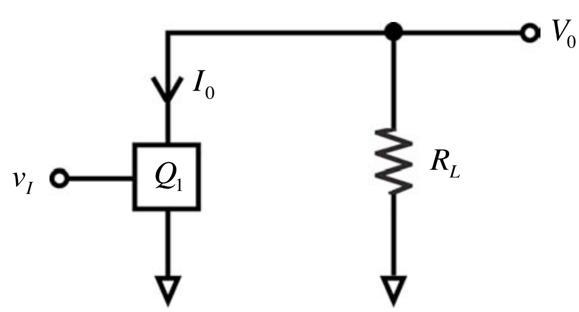
$$= \frac{q}{kT} I_C \cdot v_{BE} \rightarrow \boxed{g_m = \frac{q}{kT} I_C}$$

Don't lose track of what happened here. Our transistor just underwent a transformation.



CLASS EXERCISE: Now Your Turn

Suppose that we discovered a strange new device.



The current I_0 depends on v_I according to

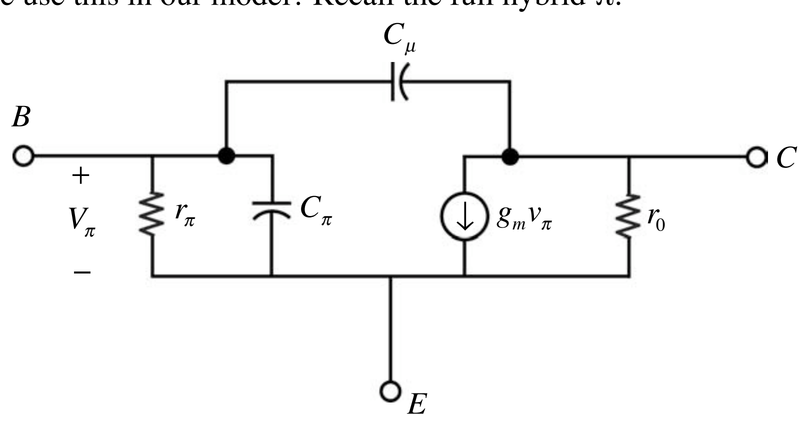
$$I_0 = \sin\left(k_0 \cdot v_I\right)$$

1) Determine the transconductance of this device.
2) Bias the circuit so that the incremental voltage gain is >0.

(Workspace)

We use this technique all over the place in analog electronics. We take devices that are deeply nonlinear, differentiate about our operating point, and from then on it's a linear world.

Where else do we use this in our model? Recall the full hybrid-π:



None of the elements or $r_\pi, C_\pi, C_\mu,$ or r_0 are linear. That is to say, for C_π it is not true that the charge we supply to C_π varies linearly with the voltage we impose on the base emitter junction.

That is, instead of

$$Q_\pi = CV_\pi,$$

We have

$$Q_\pi = f\left(V_\pi\right).$$

It doesn't matter. At the end of the day, we use device physics to get an exact form of $f\left(V_\pi\right)$. Then we differentiate, and say

$$C_\pi = \frac{dQ_\pi}{dV_\pi} = \frac{d}{dV_\pi} f\left(V_\pi\right)\Big|_{\text{Operating point}}$$

Read the text carefully, and you will see how this technique is used so often. Once you understand, you need only remember results:

$$I_C = I_S e^{\frac{qV_{BE}}{kT}}$$

$$I_B = I_C / B_F$$

$$g_m = \frac{qI_C}{kT}$$

$$c_h = g_m \tau_F$$

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. And so on.
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