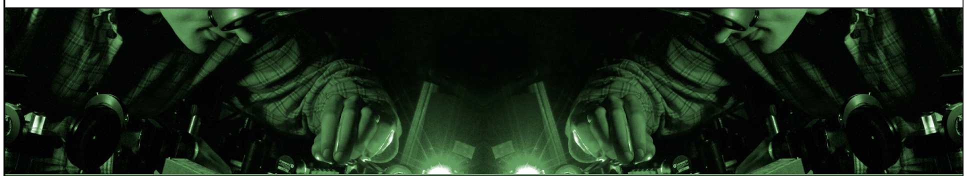

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6.453 *Quantum Optical Communication* Lecture 11

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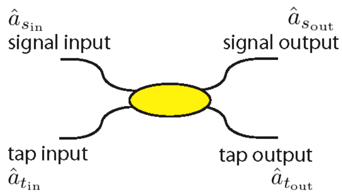
6.453 *Quantum Optical Communication* - Lecture 11

- Single-Mode Photodetection
  - Squeezed-state waveguide tap — reprise
- Single-Mode Linear Systems
  - Attenuators
  - Phase-Insensitive Amplifiers

Optical Waveguide Tap — Quantum

Fused Fiber Coupler



- Coupler is a beam splitter

$$\hat{a}_{s_{\rm out}} = \sqrt{T}\hat{a}_{s_{\rm in}} + \sqrt{1-T}\hat{a}_{t_{\rm in}}$$
$$\hat{a}_{t_{\rm out}} = \sqrt{1-T}\hat{a}_{s_{\rm in}} - \sqrt{T}\hat{a}_{t_{\rm in}}$$

- Tap input is in squeezed vacuum
- Homodyne SNR at signal input

$$\text{SNR}_{\rm in} = 4|a_{s_{\rm in}}|^2$$

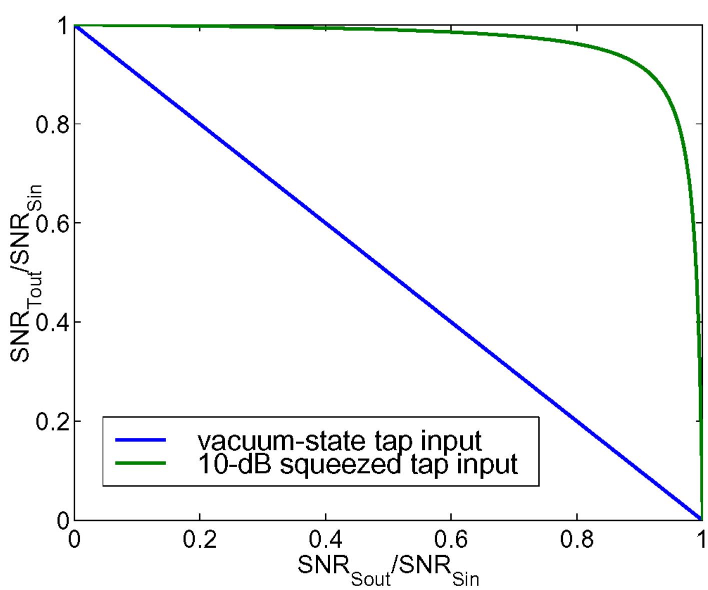
- Homodyne SNR at signal output

$$\text{SNR}_{\rm out} = \frac{4T|a_{s_{\rm in}}|^2}{T + (1-T)(\mu-\nu)^2}$$

- Homodyne SNR at tap output

$$\text{SNR}_{\rm tap} = \frac{4(1-T)|a_{s_{\rm in}}|^2}{(1-T) + T(\mu-\nu)^2}$$

Optical Waveguide Tap: SNR Tradeoff



Non-Ideal Quantum Photodetection

- Quantum Efficiency $\eta < 1$:

$$\hat{a}' \equiv \sqrt{\eta}\,\hat{a} + \sqrt{1-\eta}\,\hat{a}_\eta, \quad \hat{a}_\eta \text{ in vacuum state}$$

- Direct Detection: $\hat{a}'^\dagger \hat{a}'$ measurement
- Homodyne Detection: $\text{Re}(\hat{a}' e^{-j\theta})$ measurement
- Heterodyne Detection: $\hat{a}'$ measurement

Non-Ideal Quantum Photodetection: $\eta < 1$

- Direct Detection of Number State $|n\rangle$:

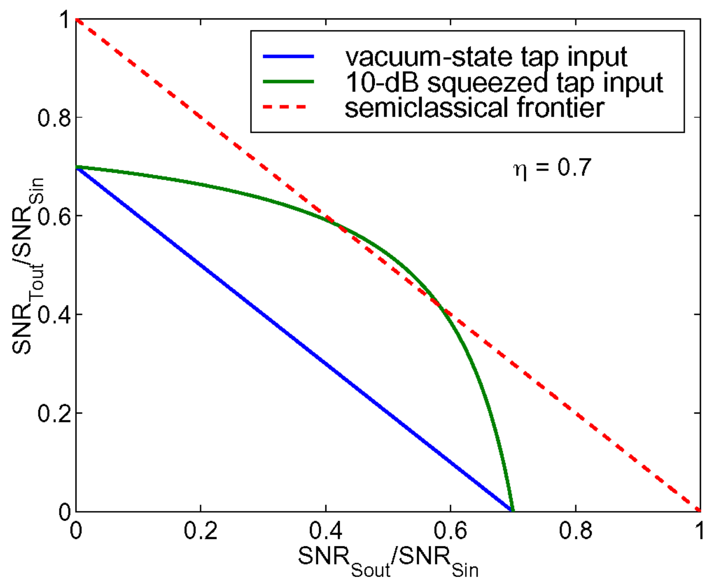
$$\Pr(\,\hat{N}' = k \mid |n\rangle\,) = \binom{n}{k} \eta^k (1-\eta)^{n-k}, \quad \text{for } 0 \le k \le n$$

- Homodyne Detection of Squeezed State $|\beta;\mu,\nu\rangle$:

$$\alpha_1' \;\sim\; N\left(\sqrt{\eta}\,(\mu-\nu)\beta, \frac{\eta(\mu-\nu)^2 + (1-\eta)}{4}\right),$$

$$\text{for } \beta, \mu, \nu \text{ real}$$

Loss is a Problem for Squeezed States



Single-Mode Linear Systems: the Attenuator

- Classical Attenuation

$$a_{\text{in}} \rightarrow \boxed{0 < L < 1} \rightarrow a_{\text{out}} = \sqrt{L}\, a_{\text{in}}$$

  - Noiseless attenuation is possible

$$\frac{\langle |a_{\text{out}}|^2 \rangle^2}{\langle \Delta(|a_{\text{out}}|^2)^2 \rangle} = \frac{\langle |a_{\text{in}}|^2 \rangle^2}{\langle \Delta(|a_{\text{in}}|^2)^2 \rangle}$$

$$\frac{\langle a_{\text{out}_\theta} \rangle^2}{\langle \Delta a^2_{\text{out}_\theta} \rangle} = \frac{\langle a_{\text{in}_\theta} \rangle^2}{\langle \Delta a^2_{\text{in}_\theta} \rangle}$$

Single-Mode Linear Systems: the Amplifier

- Classical Amplification

$$a_{\text{in}} \rightarrow \boxed{G > 1} \rightarrow a_{\text{out}} = \sqrt{G}\, a_{\text{in}}$$

  - Noiseless amplification is possible

$$\frac{\langle |a_{\text{out}}|^2 \rangle^2}{\langle \Delta(|a_{\text{out}}|^2)^2 \rangle} = \frac{\langle |a_{\text{in}}|^2 \rangle^2}{\langle \Delta(|a_{\text{in}}|^2)^2 \rangle}$$

$$\frac{\langle a_{\text{out}_\theta} \rangle^2}{\langle \Delta a^2_{\text{out}_\theta} \rangle} = \frac{\langle a_{\text{in}_\theta} \rangle^2}{\langle \Delta a^2_{\text{in}_\theta} \rangle}$$

Single-Mode Linear Systems: Quantum Case

- Input and Output are Photon Annihilation Operators:

$$\left[\hat{a}_{\text{out}}, \hat{a}^{\dagger}_{\text{out}}\right] = \left[\hat{a}_{\text{in}}, \hat{a}^{\dagger}_{\text{in}}\right] = 1$$

- Quantum Attenuator *Cannot* Obey $\hat{a}_{\text{out}} = \sqrt{L}\, \hat{a}_{\text{in}}$

- Quantum Amplifier *Cannot* Obey $\hat{a}_{\text{out}} = \sqrt{G}\, \hat{a}_{\text{in}}$

- Loss and Gain Require Additional Quantum Noise:
  to preserve the Heisenberg Uncertainty Principle

Single-Mode Linear Systems: Quantum Case

- Quantum Attenuation

$$\hat{a}_{\text{in}} \rightarrow \boxed{0 < L < 1} \rightarrow \hat{a}_{\text{out}} = \sqrt{L}\,\hat{a}_{\text{in}} + \sqrt{1-L}\,\hat{a}_L$$

- Quantum Amplification

$$\hat{a}_{\text{in}} \rightarrow \boxed{G > 1} \rightarrow \hat{a}_{\text{out}} = \sqrt{G}\,\hat{a}_{\text{in}} + \sqrt{G-1}\,\hat{a}_G^\dagger$$

Single-Mode Linear Systems: Quantum Case

- Attenuator with Coherent-State $\hat{a}_{\text{in}}$ and Vacuum-State $\hat{a}_L$:

$$\langle \Delta \hat{N}_{\text{out}}^2 \rangle = L \langle \hat{N}_{\text{in}} \rangle$$

$$\langle \Delta \hat{a}_{\text{out}_\theta}^2 \rangle = \langle \Delta \hat{a}_{\text{in}_\theta}^2 \rangle = \frac{1}{4}$$

- Amplifier with Coherent-State $\hat{a}_{\text{in}}$ and Vacuum-State $\hat{a}_G$:

$$\langle \Delta \hat{N}_{\text{out}}^2 \rangle =$$

$$[G\langle \hat{N}_{\text{in}} \rangle + (G-1)] + [2G(G-1)\langle \hat{N}_{\text{in}} \rangle + (G-1)^2]$$

$$\langle \Delta \hat{a}_{\text{out}_\theta}^2 \rangle = \frac{2G-1}{4}$$

Coming Attractions: Lecture 12

- Lecture 12:
  Single-Mode and Two-Mode Linear Systems
  - Phase-Insensitive Amplifiers
  - Phase-Sensitive Amplifiers
  - Entanglement