Problem 1.

A listener, whose head is at the origin of a coordinate system in anechoic space, attempts to distinguish a sound source located at \((r, \theta, \phi)\) from one located at \((r + \Delta r, \theta, \phi)\). In this problem, you are to assume that \(\theta = \phi = 0\) so that the sound source is in the median and horizontal planes.

a) When the source is located at \(r_{80} = 2\) m, the sound pressure \(P\) at the two ears is 80 dB SPL and that when located at \(r_{40}\) the sound pressure is 40 dB SPL. Determine the value of \(r_{40}\).

b) When distinguishing between sound pressure differences, determine the relative pressure differences \(\Delta P/P\) for

1. For broadband noise of 40 dB SPL.
2. For 500 Hz tones of 40 dB SPL.
3. For 500 kHz tones of 80 dB SPL.

c) Determine the size of the changes in \(r\) that the listener is able to discriminate when \(r = r_{80} = 2\) m and when \(r = r_{40}\)

1. For broadband noise of 40 dB SPL.
2. For 500 Hz tones of 40 dB SPL.
3. For 500 Hz tones of 80 dB SPL.

Experimental results indicate that when the stimulus is broadband noise of pressure 40 dB SPL the smallest change in \(\theta\) and \(\phi\) that listeners can discriminate are \(\Delta \theta = 2\) deg and \(\Delta \phi = 8\) deg.

d) Compare the sizes of the source displacements corresponding to the smallest discriminable changes in \(r\), \(\theta\), and \(\phi\).
Problem 2.

A sinusoidal sound source located in anechoic space far from the head in the horizontal plane at azimuth $\theta$ and elevation $\phi = 0$. The sound at the left ear is $l(t)$ and the sound at the right ear is $r(t)$ where:

\[
\begin{align*}
    l(t) &= \sqrt{2}A_L \sin (2\pi f (t - \tau_L)) \\
    r(t) &= \sqrt{2}A_R \sin (2\pi f (t - \tau_R))
\end{align*}
\]

Thus the interaural level difference is

\[
\alpha = \frac{A_L}{A_R}
\]

and the interaural time difference is

\[
\tau = \tau_L - \tau_R.
\]

a) Plot graphs of $20 \log_{10} \alpha$ as a function of $\tau$ as $\theta$ varies from 0 to $\pi/2$ for $f = 500, 1600, \text{ and } 6000 \text{ Hz}$.

When sounds are presented by earphones, sound images are typically lateralized within the head. When identical sounds are presented via earphones, the image of the sound is typically centered in the head. When the sound in the left ear has an amplitude that is greater than the sound in the right ear, the sound image shifts from the center towards the left ear. When the sound is a low frequency tone or broadband, and a time delay is introduced in the left ear, the image of the sound shifts from the center towards the right ear.

It is possible to displace the image from a centered position using an interaural time difference $\tau$ and recenter it by applying an interaural level difference $\alpha$. The time-intensity trading relation is heavily dependent on the spectral content of the stimulus, with smaller time differences required to match a given level difference when the stimulus contains only frequency components below 1400 Hz (see Fig. 25 in the Notes on Localization).

b) How do the plots you prepared in part (a) of this problem compare the data obtained from subject 2 on time-intensity trading for lowpass filtered clicks presented at 20 dB SL (included in Fig. 25 in the class notes). Compare a sinusoid of frequency 500 Hz with clicks lowpass filtered to 500 Hz, a sinusoid of frequency 1600 Hz to clicks lowpass filtered to 1400 Hz. and a sinusoid of frequency 6000 Hz to clicks lowpass filtered to 7000 Hz.

c) Some researchers speculate that the time-intensity trading ratio is a reflection of experience with the properties of real world sounds. Comment on this speculation based on the analysis you performed in part (b) of this problem.
Problem 3.

The following two paragraphs are from the section Ambiguity of Interaural Phase of the class notes.

Mills (1972) notes that a time difference between two pure tones that produces a half cycle or more of phase lead $(\pi + \phi)$ is indistinguishable from an opposite phase difference that produces a phase lag of $(\pi - \phi)$. The minimum acoustical path between the ears (roughly 23 cm in air) is greater than one half of a wavelength for frequencies in excess of 750 Hz.

For a specified (angular) frequency $\omega = 2\pi f$, for which $(\pi + \phi')$ is the maximum possible interaural phase difference for the frequency $\omega$, phase differences less than $(\pi - \phi')$ correspond to unique azimuths, but phase differences greater than $(\pi - \phi')$ can be produced by two different sound sources – one on each side of the head.

a) Assume that sound sources are located in the horizontal plane and that it takes sound 714 $\mu$s for sound to travel from one ear to the other. Give an example of two sinusoidal sound sources that would be confused (become indistinguishable) as described in the first of the above paragraphs. What is the lowest frequency for which it is possible to observe such confusions.

b) The assertion in the second of the above paragraphs is true for a ranges of frequencies. Demonstrate that you understand why it is true by determining the range of frequencies for which it is true.

Problem 4.

A person with normal hearing (e.g. Fig. 2 from the class handout) attempts to detect a sound pressure waveform $s(t)$ that is a square-wave of peak-to-peak amplitude $A$ and period $T$. The square-wave pressure is $+A$ for one-half the period and $-A$ for the other half. The Fourier series for the square-wave can be expressed as:

$$ s(t) = \frac{2A}{\pi} \sum_{n,\text{odd}}^{\infty} \frac{1}{n} \sin \frac{2n\pi t}{T} $$  \hspace{1cm} (1)

$$ = \frac{2A}{\pi} \left[ \sin \frac{2\pi t}{T} + \frac{1}{3} \sin \frac{6\pi t}{T} + \frac{1}{5} \sin \frac{10\pi t}{T} + \cdots \right] \hspace{1cm} (2) $$

a) Assume the square wave is audible when at least one of its component sinewaves is audible, i.e., at the threshold of hearing. Determine the minimum value of $A$ for which $s(t)$ is audible.
when $T = 0.2, 2.0, \text{ and } 20.0 \text{ msec.}$ In each case identify the frequency of the component that provides the cue used for detecting $s(t)$.

b) How would your answers change if the square wave is audible when the the power in one critical band is at the threshold of hearing, consistent with the findings of Fletcher (1954).

**Problem 5.**

a) In some experiments it would be convenient to use a masking noise that produced tone detection thresholds that were independent of frequency. What would be the shape of the spectrum of such a masking noise? Assume that the masked tone detection thresholds are to be well above the quiet thresholds of the tones. If the total noise power is limited to 80 dB SPL, over what frequency range would such a spectrum yield tone thresholds that were independent of frequency?

b) According to the data of Hawkins and Stevens, at a given frequency the Critical Ratio is constant, independent of the spectrum level $N_0$ of the noise, for sufficiently high values of $N_0$. At moderate sound levels, the critical ratio for 1000 Hz tones is roughly 70 Hz (18.5 dB). How much would tone detection thresholds change if the critical ratio were twice this value, 140 Hz.

c) Fletcher’s analysis of the results of his masking experiments in terms of Critical Bands has been criticized because he assumed that tones were detected when the ratio of tone power $P_T$ to noise power $N$ at the output of the critical band was unity, i.e., $P_T/N = 1$. How would Fletcher’s estimates of the critical bandwidths change if he had assumed that $P_T/N = 2.5$?

d) Moore’s data on the detection of 2000 tones in noises of various bandwidths can be used to estimate the ratio of tone power $P_T$ to noise power $N$ at the output of the critical band. Assuming that the width of the critical band is 400 Hz when the tone frequency is 2000 Hz, plot a graph of the ratio $P_T/N$ as a function of the bandwidth of the external noise bandwidth. Since Moore does not report the power spectral density of his masking noise, you should assume that his thresholds would be the same as those obtained by Hawkins and Stevens at wide bandwidths.