Problem 3.1

An electric dipole consists of two opposite polarity charges, \( \pm q \) at \( z = \pm d/2 \).

(a) Start with the electric potential of a point charge, and determine \( \Phi(r, \theta) \) for the electric dipole.

(b) Define the dipole moment as \( p = qd \) and show that in the limit where \( d \to 0 \) (while \( p \) remains finite), the electric potential is

\[
\Phi(r, \theta) = \frac{p}{4\pi \varepsilon_0} \frac{\cos \theta}{r^2}
\]

(c) What is the electric field for the dipole of part (b) with \( d \to 0 \) with \( p \) remaining finite?

(d) The electric field lines are lines that are tangent to the electric field:

\[
\frac{dr}{rd\theta} = \frac{E_r}{E_\theta}
\]

Using the result of (c), integrate this equation to find the field line that passes through the radial point \( r_0 \) when \( \theta = \pi/2 \). This analytical equation can be used to precisely plot the electric field lines.

Hint: \( \int \cot \theta d\theta = \ln(\sin \theta) + \text{constant} \)

(e) Use your favorite computer plotting routine to plot on the same plot the equipotential and electric field lines for \( 4\pi \varepsilon_0 / p = 100 \text{ volt}^{-1}\text{m}^2 \). Draw electric field lines for \( r_0 = 0.25, 0.5, 1 \) and 2 meters and draw equipotential lines for \( \Phi = 0, \pm .0025, \pm .01, \pm .04, \pm .16 \) and \( \pm .64 \) volts.
Problem 3.2

When a bird perches on a dc high-voltage power line and then flies away, it does so carrying a net charge.

(a) Why?
(b) For the purpose of measuring this net charge $Q$ carried by the bird, we have the apparatus pictured below. Flush with the ground, a strip electrode having width $w$ and length $l$ is mounted so that it is insulated from ground. The resistance, $R$, connecting the electrode to ground is small enough that the potential of the electrode (like that of the surrounding ground) can be approximated as zero. The bird flies in the $x$ direction at a height $h$ above the ground with a velocity $U$. Thus, its position is taken as $y = h$ and $x = Ut$. At time $t$, what is the effective charge distribution that will allow easy calculation of the electric scalar potential?
(c) The bird flies at an altitude $h$ sufficiently large to make it appear as a point charge. What is the potential distribution as a function of time and position $(x, y, z)$?
(d) Determine the surface charge density $\sigma_x(x, y = 0, z, t)$ on the ground plane at $y = 0$ as a function of time.
(e) At time $t$, what is the net charge, $q$, on the electrode? (Assume that the width $w$ is small compared to $h$ so that in an integration over the electrode surface, the integration in the $z$ direction is simply a multiplication by $w$.)

Hint: Let $x' = x - Ut$

Hint: $\int x \frac{dx}{[a^2 + x^2]^{3/2}} = \frac{x}{a^2 [a^2 + x^2]^{1/2}}$

(f) The current through the resistor is $dq/dt$. Find an expression for the voltage, $v$, that would be measured across the resistance, $R$.

Problem 3.3

Find the magnetic field intensity at the point $P$ shown due to the following line currents:

(a) 

(b) 

(c) 

(d) 

Problem 3.4

A constant current $K_0 \vec{I}_\phi$ flows on the surface of a sphere of radius $R$.

(a) What is the magnetic field intensity at the center of the sphere?

(Hint: $\vec{I}_\phi \times \vec{I}_r = \cos \theta \cos \phi \vec{I}_r + \cos \theta \sin \phi \vec{I}_z - \sin \theta \vec{I}_z$)

(b) Use the results of (a) to find the magnetic field intensity at the center of a spherical shell of inner radius $R_1$ and outer radius $R_2$ carrying a uniformly distributed volume current $\vec{J} = J_0 \vec{I}_\phi$. 

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A line current \( I \) of infinite extent in the \( z \)-direction is at a distance \( d \) above a perfectly conducting plane.

(a) Use the method of images to satisfy boundary conditions and find the magnetic vector potential for \( y > 0 \).

(b) What is the magnetic field for \( y > 0 \)?

(c) What is the surface current distribution that flows on the \( y = 0 \) surface?

(d) What is the force per unit length on the line current at \( y = d \)?