Problem 11.1

Interferometers and communications receivers incorporate ideal multipliers that are typically implemented using non-ideal diodes and other circuitry. An exception is digital systems in which the signals are digitized before multiplication.

a) Show that the output $v_o(t)$ of the simple adding interferometer shown here is proportional to the time average of the product of the two signals $a(t)$ and $b(t)$ after a DC term is subtracted.

b) Assume we must use a diode with $i = v^2 + cv^3$ instead of the illustrated square-law device, and that $a(t)$ and $b(t)$ are narrowband signals centered at 100 MHz. How might you change the circuit (minimally, if at all) so that the interferometer still works? Note that the same issue arises in communications receivers employing matched filters.

Problem 11.2

Assume the circuit of Problem 11.1 is an optical interferometer operating in the visible region over the wavelength range 0.5 - 0.6 microns. To obtain good coherence between the two light beams the two optical pathlengths between the adder and the target must be equal within some distance $d \text{ [m]}$. Assuming the target radiates approximately white noise over this wavelength range, what maximum value of $d$ preserves a fringe visibility that is at least ~half of maximum?

(Third problem is on second page)
Problem 11.3

A certain x-ray source at 0.5 nm wavelength is used for lithography. Assume the x-ray beam is formed by blasting a metal with an extremely high power laser in a square spot 100 nm on a side.

This spot radiates thermal radiation at 0.5 nm in all directions, including toward an x-ray lens (exotic) that focuses the radiation where desired. As part of the design phase we wish to know how correlated the x-rays from the source are in various directions. To determine this we correlate the outputs of two infinitesimal x-ray detectors as a function of their separation x,y in two dimensions, as illustrated. Assume these detectors are one meter from the source and measure power fluctuations in the 0.5-nm x-ray flux from zero to 100 MHz. Sketch the correlation function \( \phi(v_a,v_b) \) between the outputs \( v_a \) and \( v_b \) of the two infinitesimal x-ray detectors as a function of their separation x,y. Please quantitatively dimension your sketch. Hint: this is a variation of the Hanbury-Brown Twiss interferometer, and ignore any dark count.